Stability of static massive bodies in the presence of creation field

By

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Abstract.

It has been shown that a static massive body is stable in the presence of creation field. Hoyle & Narlikar (1964) have demonstrated that in the static solution the repulsive gravitational force of the C-field can be adjusted to balance the usual gravitational force inside the body of arbitrary mass. The HN-internal and external solutions are matched. The perturbed form of the metrics reduce to the unperturbed form of the metric in presence of C-field. Such solutions have relevance to the massive objects as considered by Fowler and Hoyle (1963 a,b).

Key words: Stability, Static massive bodies, Creation field.

Introduction

In General Relativity, a spherically symmetric cold body collapses into a singularity. Penrose (1965) and Hawking (1965) have pointed out that the singularity in Friedmann-Robertson-Walker models and in the gravitational collapse of massive objects is an unavoidable consequences of General Relativity. At the time of big-bang, it was assumed that all the matter and radiation present in the universe appears in its primary form. Subsequently, matter should conserve according to Einstein's field equation and change its form as the universe evolves. In C-field theory (the first alternative to General Relativity theory), there is no big-bang type singularity as in the steady state theory of Bondi and Gold (1948). The space-time represents an inflationary phase in the presence of C-field. The C-field is used to represent creation of matter. The prevention of singularities does not depend on the creation property of the field but on the negative energy density. Hoyle and Narlikar (1964) (the originators of C-field theory) emphasized that if overall energy conservation is to be maintained then primary matter creation must be accompanied by release of negative energy. The repulsive nature of the negative energy reservoir will be sufficient to prevent the singularity. They have shown that a spherically symmetric imploding cold body collapses into a space-time singularity in General Relativity and this singularity can be avoided by negative energy C-field provided internal pressure of the ordinary kind fail to provide support against gravitation and the mass of the body is sufficient large. Hoyle and Narlikar (1966) explored the possibility that the creation of matter in the universe takes place, not uniformly, as required by homogeneous steady state theory but in a discrete matter around isolated centres. These centres were called 'pockets' of creation. It has been shown that the universe can be maintained in a state of steady state expansion by C-field arising from creation in the pockets. Narlikar and Padmanabhan (1985) have investigated the solution of Einstein's field equation which admits radiation and negative-energy massless scalar creation field as a source. It has been shown that the cosmological model based on this solution satisfies all the observational tests and thus is a viable alternative to the standard big-bang model. Vishwakarma and Narlikar (2007) have discussed modeling repulsive gravity with creation.

Recently, it has been pointed out by many authors (Caldwell [2002], Gibbons [2003], Singh et al. [2003], Giacomini and Lara [2006], Paul and Paul [2008]) that phantom fields are the revival of HN-C-field. A scalar field with negative kinetic energy is called phantom field. Hoyle [1948, 1949] introduced the negative kinetic energy term in order to reconcile the homogeneous density by the creation of new matter. Later, it was formulated by Hoyle and Narlikar [1964] in the context of steady state theory of universe which is popularly known as Creation or C-field theory.

In this paper, we have shown that a static universe can be stable in the presence of C-field. Considering static external solution obtained by HN (1964), we make small perturbation of the metric and then find that perturbations die with time. We consider two cases :

Case (i) : Interior Solution

We consider the line-element inside the body in the form as

$$ds^2 = dt^2 - S^2\left[\frac{dr^2}{1-\alpha r^2} + r^2 d\Omega^2\right] \qquad \ldots(1)$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\Phi^2$

Following Hoyle and Narlikar (1964), the modified Einstein's field equation is given by

$$R^{ik} - ½Rg^{ik} = -8\pi G\left[T_n^{ik} - f\left(C^i C^k - ½g^{ik}C^l C_l\right)\right] \qquad \ldots(2)$$

where T_n^{ik} is the normal tensor, f is the coupling constant and C-field satisfies the source equation :

$$f\,\dot{C}_{;i} = J^i_{;i}, J^i = \rho \frac{dx^i}{ds},\ T^{ik}_{;k} = f\,\dot{C}C^k_{;k} \qquad \ldots(3)$$

Now, we describe the HN-internal solution representing oscillatory solution.

The function S satisfies the differential equation

$$\dot{S}^2 = P(S) \text{ where } P(S) = \frac{\alpha}{S} - \alpha - \frac{\beta}{S^4}$$

with the density (ρ) and $\dot{C}$ given as

$$\rho = \frac{3\alpha}{8\pi G} S^{-3},\ \dot{C} = \frac{A}{S^3}$$

where $\alpha = 1, -1, 0$ and A is a constant, $\beta = \frac{4\pi G f A^2}{3}$.

The static solution can always be adjusted to make S = 1 at the beginning of the implosion but for a static solution such a transformation has no significance.

For a static solution with S = Sc (say), the quartic P(S) has a double zero at S = Sc. Thus P (Sc) = 0, P'(Sc) = 0. This gives

$$Sc = 3/4 \qquad \ldots(4)$$

$$\frac{4\pi G f}{3} A^2 = \frac{27}{256}\alpha \qquad \ldots(5)$$

i.e. $$A = \left(\frac{3}{4\pi G f}\right)^{1/2} \left(\frac{27\alpha}{256}\right)^{1/2} \equiv Ac\ \text{(say)} \qquad \ldots(6)$$

For A < Ac, P(S) has two real positive zeros S_1 and S_2 and S oscillates between S_1 and S_2. For A > Ac, there is no physical solution at all.

Now we consider the perturbation of HN internal solution (1964).

Let us consider

$$S = Sc\,[1 + \eta(r, t)] \qquad \ldots(7)$$

Thus, we have $S = \frac{3}{4}(1+\eta)$ as $Sc = \frac{3}{4}$ in the HN-solution.

$$\dot{S} = \frac{3}{4}\dot{\eta}\,,\ \ddot{S} = \frac{3}{4}\ddot{\eta} \qquad \ldots(8)$$

Since

$$\dot{S}^2 = \frac{\alpha}{S} - \alpha - \frac{\beta}{S^4} \text{ as given by (HN-1964)} \qquad \ldots(9)$$

Therefore, we have

$$2\ddot{S} = -\frac{\alpha}{S^2} + \frac{4\beta}{S^{-5}} \qquad \ldots(10)$$

This leads to

$$\frac{3}{2}\ddot{\eta} = -\frac{16\alpha(1+\eta)^{-2}}{9} + 4\beta\left(\frac{4}{3}\right)^5 (1+\eta)^{-5}$$

$$= -\frac{16\alpha}{9}(1-2\eta) + \frac{4^6\beta}{3^5}(1-5\eta) \qquad \ldots(11)$$

Now we have

$$\ddot{\eta} = \left(\frac{64\alpha}{27} - \frac{4^6 \text{ x } 10\beta}{3^6}\right)\eta + \left(\frac{2\beta\, 4^6}{3^6} - \frac{32\alpha}{27}\right) \qquad \ldots(12)$$

This leads to

$$\ddot{\eta} = -\gamma\eta \qquad \ldots(13)$$

where

$$\alpha = \frac{256\beta}{27} \qquad \ldots(14)$$

and

$$\gamma = \frac{4^6 \text{ x } 10\beta}{3^6} - \frac{64\alpha}{27} \qquad \ldots(15)$$

$$= \frac{4^6 \text{ x } 10\beta}{3^6} - 4^3 \text{ x } \frac{4^4\beta}{3^6}$$

$$= \frac{4^6\beta}{3^6}(10-4) = \frac{2 \text{ x } 4^6\beta}{3^5} \qquad \ldots(16)$$

Thus

$$\gamma > 0 \text{ as } \beta = \frac{4\pi G \text{ f } A^2}{3} > 0 \qquad \ldots(17)$$

Equation (1.13) leads to

$$\eta = \text{a} \sin \sqrt{\gamma}(t + t_0) \qquad \ldots(18)$$

where a is arbitrary constant. Thus, η satisfies an oscillatory solution and

$$S = \frac{3}{4}[1 + \text{a} \sin \sqrt{\gamma}\,(t + t_0)] \qquad \ldots(19)$$

$$\dot{C} = \frac{A}{S^3} = \frac{64A}{27}(1+\eta)^{-3}$$

$$= \frac{64A}{27}(1 - 3\eta) \qquad \ldots(20)$$

Thus $\dot{C}$ oscillates as η oscillates between two constants showing η is bounded and η is small compared to 1.

Case (ii) : Exterior Solution

We consider de-Sitter line-element in the form

$$ds^2 = (1 - H^2R^2)dT^2 - \frac{dR^2}{1 - H^2R^2} - R^2(d\theta^2 + \sin^2\theta d\phi^2) \qquad \ldots (21)$$

where H is Hubble constant given by

$$H^{-1} = \sqrt{\frac{3}{4\pi G f}} \qquad \ldots (22)$$

Considering the modification produced by the body, HN found the static external line-element as

$$ds^2 = e^N dT^2 - \overline{e}^N dR^2 - R^2(d\theta^2 + \sin^2\theta d\phi^2) \quad \text{... (23)}$$

where $e^N = 1 - \frac{2GM}{R} - H^2R^2$. The line-element (23) approaches the line-element (21) for large R.

Now we consider the perturbation of the metric (23) and find that perturbations die with time; the perturbed form of the metric reduces to the metric (23).

The perturbation of the metric (23) is assumed in the form

$$ds^2 = -(\overline{e}^N + h_{11})dR^2 - R^2(d\theta^2 + \sin^2\theta d\phi^2) + (e^N + h_{44})dT^2 \quad \text{... (24)}$$

where h_{11}, h_{44} are functions of T only. For the metric (24),

$$\sqrt{-g} = \left(1 + \frac{e^N h_{11} + e^{-N} h_{44}}{2}\right) R^2 \sin\theta \quad \text{... (25)}$$

$$\log\sqrt{-g} = \frac{e^N h_{11} + e^{-N} h_{44}}{2} + 2\log R + \log\sin\theta \quad \text{... (26)}$$

$$\frac{\partial}{\partial T}\log\sqrt{-g} = \frac{e^N \dot{h}_{11} + e^{-N} \dot{h}_{44}}{2} \quad \text{... (27)}$$

$$\frac{\partial^2}{\partial T^2}\log\sqrt{-g} = \frac{e^N \ddot{h}_{11} + e^{-N} \ddot{h}_{44}}{2} \quad \text{... (28)}$$

In the following calculations, the squares and higher powers of h_{ij} and their derivatives are neglected, the expression for R_{ik} can then be linearized. The coordinates x^{μ} are chosen in such a way that the lines x^{μ} = constant, are orthogonal to the surface C = t = constant. The flow vector of matter need not be in the direction of $u_0^i = (0,0,0,1)$. The perturbed form of T^{ik} is taken in the form

$$T^{ik} = \rho_0 u_0^i u_0^k + \rho_1 u_0^i u_0^k + \rho_0 (u_0^i u_1^k + u_1^i u_0^k) \qquad ... (29)$$

and

$$J^i = \rho_0 u_0^i + \rho_1 u_0^i + \rho_0 u_1^i \qquad ... (30)$$

For the convenient, we choose,

$$\rho_0 = 1, K = 6, Kf = 6, K = 8\pi G, C^\mu = (u_1^\mu, 1 + u_1^4), \mu = 1,2,3$$

The modified Einstein field equation (2) in the presence of C-field, can be written as

$$R_{ik} = -6T_{ik} + 3Tg_{ik} + 6e^{2N} C_i C_k \qquad ... (31)$$

where, $T^{44} = 1 + \rho_1 + 2u_1^4$ ... (32)

$$T = g_{ik} T^{ik} = (1 + \rho_1 + 2u_1^4)(e^N + h_{44}) \qquad ... (33)$$

The equation

$$T^{ik}_{;k} = C^i C^k_{;k}$$

leads to

$$\frac{\partial u_1^4}{\partial T} + e^{2N} u_1^4 \frac{\partial N}{\partial x^\mu} + e^{-N} \frac{\dot{h}_{44}}{2} = 0 \qquad ... (34)$$

$$\frac{\partial u_1^\mu}{\partial T} + \frac{\partial N}{2\partial x^\mu} u_1^\mu + \frac{e^N}{2} \frac{\partial h_{44}}{\partial x^\mu} = 0 \qquad ... (35)$$

$$\frac{\partial \rho_1}{\partial T} + e^{2N} \rho_1 \frac{\partial N}{2\partial x^\mu} + \rho_1 \frac{e^{-N} \dot{h}_{44}}{2} + \frac{\partial u_1^\mu}{\partial x^\mu} = 0 \qquad ... (36)$$

The normalizing condition on the flow vector gives

$$g_{ik} (u_0^i + u_1^i)(u_0^k + u_1^k) = 1 \qquad ... (37)$$

This leads to

$$g_{44} (u_0^4 + u_1^4)(u_0^4 + u_1^4) = 1$$

Thus we have

$$e^{n}u_1^4 + \frac{h_{44}}{2} = \frac{1}{2} - \frac{e^N}{2} \qquad \text{... (38)}$$

Equation (1.38) leads to

$$(1+N)\dot{u}_1^4 + \frac{\dot{h}_{44}}{2} = 0 \text{ (since N is function of R only)} \qquad \text{... (39)}$$

From (1.39), we have

$$\dot{u}_1^4 = 0, \dot{h}_{44} = 0 \qquad \text{... (40)}$$

Using (40) in (34), we have

$$u_1^4 \frac{\partial N}{\partial x^\mu} = 0$$

which leads to

$$u_1^4 = 0 \qquad \text{... (41)}$$

Now equation (35) leads to

$$u_1^\mu = \xi(R)e^{-aT} \qquad \text{... (42)}$$

where a is the function of R.

Thus equation (36) leads to

$$\rho_1 = Ae^{-bT} + Be^{-aT} \qquad \text{... (43)}$$

where A and B are function of R only.

Equation (31) leads to

$$R_{44} = -6T_{44} + 3Tg_{44} + 6e^{2N}C_4C_4 \qquad \text{... (44)}$$

where

$$R_{44} = \frac{e^N \ddot{h}_{11}}{2} + e^N \dot{h}_{11} + 3e^{2N} \qquad \text{... (45)}$$

$$T^{44} = 1 + \rho_1 + 2u_1^4 = 1 + \rho_1 (Since\, u_1^4 = 0) \qquad \text{... (46)}$$

$$T = g_{ik} T^{ik} = (1 + \rho_1) e^N , T_{44} = e^{2N} (1 + \rho_1) \qquad \text{... (47)}$$

Thus equation (44) leads to

$$\ddot{h}_{11} + 2\dot{h}_{11} = -6e^N \left(Ae^{-bT} + Be^{-aT} \right) \qquad \text{... (48)}$$

Equation (48) leads to

$$h_{11} = le^{-\alpha T} + m \;\; e^{-\beta T} + ne^{-2T} \qquad \text{... (49)}$$

where l, m, n are functions of R only.

Conclusion

For large value of T,

$$h_{11} \to 0, \rho_1 \to 0, u_1^{\mu} = 0$$

Thus the static model is stable in presence of the C-field. The perturbed form of the metric reduces to the unperturbed form of the metric in the presence of C-field. Such solutions have relevance to the existence of massive objects. The C-field acts so as to prevent a gravitational collapse of a massive objects to a singular state. A general time-dependent solution of the problem had been considered by Hoyle and Narlikar (1964). They had obtained an internal solution but had not been able to get the external solution in explicit form. This solution describes a massive object undergoing oscillations under the opposing forces of the C-field and gravity. The limiting form of such solutions was discussed by them in a static form. Our work above shows that the limiting static solution is stable, thus leading to a concidence that the general oscillating solution would likewise be so.

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