STRING COSMOLOGY IN CYLINDRICALLY SYMMETRIC SPACE-TIME WITH STIFF FLUID AND PETROV TYPE I DEGENERATE

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Abstract.

A model formed by massive string and stiff perfect fluid is studied in the context of General Relativity. The model is used as a source with cylindrically symmetric space-time. To get the deterministic solution in terms of cosmic time t, we use Petrov Type I degenerate condition, the degeneracy being in yz-plane. The particular case for string dust is also discussed. If B = C then it gives unreaslistic solution as in Petrov Type I degenerate condition when degeneracy in yz-plane, then B = C is not allowed. Anisotropy is maintained in the model throughout, it is due to the presence of string. If string disappears then the anisotropy also disappears. We have models of a universe that evolve from a massive string dominated era to a pure geometric string dominated era. The model is in decelerating phase for massive string but represents decelerating and accelerating phases both for geometric string.

Introduction

The generation of stiff fluid solution from a vacuum solution is systematically given by Krishinski[1997]. The solution of Einstein's field equations are A_2 symmetric and whose source is stiff fluid given by

$$\rho = p$$

where A_2 symmetry implies that there exists two space - like killing vector fields which commute. The equation of state $\rho = p$ was first proposed by Zel'dovich[1970] in the study of early universe. The velocity of sound is equal to the velocity of light so no material in this universe could be more stiff. Such stiffness is conceivable at the very high densities just after the big bang. The energy-momentum tensor of non-rotating 'stiff fluid' is algebraically equivalent to a massless scalar field.

Stiff fluid cosmological models create more interest in the study because for these models, the speed of light equals to the speed of sound and its governing equations have the same characteristic as those of gravitational field (Zel'dovich [1970], Barrow [1978]) in his investigation has also pointed out that entropy level of the universe makes it likely that its initial state was isotropic and quiescent ($p = w\rho$, $w \epsilon$ (-1, 0) rather than chaotic only if the equation of state for high density matter tends to stiff $\rho = p$ (ρ being the matter density, p the isotropic pressure). Keeping in view of the importance of stiff fluid models, the cosmological models for stiff fluid distribution are also investigated by many authors viz. Tabensky and Jamorano[1975], Wainwright et al.[1979], Mak and Harko[2001, 2004], Bali et al.[2003, 2008] in different contexts.

String theory is a theory for calculating scattering amplitudes between asymptotic states where the fundamental objects are one dimensional string. These amplitudes are found in low-energy limit assuming its form as an effective field theory (Fradkin and Tseytlin [1985]).

The objective of string theory is to promote better understanding of the evolution in early stages of universe after its beginning as a singular event. The universe passes through a string era and evolves into its present state after formation of matter. This suggests the possibility that after the formation of matter, the universe might have passed through a state with its content in the form of matter with a remnant string cloud as a transitory state with energy momentum tensor

After the formation of matter, the string cloud in the lemnant form may consist of purely geometric strings with energy density same as the string tension. There is no reliable information about the equation of state of matter in the early stages of evolution of matter. Accordingly, it is customary to assign a suitable equation of state for matter and explore the implications on the evolution of the respective cosmic model of universe. In this study, we have assumed that the matter distribution in this state to be in the form of stiff fluid ρ = p as the equation of state and studied the subsequent evolution of the model of universe. Letelier[1983] obtained string cosmological models of Bianchi Type-I and Kantowski-Sachs space-times stipulating the form of energy-momentum tensor

$$T_{ij} = \rho v_i v_j - \lambda x_i x_j \text{ with } v^i v_j = -x^i x_j = -1 \text{ and } v^i x_i = 0$$

For massive string cloud (with particle attached with string) $\rho = \rho_p + \lambda$. Here ρ_p and λ devote particle density and string tension respectively.

The period of transition of the universe from string era to matter era may have witnessed evolution of stresses which may have in the end resulted in emergence of isotropic fluid pressure. Accordingly, the energy-momentum tensor during this period which saw emergence of fluid matter in the presence of remnant string cloud may be described by the expression

$$T_{ij} = (\rho + p) v_i v_j + p g_{ij} - \lambda x_i x_j$$

in view of $\rho = \rho_p + \lambda$

A number of cosmological models to understand the early stage of evolution of the universe have been obtained prescribing a stiff fluid as equation of state for its matter content. Accordingly, it will be appropriate to stipulate $\rho = p$ as equation of state for the fluid content in the presence of remnant string cloud in the universe in its early stages of evolution. We also use Petrov Type I degenerate condition which is the next priority after equation of state to solve non-linear differential equations. All the strings are assumed to be along the same axis because the degeneracy is in yz-plane so strings directions are along x-axis. If strings are taken in different directions then it will be very difficult to solve non-linear differential equations. Therefore directions of the strings are supposed to be isotropic keeping in view of modern universe scenario.

The grand unified theories (Kibble [1976]; Vilenkin [1981]) predict existence of strings in the early universe. String structures account for density fluctuations and lead to structure formation in later stages of cosmic evolution (Zel'dovich [1980]). Accordingly the string hypothesis is expected to lead to cosmological models providing vital clues about nature of early cosmic evolution. Stachel[1980] considered a mass less (geometric) strings in this respect to examine their relevance in cosmic evolution. Later on, several studies (Banerjee et al. [1990], ... Tikekar [1999]) aimed at exploring relevance of cosmological models based on string hypothesis have brought out various aspects of early cosmic evolution.

In this paper, Bianchi Type I cosmological models describing a universe of stiff fluid distribution in the presence of massive strings and its role in cosmic evolution are investigated. The degeneracy of Petrov type-I condition is assumed in yz-plane. In special case, string dust model is considered. The physical and geometrical aspects of the model together with singularities are discussed. We also find that the model is in decelerating phase for massive string but represents decelerating and accelerating phases both for geometric string.

Petrov Conditions

A gravitational field is characterized by Riemann curvature tensor and in vacuum is same as conformal Weyl tensor defined as

$$C_{hijk} = R_{hijk} + \frac{1}{n-2}\left(g_{hj}R_{ik} + g_{ik}R_{hj} - g_{hk}R_{ij} - g_{ij}R_{hk}\right)$$

$$- \frac{R}{(n-1)(n-2)}\left(g_{hj}g_{ik} - g_{hk}g_{ij}\right) \qquad \ldots(1)$$

Weyl tensor is trace-free tensor in vacuum space times since

$$C_{hk} = g^{ij}C_{hijk} = 0 \qquad \ldots(2)$$

Accordingly, classification of vacuum gravitational field follows from that of its Weyl tensor. The classification of gravitational fields known as 'Petrov Classification' is very much useful in examining role of free gravitational fields especially as describing field of gravitational radiation (Petrov [1969]).

For complete determination of physical quantities and realistic model, we use that the universe is filled with stiff fluid. It is well known that although the nature of expansion in the model at each point is determined by the distribution of matter, the model is also affected by free gravitational field through its effect on the expansion, the vorticity and the shear in the fluid flow. A prescription of such a field may therefore be made on an apriority basis. We therefore choose the free gravitational field to be Petrov Type I degenerate which is the next order in the hierarchy of Petrov classification.

Derivation of Space-time

We consider cylindrically symmetric line-element in the form

$$ds^2 = A^2(dx^2 - dt^2) + B^2 dy^2 + C^2 dz^2 \qquad \text{... (3)}$$

with metric coefficients A, B, C as functions of time *t*-measured as cosmic time by all commoving stationary observers. This form of line-element was used by Marder[1958] to investigate cylindrical waves to find exact solutions representing radiation from infinite cylinder. The energy momentum tensor for perfect fluid distribution in the presence of massive string proposed by Letelier[1983] is taken in the form

$$T_i^{\ j} = (\rho + p)\nu_i \nu^j + p g_i^{\ j} - \lambda x_i x^j \qquad \text{... (4)}$$

with $\rho = \rho_p + \lambda$, ν_i and x^i respectively denote the 4-velocity field of the fluid and the string direction, which satisfy

$$\nu^i \nu_i = -x^i x_i = -1 \qquad \text{... (5)}$$

$$\nu^i x_i = 0 \qquad \text{... (6)}$$

p denotes isotropic fluid pressure, ρ denotes proper energy density and λ the string tension density, ρ_p enters into the stress energy tensor as simply an additional dust component. We assume that the co ordinate system is co-moving and so that

$$\nu^i = \left(0,0,0,\frac{1}{A}\right), x^i = \left(\frac{1}{A},0,0,0\right) \qquad \text{... (7)}$$

The Einstein's field equations in the geometrized unit (c=1, G=1)

$$R_i^{\ j} - ½ R g_i^j = -8\pi T_i^{\ j} \qquad \text{... (8)}$$

imply the following relations connecting dynamical variables with metric parameters :

$$\frac{1}{A^2}\left[\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} - \frac{A_4 B_4}{AB} - \frac{A_4 C_4}{AC}\right] = -8\pi(p - \lambda)\,, \qquad \text{... (9)}$$

$$\frac{1}{A^2}\left[\frac{C_{44}}{C} + \left(\frac{A_4}{A}\right)_4\right] = -8\pi p\,, \qquad \text{... (10)}$$

$$\frac{1}{A^2}\left[\frac{B_{44}}{B}+\left(\frac{A_4}{A}\right)_4\right]=-8\pi p\,, \qquad \text{... (11)}$$

$$\frac{1}{A^2}\left[\frac{A_4B_4}{AB}+\frac{A_4C_4}{AC}+\frac{B_4C_4}{BC}\right]=8\pi\rho\,. \qquad \text{... (12)}$$

From equations (10) and (11) it follows that

$$\frac{B_{44}}{B}-\frac{C_{44}}{C}=0 \qquad \text{... (13)}$$

Equation (9) — (12) is a system of four equations relating three metric potential A, B, C with three dynamic physical parameters λ, ρ and *p*. A deterministic model of the universe, follows if it is stipulated that the universe is filled with stiff fluid (ρ = *p*) and the free gravitational field is Petrov Type-I degenerate with degeneracy in the yz-plane. The requirement $C_{12}{}^{12}=C_{13}{}^{13}$ implies the following relation

$$\frac{B_{44}}{B}-\frac{C_{44}}{C}=\frac{2A_4}{A}\left(\frac{B_4}{B}-\frac{C_4}{C}\right). \qquad \text{... (14)}$$

The non-vanishing components of conformal curvature tensor ($C_{hi}{}^{jk}$) for the metric (3) have the following explicit expressions:

$$C_{34}{}^{34}=C_{12}{}^{12}=\frac{1}{6A^2}\left[\frac{A_{44}}{A}+\frac{B_{44}}{B}-2\frac{C_{44}}{C}+3\frac{A_4C_4}{AC}-3\frac{A_4B_4}{AB}+\frac{B_4C_4}{BC}-\frac{A_4{}^2}{A^2}\right], \qquad \text{... (15)}$$

$$C_{24}{}^{24}=C_{13}{}^{13}=\frac{1}{6A^2}\left[\frac{A_{44}}{A}+\frac{C_{44}}{C}-2\frac{B_{44}}{B}+3\frac{A_4B_4}{AB}-3\frac{A_4C_4}{AC}+\frac{B_4C_4}{BC}-\frac{A_4{}^2}{A^2}\right], \qquad \text{... (16)}$$

$$C_{14}{}^{14}=C_{23}{}^{23}=\frac{1}{6A^2}\left[\frac{B_{44}}{B}+\frac{C_{44}}{C}-2\frac{A_{44}}{A}+2\frac{A_4{}^2}{A^2}-2\frac{B_4C_4}{BC}\right]. \qquad \text{... (17)}$$

Using B ≠ C as degeneracy is in yz-plane is assumed in Petrov Type-I degenerate condition. It follows from (13) and (14) that

$$\frac{A_4}{A} = 0$$

which implies

$$A = \alpha \text{ (a constant).} \qquad \text{... (18)}$$

Equations (11) and (12) together with stiff fluid condition $\rho = p$ imply

$$\left(\frac{A_4}{A}\right)_4 + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC} = 0 \qquad \text{... (19)}$$

Equations (14) and (18) lead to

$$C^2\left(\frac{B}{C}\right)_4 = L \text{ (a constant)} \qquad \text{... (20)}$$

From equations (18) and (19), we have

$$\frac{B_{44}}{B} + \frac{B_4 C_4}{BC} = 0 \qquad \text{... (21)}$$

which can be written as

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + 2\frac{B_4 C_4}{BC} = \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = \frac{B_{44}}{B} + \frac{B_4 C_4}{BC}$$

which leads to

$$\frac{(BC)_{44}}{BC} = 0 \text{ using (13) and (21)} \qquad \text{... (22)}$$

To find the solution, we assume that BC = μ and B/C = ν in (3.15) and (22). Thus, we have

$$\frac{\nu_4}{\nu} = \frac{L}{\mu}, \qquad \text{... (23)}$$

$$\mu_{44} = 0. \qquad \text{... (24)}$$

Equation (24) leads to

$$\mu = at + b \qquad \text{... (25)}$$

where a and b are constants of integration.

From equations (25) and (23), we have

$$\nu = N(at+b)^{L/a} \qquad \text{... (26)}$$

Thus,

$$B^2 = \mu\nu = N(at+b)^{1+\frac{L}{a}} \qquad \text{... (27)}$$

and

$$C^2 = \mu/\nu = \frac{1}{N}(at+b)^{1-\frac{L}{a}} \qquad \text{... (28)}$$

On using new coordinates *X*, *Y*, *Z* and *T* defined by

$$\alpha x = X, at+b = T, \sqrt{N}y = Y, \frac{1}{\sqrt{N}}z = Z$$

the metric (3) leads to the form

$$ds^2 = dX^2 - \frac{dT^2}{a^2} + T^{\frac{a+L}{a}}dY^2 + T^{\frac{a-L}{a}}dZ^2 \qquad \text{... (29)}$$

Physical and Geometric Features

The energy density ρ, the pressure p, the string tension density λ, the particle density ρ_p, the expansion θ and shear tensor σ for the model of (29) have following explicit expressions :

$$8\pi\rho = 8\pi p = \frac{1}{4\alpha^2}\left(\frac{a^2 - L^2}{T^2}\right), \quad \text{... (30)}$$

$$8\pi\lambda = -\frac{1}{4\alpha^2}\left(\frac{a^2 - L^2}{T^2}\right), \quad \text{... (31)}$$

$$8\pi\rho_p = \frac{1}{2\alpha^2}\left(\frac{a^2 - L^2}{T^2}\right), \quad \text{... (32)}$$

$$\theta = \frac{a}{\alpha T}, \quad \text{... (33)}$$

$$\sigma^2 = \frac{a^2 + 2L^2}{18\alpha^2 T^2}. \quad \text{... (34)}$$

The spatial volume R^3, the mean Hubble parameter H and deceleration parameter q are given by

$$R^3 = \alpha^2 T \quad \text{... (35)}$$

$$H = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{a}{3\alpha T} \quad \text{... (36)}$$

$$q = -\frac{\frac{R_{44}}{R}}{\frac{R_4{}^2}{R^2}} = 2 > 0 \quad \text{... (37)}$$

Special model : String Dust Model (ρ_p=0) as considered by Stachel(1980).

For massive string, $\rho = \rho_p + \lambda$

In view of $\rho_p = 0$, we have $\rho = \lambda$. Now using stiff fluid condition $\rho = p$, equation (9) leads to the following relation:

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} - \frac{A_4 B_4}{AB} - \frac{A_4 C_4}{AC} = 0 \quad \text{... (38)}$$

Use of Petrov Type degeneracy condition in yz-plane ($B \neq C$) in equation (14) leads to

$$A = \alpha \text{ (constant)} \quad \text{... (39)}$$

The equation (38) then implies

$$\frac{(BC)_{44}}{BC} = \frac{B_4 C_4}{BC} \quad \text{... (40)}$$

We write $BC = \mu$ and $\frac{B}{C} = \nu$. Then equation (40) assumes the form

$$\frac{\mu_{44}}{\mu} = \frac{1}{4}\left(\frac{{\mu_4}^2}{\mu^2} - \frac{{\nu_4}^2}{\nu^2}\right) \quad \text{... (41)}$$

In terms of these variables, equation (41) on using (23) leads to

$$2\mu_{44} - \frac{1}{2}\frac{{\mu_4}^2}{\mu} = -\frac{L^2}{2\mu} \quad \text{... (42)}$$

Equation (42) implies

$$\left(\frac{d\mu}{dt}\right)^2 = L^2 + M\mu^{1/2} \quad \text{... (43)}$$

<u>Two cases arise</u> :

<u>Case (i)</u> : If $L = 0$ then

$$\mu = (at+b)^{4/3} \qquad \text{... (44)}$$

where $a = \frac{3\sqrt{M}}{4}, b = \frac{3N}{4}$, with N as a constant of integration. Use of (44) in (23) determines

$$\nu = \exp\left[\frac{-3L(at+b)^{-1/3}}{a}\right] \qquad \text{... (45)}$$

After suitable transformation of coordinates, the metric (3) leads to

$$ds^2 = \alpha^2 dx^2 - \frac{\alpha^2 dT^2}{a^2} + T^{4/3}\left[\exp\left(\frac{-3LT^{-1/3}}{a}\right)dy^2 + \exp\left(\frac{3LT^{-1/3}}{a}\right)dz^2\right] \qquad \text{... (46)}$$

where T = at + b.

The matter density ρ, the string tension λ, the expansion θ, the mean Hubble parameter H, the spatial volume R^3 and deceleration parameter q for the model (46), are given by

$$\rho = \lambda = \frac{1}{4\alpha^2}\left[\frac{16a^2}{9T^2} - \frac{L^2}{T^{8/3}}\right], \qquad \text{... (47)}$$

$$\theta = \frac{4a}{\alpha T}, \qquad \text{... (48)}$$

$$H = \frac{4a}{9\alpha T}, \qquad \text{... (49)}$$

$$R^3 = \alpha T^{4/3}, \qquad \text{... (50)}$$

$$q = -\frac{63}{8} < 0 \qquad \text{... (51)}$$

The physical reality requirement ρ > 0 leads to

$$\frac{L^2}{16a^2} < T^{2/3} \qquad \text{... (52)}$$

<u>Case (ii)</u> : If L ≠ 0 then equation (43) leads to

$$\left(\frac{d\mu}{dt}\right)^2 = f^2 = L^2 + M\mu^{1/2} \qquad \text{... (53)}$$

and (23) leads to

$$\frac{dv}{v} = \frac{L}{\mu}\frac{dt}{d\mu}d\mu = \frac{L}{\mu}\frac{d\mu}{\sqrt{L^2 + M\mu^{1/2}}} \qquad \text{... (54)}$$

implying

$$\log v = \int \frac{L d\tau}{\tau\sqrt{L^2 + M\tau^{1/2}}} \text{ as } \mu = \tau \qquad \text{... (55)}$$

where M is constant of integration and M > 0.

Equation (55) can be written as

$$\log v = \int \frac{L\tau^{-1/2} d\tau}{\tau^{1/2}\sqrt{L^2 + M\tau^{1/2}}}$$

which leads to

$$\log v = \int \frac{4L\xi d\xi}{(\xi^2 - L^2)\xi} \qquad \text{... (56)}$$

where $L^2 + M\tau^{1/2} = \xi^2$

$$= 4L\int \frac{d\xi}{\xi^2 - L^2} \qquad \text{... (57)}$$

which leads to

$$\log \nu = 2\log\left(\frac{\xi - L}{\xi + L}\right)$$

This again leads to

$$\nu = \left(\frac{\sqrt{L^2 + M\tau^{1/2}} - L}{\sqrt{L^2 + M\tau^{1/2}} + L}\right)^2 \qquad \text{... (58)}$$

Thus,

$$B^2 = \mu\nu = \tau\left(\frac{\sqrt{L^2 + M\tau^{1/2}} - L}{\sqrt{L^2 + M\tau^{1/2}} + L}\right)^2 \qquad \text{... (59)}$$

and

$$C^2 = \frac{\mu}{\nu} = \tau\left(\frac{\sqrt{L^2 + M\tau^{1/2}} - L}{\sqrt{L^2 + M\tau^{1/2}} + L}\right)^{-2} \qquad \text{... (60)}$$

The metric (3) leads to the form

$$ds^2 = \alpha^2\left[dx^2 - \left(\frac{dt}{d\mu}\right)^2 d\mu^2\right] + \tau\left(\frac{\sqrt{L^2 + M\tau^{1/2}} - L}{\sqrt{L^2 + M\tau^{1/2}} + L}\right)^2 dy^2$$

$$+\ \tau\left(\frac{\sqrt{L^2 + M\tau^{1/2}} - L}{\sqrt{L^2 + M\tau^{1/2}} + L}\right)^{-2} dz^2 \qquad \text{... (61)}$$

which again leads to

$$ds^2 = \alpha^2\left[dx^2 - \frac{d\tau^2}{L^2 + M\tau^{1/2}}\right] + \tau\left(\frac{\sqrt{L^2 + M\tau^{1/2}} - L}{\sqrt{L^2 + M\tau^{1/2}} + L}\right)^2 dy^2$$

$$+\ \tau\left(\frac{\sqrt{L^2 + M\tau^{1/2}} - L}{\sqrt{L^2 + M\tau^{1/2}} + L}\right)^{-2} dz^2 \qquad \text{... (62)}$$

as $\mu = \tau$

The matter density ρ, the string tension λ, the particle density the expansion θ, the shear σ, the mean Hubble parameter (H), the spatial volume R^3 and deceleration parameter q for the model (62) are given by

$$8\pi\rho = \frac{M}{4\alpha^2\tau^{3/2}} = 8\pi\lambda,\ \rho_p = 0 \qquad \text{... (63)}$$

$$\theta = \frac{\sqrt{L^2 + M\tau^{1/2}}}{\alpha\tau}, \qquad \text{... (64)}$$

$$\sigma = \frac{\sqrt{3(1+3L^2)}}{2\alpha\tau}\sqrt{L^2+M\tau^{1/2}}, \qquad \text{... (65)}$$

$$H = \frac{\sqrt{L^2 + M\tau^{1/2}}}{3\alpha\tau}, \qquad \text{... (66)}$$

$$R^3 = \alpha^2\tau, \qquad \text{... (67)}$$

$$q = -\frac{3M}{L_1\alpha\tau^{1/2}\left(L^2 + M\tau^{1/2}\right)} - \frac{1}{M^2} \qquad \text{... (68)}$$

where M and α are positive constants.

To Examine Interaction Between Perfect Fluid And String

Conservation equation $T^{j}_{i;j} = 0$ leads to

$$\frac{\partial T_i^j}{\partial x^k} + T_i^h\Gamma_{hj}^j - T_h^j\Gamma_{ij}^h = 0$$

which leads to

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \lambda \dot{A}/A$$

where overhead dot indicate differentiation with respect to t.

This shows that there is interaction between perfect fluid and string. The conservation law for perfect fluid leads to $\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0$. The conservation law for string leads to

$$\dot{\rho} + \rho\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \lambda \dot{A}/A \, .$$

Conclusion

For the model (29), the reality condition $\rho > 0$ leads to $a > L$. The model (29) starts with a big bang at $T = 0$ and the expansion in the model decreases as time increases. The energy density $\rho \to \infty$ when $T \to 0$ and $\rho \to 0$ when $T \to \infty$. The particle density ρ_p behaves in the same way as energy density ρ. Since $\frac{\sigma}{\theta} \neq 0$, therefore, anisotropy is maintained throughout. The deceleration parameter $q > 0$ implies that the model is in decelerating phase for massive string. The mean Hubble parameter is initially large and decreases as time increases. The model (29) has a point type singularity at $T = 0$ when $a > L$ but has Cigar type singularity at T =

0 if a < L (MacCallum 1971). The stiff fluid condition with Petrov Type I degenerate condition lead to massive string model.

The model (46) starts expanding with a big bang at T = 0 and the expansion decreases as T increases. The spatial volume increases as time increases indicating inflationary scenario in the cosmic evolution. Thus reality condition $\rho > 0$ is satisfied as indicated by the expression (52). Since $q < 0$, therefore the model (46) represents an accelerating universe. Also $\frac{\sigma}{\theta} \neq 0$, therefore the anisotropy is maintained throughout. The model (46) has Barrel Type singularity at T = 0 (MacCallum 1971).

The model (62) starts with a bigbang at $\tau = 0$ with its expansion decreasing as τ increases. The energy density $\rho \to \infty$, as $\tau \to 0$ and $\rho \to 0$ when $\tau \to \infty$. The spatial volume increases as τ increases. Since deceleration parameter $q < 0$, hence the model (62) represents an accelerating universe. $\frac{\sigma}{\theta} \neq 0$ leads to anisotropy is maintained throughout. The model (62) has Barrel Type singularity at $\tau = 0$ (MacCallum 1971).

The anisotropy is maintained in the models (29) and (62) due to the presence of string. As soon as string disappears, the anisotropy also disappears (Letelier 1983). Thus we have models of a universe that evolve from a massive string dominated era to a pure geometric string dominated era.

The condition B = C is not possible case in Petrov Type-I degenerate condition when degeneracy is in yz-plane, It also leads to unrealistic condition $\rho < 0$.

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