

# Fast Light, Fast Neutrinos?

Kevin Cahill

*Department of Physics & Astronomy, University of New Mexico, Albuquerque, New Mexico*  
*Physics Department, Fudan University, Shanghai, China\**

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In certain media, light has been observed with group velocities faster than the speed of light. The recent OPERA report of superluminal 17 GeV neutrinos may describe a similar phenomenon.

Over the past decade, physicists have observed light moving through certain media with group velocities faster than the speed of light in vacuum [1]. The OPERA collaboration [2] may have detected the neutrino analog of this “fast light” phenomenon.

The group velocity of an amplitude

$$A(x, t) = \int e^{i(k \cdot x - \omega t)} B(k) dk \quad (1)$$

is determined by the condition that the phase remain constant as a function of wave-number,  $v_g = d\omega/dk$ . When neutrinos traverse a medium with a complex index of refraction  $n$ , scattering makes the wave number  $k$  complex with a positive imaginary part. The real frequency  $\omega$  is related to the complex wave number  $k$  by  $k = n\omega/c$ . If  $n_r$  is the real part of the index of refraction, then the group velocity is

$$v_g = \frac{d\omega}{dk_r} = \frac{c}{n_r + \omega dn_r/d\omega}. \quad (2)$$

The index of refraction  $n$  is related to the forward scattering amplitude  $f$  and the density  $N$  of scatterers by [3]

$$n(\omega) = 1 + \frac{2\pi c^2}{\omega^2} N f(\omega) \quad (3)$$

in which for simplicity I replaced  $k^2$  by  $\omega^2/c^2$ , which introduces an error of less than  $10^{-19}$  for 17 GeV neutrinos with a mass of less than  $2 \text{ eV}/c^2$  [2].

Group velocities faster than  $c$  can occur when the frequency  $\omega$  is near a resonance in the total cross-section. For instance, if the amplitude for forward scattering is of the Breit-Wigner form

$$f(\omega) = f_0 \frac{\Gamma/2}{\omega_0 - \omega - i\Gamma/2} \quad (4)$$

then the real part of the index of refraction is

$$n_r(\omega) = 1 + \frac{\pi c^2 N f_0 \Gamma (\omega_0 - \omega)}{\omega^2 [(\omega - \omega_0)^2 + \Gamma^2/4]} \quad (5)$$

and by (2) the group velocity is

$$v_g = c \left[ 1 + \frac{\pi c^2 N f_0 \Gamma \omega_0}{\omega^2} \frac{[(\omega - \omega_0)^2 - \Gamma^2/4]}{[(\omega - \omega_0)^2 + \Gamma^2/4]} \right]^{-1} \quad (6)$$

which is superluminal if  $(\omega - \omega_0)^2 < \Gamma^2/4$ .

More generally, we may use the optical theorem and the regularized Kramers-Kronig formula

$$n_r(\omega) = 1 + \frac{cN}{\pi} \int_0^\infty \frac{\sigma_t(\omega') - \sigma_t(\omega)}{\omega'^2 - \omega^2} d\omega' \quad (7)$$

in which  $\sigma_t$  is the total cross-section to write the group velocity in terms of the principal part of an integral

$$\frac{c}{v_g(\omega)} = 1 + \frac{cN}{\pi} P \int_0^\infty \frac{[\sigma_t(\omega') - \sigma_t(\omega)] (\omega'^2 + \omega^2)}{(\omega'^2 - \omega^2)^2} d\omega' \quad (8)$$

which shows the effect of scattering on group velocities. Just as the scattering of photons by atoms can cause fast light [1], so too the scattering of neutrinos by electrons and quarks may make neutrino group velocities that are faster than the speed of light. The  $\nu_\mu$ -nucleon charged-current total cross-section rises linearly up to 300 GeV [4] and makes a positive contribution to the integral (8). Yet the OPERA Collaboration [2] may have discovered “fast neutrinos”—neutrinos with group velocities faster than the speed of light [5]. Their high group velocity  $(v - c)/c = 2.48 \times 10^{-5}$  may arise from a resonance in neutrino-electron and/or neutrino-quark scattering at an energy  $\omega_0$  somewhere near 17 GeV.

A group velocity faster than  $c$  doesn't violate special relativity, but a superluminal signal velocity would [1].

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\* cahill@unm.edu

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