Are Singularities Integral to General Theory of Relativity?

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Abstract

Since the 1960s the general relativists have been deeply obsessed with the possibilities of GTR singularities - blackhole as well as cosmological singularities. Senovilla, for the first time, followed by others, showed that there are cylindrically symmetric cosmological space-times which are free of singularities. On the other hand, Krori et al. have presently shown that spherically symmetric cosmological space-times - which later reduce to FRW space-times may also be free of singularities. Besides, Mitra has in the mean-time come forward with some realistic calculations which seem to rule out the possibility of a blackhole singularity. So whether singularities are integral to GTR seems to come under a shadow.

Keywords: General Relativity, Singularity Free Model, Black Hole, Big Bang, Gravitational Collapse, Cosmology

Einstein took ten long years to formulate general theory of relativity (GTR) which has been the most significant landmark in the twientieth century physics. But singularities have robbed GTR of some of its lustre since the 1960s. There are sceptics who tend to believe that singularities are integral to GTR. But is it so? This is the central theme of this short essay.

Mitra (1998, 2000, 2001) has for the first time come forward with some realistic calculations with the claim the spherically symmetric blackholes should not exist at all. He has shown that in the final (f) state of continued spherical gravitational collapse, the gravitational mass of a physical fluid is $M_f \to 0$ so that the value of $2M_f/R_f \to 1$ $(G = c = 1)$ rather than $2M_f/R_f < 1$, indicating approach to a zero -mass situation. In other words, as the gravitational collapse progresses, the mass continues to dissipate as radiation and ultimately reduces to zero, there occurring no possibility for a trapped surface to be formed at all.

Mitra has pointed out that all the idealized calculations by Oppenheimer & Snyder (1939) and some of the crucial assumptions made by Hawking & Ellis (1973) have consoloditated the notion of blackholes. He claims that once the faulty assumptions are removed, their work too would not allow the formation of blackholes.

It may be mentioned that Senovilla (1998) has also noted that the final state of gravitational collapse may be singularity free.

On the other hand, Senovilla et al. (1990, 1992, 1998) have, for the first time, derived cylindrically symmetric singularity- free cosmological solutions. Dadhich et al. (1993a,b) have followed the pioneers with some more such solutions. However, later Dadhich (1997) has succeeded in presenting a form of singularity-free spherically symmetric cosmology. But the physical singnificance of this cosmology is too limited, for it does not reduce to FRW cosmology. However, this is no doubt, a step forward in the sense that singularity-free spherically cosmological solutions are not impossible in GTR.

Krori, Dutta and Bhowmik (2002) appear now to have carried further Senovilla, Dadhich et al.'s exercises toward the cherished goal. They have adopted the following metric:

$$ds^2 = [1 - \gamma \mathrm{sech}(\alpha r)\mathrm{sech}(\beta t)]^2 - [1 - kt\tanh(\beta t) + (\eta \mathrm{sech}(\alpha r) + \xi)\mathrm{sech}(\beta t)]\,(dr^2 + r^2 d\Omega^2) \tag{1}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$, and $\alpha, \beta, \gamma, k, \eta \& \xi$ are constants. The corresponding energy momentum tensor is

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu} + \Delta p[c_\mu c_\nu + \frac{1}{3}(g_{\mu\nu} - u_\mu u_\nu)] + 2qc_\mu u_\nu \quad (2)$$

where u_μ is a time like velocity vector, c_μ is the space-like velocity vector given by $c_\mu = \sqrt{g_{11}}\delta^1_\mu$, ρ is the energy density, p is the isotropic pressure, Δp is the pressure anisotropy and q is the heat flux.

We note here some of the salient features of this cosmology.

• 1. The metric is singularity free and holds for the following ranges of coordinates:

$$-\infty \leq t \leq \infty, \quad 0 \leq r \leq \infty, \quad 0 \leq \theta \leq \pi, 0 \leq \phi, 2\pi$$

• 2. All the energy conditions in Wald (1984) are satisfied in the pre-FRW era.

• 3. With an adopted value of $k = 20$ cm^{-1}, $\alpha = \beta = 10^3$ cm^{-1}, $\gamma = 0.3$ and $\eta = \xi = 0.1$, we have seen how ρ varies with r, t: essentially it satisfies all physical conditions and ultimately attain a value appropriate to be a FRW *Radiation Cosmology*, i.e., $\rho = 3p$ (Krori & Dutta 2003).

• 4. Finally they have carried out a geodesic study of the metric exhaustively and no unphysical effect has come to our notice.

We now colclude: The theme of this short essay is to highlight a question:

Is it worthwhile now for the general relativists to turn around and have a second thought over whether GTR which is a marvel of the last century is indeed healthy and is not suffering from any singularity malaise?

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I. EDITORIAL NOTE

This article is essentially a reproduction of the Gravity Research Foundation Essay written by Krori & Dutta in 2003; it was not published anywhere else in the past.

II. ABOUT PROF. K.D. KRORI

Prof. K.D. Krori happens to be the seniormost living general relativist from India. More importantly, he is a renowned and prolific relativist; NASA Astrophysical Data Base shows he has coathored (often with his numerous students & coworkers) 107 research articles and many of which are in top journals such as General Relativity & Gravitation, Classical & Quantum Gravity, J. Phys. A., J. Mathematical Physics and Physical Review D. In 1975, he (along with J. Barua) presented the maiden "Singularity Free Solution of a Charged Fluid Sphere in General Relativity" (J. Phys. A. 8, 508, 1975), and this work is often referred to as the "*Krori -Barua Solution*". They also discovered the generalization of Vaidya's radiating metric for a collapsing *charged* sphere (J. Phys. A., 7, 2125, 1974).

He passed out M.Sc. (Physics) from the Calcutta University in 1953 when its Physics Department hosted many great physicists including Prof. S.N. Bose, Prof. M.N. Saha and Prof. Sisir Mitra. In 1955, Krori joined Cotton College of Guwahati, Assam, as a lecturer. Cotton college happens to be the most prestigious undergraduate college in North -Easterern part of India comprising 7 states. Krori, practically, earned his Ph.D. under his own guidance from Jadavpur University; and since he did not belong to any "school", his work has not been properly appreciated in India. Later, he became first the Head of the Physics Department of Cotton College and then its principlal. During this period, he turned Cotton College into one of the major research centres of General Relativity in India.

Though, Prof. Krori, himself has done lot of research using the *the mathematical paradigm* of "Black Holes", as his physical intuition got matured, started doubting the physical existence of both "Black Holes" and "Big Bang" singularities. He came to the conclusion that the appearance of singularities could be an artifact of simplistic models devoid of physical content. Incidentally, "Singularity Theorems" of Hawking, Penrose & Ellis were built upon "Raychowdhuri" Equations due to the renowned Indian relativist Prof. A.K. Raychoudhuri. Ironically, towards the final stages of his research career, Prof. Raychoudhuri too started doubting the validity of the "Singularity Theorems" and thought that "trapped surfaces" may not arise in physically valid models.

Prof. Krori received the "Eminent Physicist" award from Physics Academy in 1991. Before that, in 1990, he received Dr. H.C. Bhuyan Award in Physical Sciences. He has also been a member of the Editorial Board of *Indian Journal of Physics*. In 2010, he published a monograph "Fundamentals of Special & General Relativity" (Prentice Hall, New Delhi 2010, ISBN-9781-81-203-3867-8) meant for both M.Sc. as well as research students.