# C-FIELD COSMOLOGY WITH VARIABLE GRAVITATIONAL CONSTANT IN 5-DIMENSIONAL HOMOGENEOUS DUST UNIVERSE

**by**

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**Abstract**.

We have extended the pioneer work of Hoyle and Narlikar of creation field theory in the frame work of higher dimensional space-time and investigated C-field cosmological model with variable G in five dimensional homogeneous dust universe. To get the deterministic model of the universe, we have assumed $G = R^n$, $p = 0$ where R is the scale factor, p the isotropic pressure and n is a constant. We have also discussed the special case for $n = -2$ to get the results related with astronomical observations. We find that the universe passes through a singular event at $t = 0$ and mean density of matter tends to zero when $t \to \infty$. In C-field, universe exists for all the time with uniform mass density. The model is rather puzzling in the sense that the contraction is followed by an expansion, the mass density finally reaches a steady state value even both R and A (metric potentials) approach infinity which is consistent with steady state theory where matter creation via creation field maintains the balance. In particular, the model has uniform density and creation filed increases with time as given in HN Theory (1964).

**Introduction**

The study of cosmological models in higher dimensional theories create more interest in the study because these models have been very helpful in the attempts to unify gravity and other forces in nature (Chatterjee and Bhui [1999]). In the cosmological contexts, the higher dimensional theories have relevance in the early universe when all the spatial dimensions including the extra one are treated in the same way before the universe underwent the compactification transition. The compactification of the extra dimension produces a large amount of entropy during the contraction process. This leads to an alternative to the horizon and flatness problems as compared to the usual inflationary scenario (Alvarez and Gavela [1983]). Marciano [1984] has pointed out that the experimental detection of the time variation of the fundamental constants can provide strong evidence for the existence of extra dimensions. Various authors viz. Appelquist and Chodos [1983], Randjbar-Daemi et al. [1984], Rahman et al. [2002], Singh et al. [2004], Mohanty and Sahoo [2008] have studied cosmological models in the frame work of 5-dimensional space-time.

The importance of gravitation on the large scale is due to the short range of strong and weak forces and electromagnetic forces become weak due to global neutrality of matter as pointed by Dicke and Peebles [1965]. Dirac [1937] proposed a theory with variable gravitational constant motivated by the occurrence of large number hypothesis. Demarque et al. [1994] considered an ansaz in which

$G \, \alpha \, t^{-n}$ and

$$\left| \frac{\dot{G}}{G} \right| < 2 \, x \, 10^{-11} \, yr^{-1} \text{ with } | \, n \, | < 0.1$$

Barrow [1978] assumed that $G \alpha t^{-n}$ and obtained from helium abundances for

$-5.9 \text{ x } 10^{-3} < n < 7 \text{ x } 10^{-3}$,

$$\left| \frac{\dot{G}}{G} \right| < (2 \pm 9.3) \text{ h x } 10^{-12} \text{ yr}^{-1}$$

by assuming a flat universe. Subsequently, alternative theories of gravity were developed to generalize Einstein's general theory of relativity by including variable G and satisfying conservation equation (Brans and Dicke [1961], Canuto et al. [1977]).

The big-bang model based on Einstein's field equations successfully explains the three important observations in Astronomy: (i) the phenomena of expanding universe, (ii) primordial nucleosynthesis, (iii) the observed isotropy of the cosmic back ground radiation. However, the big-bang model is known to have the short coming in the following aspects: (i) the model has singularity in the past and possible one in future, (ii) the conservation of energy is violated, (iii) it leads to a very small particle horizon, (iv) no consistent scenario exists that explains the origin, evolution and characteristic of structures in the universe at small scale, (v) flatness problem.

Therefore, alternative theories were proposed from time to time – the best well known theory was Steady State Theory of Bondi and Gold [1948]. In this theory, the universe does not have any singular beginning nor an end on the cosmic time scale and the statistical properties of the large scale features of the universe do not change. To maintain the constancy of mass density, they envisaged a very slow but continuous creation of matter is contrast to explosive creation at t = 0 of the standard model. But it suffered a serious disqualification for not giving any physical justification for the phenomenon of continuous creation of matter and the principle of conservation of energy is sacrificed in this formation. To overcome this difficulty, Hoyle and Narlikar [1964] adopted a field theoretic approach by introducing a massless and chargeless scalar field C in Einstein-Hilbert action to account for the creation of matter. In C-field theory, there is no big-bang type singularity as in the earlier steady state theory of Bondi and Gold. C-field (a negative energy field) has solved the problem of horizon and flatness faced by the big-bang model. Narlikar [1973] pointed out that matter creation is accomplished at the expense of negative energy C-field. Narlikar and Padmanabhan [1985] have investigated a solution of Einstein's field equations which admits radiation and a negative energy massless scalar creation field as a source. This solution is free from singularity and provides a natural explanation to the particle horizon and flatness problem. In an series of papers Hoyle et al. [1993, 1994a, 1994b] developed quasi steady state cosmology (QSSC) with a intention of offering viable alternative to the hot big-bang cosmology (HBCC). QSSC explains the temperature and anisotropies of microwave back ground radiation. Indeed the motivation of QSSC is to replace the singular event of bigbang cosmology (Banerjee and Narlikar [1997]). Vishwakarma and Narlikar [2007] have discussed modeling repulsive gravity with creation. Recently Bali and Tikekar [2007], Bali and Kumawat [2009] have investigated C-field cosmological models for dust distribution with variable gravitational constant in FRW space-time. Chaterjee and Banerjee [2004] extended the pioneer work of Hoyle and Narliker [1964] in the frame work of 5-dimensional space-time where gravitational constant is treated as constant.

An important achievement of HN theory is that it admits the possibility of an ever existing expanding universe with constant density of matter. The constancy of matter density is possible due to presence of an appropriate creation field-(C-field) with negative energy. We have studied the implications of HN Theory in higher dimensional space-times and possibilities of accommodating time dependence of gravitational constant G. One would like to know if HN Theory in higher dimensional set up can always admit presence of an ever existing expanding universe with constant density of matter and what are the implications of time variation of G.

At the same time an alternative ideas were evolving about black holes. Observations of the inner quasar structure at scales of several R_G became technically feasible for quasars, using techniques of gravitational microlensing and reverberation. Thus studying the UV-optical luminous inner structure immediately produced evidence of a magnetic propeller and favoured the MECO (Magnetic Eternally Collapsing Object) family of solutions that followed the basic (Mitra (2000, 2002)) ECO solutions of the Einstein-Maxwell field equations for an object with finite mass and intrinsic dipole magnetic field (Schild, Leiter & Robertson 2006, 2008; Lovegrove, Schild & Leiter 2011). This new direction of black hole theory is based upon a deeper consideration of Quantum Electrodynamics and its role in producing a pressure at the event horizon that halts the collapse to singularity and produces instead a surface of very high redshift. Recently Bali

(2012) investigated avoidance of singularity in exterior of spherically symmetric space-time in C-field cosmology.

In the present work, we have extended the pioneer work of Hoyle and Narlikar [1964] in the frame work of five dimensional space-time and investigated C-field cosmological model with variable G in five dimensional homogeneous dust universe. We find that the universe passes a singular event at t=0 and mean density of matter tends to zero when $t \rightarrow \infty$. In C-field, universe exists for all time with uniform mass density. In particular, we have shown that matter density is constant and creation field increases with time as given in HN Theory [1964].

**The Metric and Field Equations**

We consider a spatially flat 5-dimensional homogeneous cosmological model in the form

$$ds^2 = dt^2 - R^2(t)[dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2] - A^2(t)\, dy^2 \qquad \ldots(1)$$

where R(t) is the scale factor for the 3D space and A(t), that for the extra dimension.

Einstein's field equation by introduction of C-field is modified as

$$R_i^j - \frac{1}{2} R\, g_i^j = -8\pi G\, [{}^m T_i^j + {}^c T_i^j] \qquad \ldots(2)$$

The energy momentum tensor for matter is given by

$${}^m T_i^j = (\rho + p)\, v_i\, v^j - p\, g_i^j \qquad \ldots(3)$$

and

$${}^c T_i^j = -f \left[ C_i\, C^j - \frac{1}{2} g_i^j C^\alpha C_\alpha \right] \qquad \ldots(4)$$

where f > 0 and $C_i = \frac{dC}{dx^i}$ and ${}^cT_i^j$ is given by Hoyle and Narlikar [16].

The field equation (2) for the metric (1) leads to

$$\frac{3\dot{R}^2}{R^2} + \frac{3\dot{R}}{R}\frac{\dot{A}}{A} = 8\pi G(t)\left[\rho - \frac{1}{2} f \dot{C}^2\right] \qquad \ldots(5)$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{2\dot{R}}{R}\frac{\dot{A}}{A} + \frac{\ddot{A}}{A} = 8\pi G\left[\frac{1}{2} f \dot{C}^2 - p\right] \qquad \ldots(6)$$

$$\frac{3\ddot{R}}{R} + \frac{3\dot{R}^2}{R^2} = 8\pi G\left[\frac{1}{2} f \dot{C}^2 - p\right] \qquad \ldots(7)$$

**Solution of Field Equations**

Following Hoyle and Narlikar [1964], we have also taken a zero pressure matter field. From Bianchi identity

$$(G\, T_i^j)_{;j} = 0 \qquad \ldots(8)$$

we further get

$$\left[\dot{G}\rho + \dot{\rho}G - \frac{1}{2} f \dot{C}^2 \dot{G}\right] + G\,\rho\left(\frac{3\dot{R}}{R} + \frac{\dot{A}}{A}\right) = Gf\dot{C}\left[\ddot{C} + \dot{C}\left(\frac{3\dot{R}}{R} + \frac{\dot{A}}{A}\right)\right] \qquad \ldots(9)$$

which yields $\dot{C} = 1$ when used in the source equation, $C^i_{;i} = \frac{n}{f}$

Using $\dot{C} = 1$ in equation (5) and (7), we have

$$8\pi G\rho = \frac{3\dot{R}^2}{R^2} + \frac{3\dot{R}}{R}\frac{\dot{A}}{A} + 4\pi G f \qquad \ldots(10)$$

$$\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} = \frac{4\pi f}{3} G(t) \qquad \ldots(11)$$

To obtain the deterministic solution of equation (11), we assume

$$G = R^n \qquad \ldots(12)$$

where n is a constant and R is scale factor.

Equations (11) and (12) lead to

$$\ddot{R} + \frac{\dot{R}^2}{R} = \frac{k^2}{2} R^{n+1} \qquad \ldots(13)$$

where

$$k^2 = \frac{8\pi f}{3} \qquad \ldots(14)$$

Let us assume $\dot{R} = F(R)$.

This leads to $\ddot{R} = F F'$ with $F' = \frac{dF}{dR}$. Thus equation (13) leads to

$$\frac{dF^2}{dR} + \frac{2F^2}{R} = k^2 R^{n+1} \qquad \ldots(15)$$

which leads to

$$F^2 = \frac{k^2 R^{n+2}}{(n+4)} + \frac{L}{R^2} \qquad \ldots(16)$$

where L is constant of integration.

From equation (16), we have

$$\frac{R\,dR}{\sqrt{R^{n+4} + \frac{L}{k^2}(n+4)}} = \frac{k}{\sqrt{n+4}}dt \qquad \ldots(17)$$

To obtain the determinate value of R in terms of cosmic time t, we assume n = −2 in equation (17). Thus, we have

$$\frac{R\,dR}{\sqrt{R^2 + \frac{2L}{k^2}}} = \frac{k}{\sqrt{2}}dt \qquad \ldots(18)$$

From equation (18), we have

$$R^2 = \left(\frac{k}{\sqrt{2}}t + N\right)^2 - \frac{2L}{k^2} \qquad \ldots(19)$$

where N is constant of integration. Thus we have

$$G = R^{-2} = \left[\left(\frac{k}{\sqrt{2}}t + N\right)^2 - \frac{2L}{k^2}\right]^{-1} \qquad \ldots(20)$$

From equation (6) and (7), we have

$$\ddot{A} + 2\dot{A}\frac{\dot{R}}{R} - A\left(\frac{\ddot{R}}{R} + \frac{2\dot{R}^2}{R^2}\right) = 0 \qquad \ldots(21)$$

To solve equation (21), we assume

$$A(t) = R(t)\, u(t) \qquad \ldots(22)$$

Equations (21) and (22) lead to

$$R\,\ddot{u} + 4\dot{R}\,\dot{u} = 0 \qquad \ldots(23)$$

From equation (23), we have

$$\dot{u} = \frac{\beta}{R^4} = \frac{\beta}{\left[\left(\frac{k}{\sqrt{2}}t + N\right)^2 - \frac{2L}{k^2}\right]^2} \qquad \ldots(24)$$

where β is the constant of integration.

From equation (24), we have

$$u = \frac{\beta k^2}{2L^{3/2}}\left\{\frac{\frac{\sqrt{2L}}{k}\left[\frac{k}{\sqrt{2}}t + N\right]}{2\left[\frac{2L}{k^2} - \left(\frac{k}{\sqrt{2}}t + N\right)^2\right]} + \frac{1}{2}\log\left[\frac{\frac{k}{\sqrt{2}}t + N + \frac{\sqrt{2L}}{k}}{\sqrt{\frac{2L}{k^2} - \left(\frac{k}{\sqrt{2}}t + N\right)^2}}\right]\right\} \qquad \ldots(25)$$

From equations (19), (22) and (25), we have

$$A = \frac{\beta k^2}{2L^{3/2}}\sqrt{\left(\frac{k}{\sqrt{2}}t+N\right)^2-\frac{2L}{k^2}}\left\{\frac{\frac{\sqrt{2L}}{k}\left(\frac{k}{\sqrt{2}}t+N\right)}{2\left[\frac{2L}{k^2}-\left(\frac{k}{\sqrt{2}}t+N\right)^2\right]}\right.$$

$$\left.+\frac{1}{2}\log\left[\frac{\frac{k}{\sqrt{2}}t+N+\frac{\sqrt{2L}}{k}}{\sqrt{\frac{2L}{k^2}-\left(\frac{k}{\sqrt{2}}t+N\right)^2}}\right]\right\} \qquad \ldots(26)$$

where L > 0.

From equations (10), (19), (20) and (26), we have

$$8\pi\rho = \frac{\frac{3k^2}{2}\left(\frac{k}{\sqrt{2}}t+N\right)^2}{\left[\left(\frac{k}{\sqrt{2}}t+N\right)^2-\frac{2L}{k^2}\right]^2}+\frac{3k}{\sqrt{2}}\left(\frac{k}{\sqrt{2}}t+N\right)\left\{\frac{\frac{k}{\sqrt{2}}\left(\frac{k}{\sqrt{2}}t+N\right)}{\left[\left(\frac{k}{\sqrt{2}}t+N\right)^2-\frac{2L}{k^2}\right]}\right.$$

$$\left.+\frac{\frac{\frac{4L^{3/2}}{k^2}+\sqrt{L}\left[\frac{k}{\sqrt{2}}t+N\right]^2}{2\left[\frac{4L}{k^2}-\left(\frac{k}{\sqrt{2}}t+N\right)^2\right]^2}+\frac{\frac{1}{2}\left(\frac{4L}{k\sqrt{2}}+\sqrt{L}\left(\frac{k}{\sqrt{2}}t+N\right)\right)}{\left(\frac{k}{\sqrt{2}}t+N+\frac{\sqrt{2L}}{k}\right)\left(\frac{2L}{k^2}-\left(\frac{k}{\sqrt{2}}t+N\right)^2\right)}}{\frac{\frac{\sqrt{2L}}{k}\left(\frac{k}{\sqrt{2}}t+N\right)}{2\left[\frac{4L}{k^2}-\left(\frac{k}{\sqrt{2}}t+N\right)^2\right]}+\frac{1}{2}\log\left(\frac{\frac{k}{\sqrt{2}}t+N+\frac{\sqrt{2L}}{k}}{\sqrt{\frac{2L}{k^2}-\left(\frac{k}{\sqrt{2}}t+N\right)^2}}\right)}\right\}+4\pi f\,\dot{C}^2 \qquad \ldots(27)$$

To get the deterministic value of $\dot{C}$, we assume k = 1, L = 0, N = 0 and using equations (19), (20), (26) and (27) into equation (9), we have

$$\dot{C}^2 = \frac{3}{8\pi f} = 1 \text{ where } 8\,\pi f = 3. \qquad \ldots(28)$$

Equation (28) leads to $\dot{C} = 1$ which matches with the result with Hoyle-Narlikar cosmology [16].

Equation (28) leads to

$$C = t \qquad \ldots(29)$$

Here, we find $\dot{C} = 1$, which agrees with the value used in source equation. Thus creation field C is proportional to time.

**Physical and Geometrical Features**

The gravitational constant (G), the scale factor $\bar{a}(t)$ and the deceleration parameter (q), are given by

$$G = \left[\left(\frac{k}{\sqrt{2}}t + N\right)^2 - \frac{2L}{k^2}\right]^{-1} \qquad \ldots(30)$$

$$\bar{a}(t) = \left[\left(\frac{k}{\sqrt{2}}t + N\right)^2 - \frac{2L}{k^2}\right] \qquad \ldots(31)$$

$$q = \frac{2L}{k^2\left(\frac{k}{\sqrt{2}}t + N\right)^2} \qquad \ldots(32)$$

where L > 0.

In particular if we choose k = 1, N = 0, L = 0, $\dot{C}=1$, we have

$$G = \frac{2}{t^2} \qquad (33)$$

$$\dot{G} = -\frac{4}{t^3} \qquad (34)$$

$$\left|\frac{\dot{G}}{G}\right| = \frac{2}{t} \qquad (35)$$

$$\frac{\dot{\bar{a}}}{\bar{a}} = \frac{2}{t} \tag{36}$$

$$8\pi\rho = 3 + 4\pi f \tag{37}$$

$$q = 0 \tag{38}$$

$$C = t \tag{39}$$

**Conclusion**

The gravitational constant decreases with time. The deceleration parameter $q > 0$ indicates that universe passes through decelerating phase. $G \to \infty$ when $t \to 0$ and $G \to 0$ when $t \to \infty$. The creation field increases with time. The matter density decreases with time and creation of matter maintains the matter density uniform through out as the universe exists for all times in HN theory [1964]. Creation field is also useful to develop quasi steady state cosmology (QSSC) with a intention of offering viable alternative to the hot big bang cosmology and to replace the singular state of big bang cosmology (Hoyle et al. [1993, 1994a, 1994b]), Banerjee and Narlikar [1997]. This model is rather puzzling in the sense that as the contraction is followed by an expansion, the mass density finally reaches a steady value even though both R and A approach infinity which is consistent with steady state theory where matter creation via the creation field maintains the balance. When $k = 1$, $N = 0$, $L = 0$ then the matter density is constant and creation field increases with time. This result matches with HN theory [1964]. The deceleration parameter $q = 0$ which indicates that the particle moves with uniform velocity and scale factor increases with time. Thus the model leads to Milne universe [1964] for flat potential in four dimensional space-time.

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