Message 13/51 apjelw@bonnie.astro.ucla.edu   Jul 17, 97 12:31:13 pm -0700

Return-Path: <apjelw@bonnie.astro.ucla.edu>
Date: Thu, 17 Jul 1997 12:31:13 -0700
To: ir118@sdcc3.ucsd.edu, rschild@rudy.harvard.edu
Subject: my comments on your revised 35946
X-VMS-To: SMTP%"ir118@sdcc3.ucsd.edu", SMTP%"rschild@rudy.harvard.edu"
X-VMS-Cc: APJELW

% Dear Drs. Gibson & Schild:
%
% While I really wish that you had done this, I felt that somebody needed
% to look at the partial differential equations that govern the motion of
% fluid elements. So I did. **I find that there is no instability at your**
% scales. Not being an expert, I took a long time, and I may have done it
% wrong, but unless you can do it better your paper will not be accepted.
%

Note (CHG): The linear stability perturbation analysis of Professor Wright is correct. It is simply not relevant to turbulence, which is an intrinsically nonlinear process. Turbulent processes must be described differently.

% -- Edward L. (Ned) Wright, Scientific Editor (310)825-5755
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%

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\documentstyle[12pt,aaspp4]{article}
\newcommand{\bc}{\begin{center}}       \newcommand{\ec}{\end{center}}
\newcommand{\bn}{\begin{enumerate}}
\newcommand{\en}{\end{enumerate}}
\newcommand{\be}{\begin{equation}}     \newcommand{\ee}{\end{equation}}
\newcommand{\bea}{\begin{eqnarray}}    \newcommand{\eea}{\end{eqnarray}}
\newcommand{\vs}{{\it vs.}}            \newcommand{\etal}{{\it et al.}}
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\setlength{\oddsidemargin}{0in}
\setlength{\parskip}{12pt}
\begin{document}
\pagestyle{plain}
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The equations for hydrodynamics combined with gravity in a stationary background are:

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\be
\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho \vec{v}) = 0
\label{eq:conmass}
\ee
for conservation of mass, and
\be
\rho\left(\frac{\partial\vec{v}}{\partial t} +
(\vec{v}\cdot\vec{\nabla})\vec{v}\right)
= -\rho\vec{\nabla}\phi -\vec{\nabla}P + \eta\nabla^2\vec{v} +
(\zeta + \eta/3)\vec{\nabla}(\vec{\nabla}\cdot\vec{v})
\ee
for conservation of momentum (or $F= ma$), and Poisson's equation:
\be
\nabla^2\phi = 4\pi G\rho
\ee
Note that $\eta$ and $\zeta$ are the dynamical viscosity coefficients,
and both must be non-negative.

A thermal conduction or entropy equation is also needed.
The temperature will rise because of three effects: one is the conduction
of heat, the second is the energy input from viscous dissipation, the third
is adiabatic compression.  These give
\be
\frac{\partial T}{\partial t}
+\vec{v}\cdot\vec{\nabla}T
= \frac{1}{C_v}\left(\kappa \nabla^2 T
+\sum_{ik}{\sigma_{ik}\frac{\partial v_k}{\partial x_i}}\right) + (\gamma-1)
\frac{T\partial\rho}{\rho\partial t}
\ee
where $\gamma$ is the ratio of specific heats,
$\sigma$ is the stress tensor, and the term with $\sigma$ represents viscous
friction.

Now linearize using $\delta = \Delta\rho/\rho_\circ$,
$\tau = \Delta T/T_\circ$, $\Delta P = P_\circ(\delta+\tau)$, and by
dropping all the $\vec{v}\cdot\vec{\nabla}$ terms plus the viscous
dissipation which are all second order.  Finally assume that all
time and spatial variations have the form $\exp(i\vec{k}\cdot\vec{x}-st)$.
For velocities perpendicular to $\vec{k}$ this gives
\bea
-s v & = & -\frac{\eta}{\rho_\circ} k^2 v
\nonumber\\
-s \tau & = & -\frac{\kappa}{\rho_\circ C_v} k^2 \tau
\eea
so these modes are damped at a rate of $s = \nu k^2$ and
$\nu_T k^2$ where $\nu$ is the
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kinematic viscosity η/ρ and $\nu_T = \kappa/(\rho C_v)$ is the thermal diffusivity.
For velocities parallel to $\vec{k}$
there are density variations which couple to temperature and velocity, giving
\bea
-s v & = & -ik\frac{P_\circ}{\rho_\circ}(\delta + \tau) + i
\frac{4\pi G\rho_\circ}{k}\delta - \frac{(4/3)\eta + \zeta}{\rho_\circ} k^2 v
\nonumber\\
-s \delta & = & -ik v
\nonumber\\
-s\tau & = & -\nu_T k^2 \tau -(\gamma-1)s\delta
\eea
Change the definition of ν to $[(4/3)\eta+\zeta]/\rho$,
and eliminate τ and v, giving
\be
\left(s^3 - \left[(\nu+\nu_T)k^2\right]s^2 +
\left[\frac{\gamma P_\circ}{\rho_\circ}k^2
-4\pi G \rho_\circ - \nu\nu_T k^4\right]s
-\left[\nu_T k^2(k^2\frac{P_\circ}{\rho_\circ}-4\pi G\rho_\circ)\right]\right)
\delta = 0
\ee
This gives a cubic polynomial to solve for s.
Defining $c_\circ = \sqrt{P_\circ/\rho_\circ}$, $s_\circ =
\sqrt{4\pi G\rho_\circ}$, $k_\circ = s_\circ/c_\circ$, $y = k/k_\circ$,
$z = s/s_\circ$, $N_T = \nu_T s_\circ/c_\circ^2$
and $N = \nu s_\circ/c_\circ^2$ gives
\be
z^3 - [(N+N_T)y^2]z^2 - [1 - \gamma y^2 - N N_T y^4]z + N_T y^2(1-y^2) = 0
\ee
For argon at STP I get $s_\circ = 3.87 \times 10^{-5}$~sec^{-1},
$c_\circ = 2.38 \times 10^4$~cm/sec, and $N_T = 1.97 \times 10^{-14}$.
I haven't found ζ in my handbooks, so I just used $\nu = \eta/\rho$
to get $N = 8.05 \times 10^{-15}$.

It is pretty simple to solve for z, and for $y > 1/\sqrt{\gamma}$ there is
one real root and two complex conjugate roots. All have positive real parts,
so all are damped modes. There is no instability except for the Jeans
instability at $y < 1/\sqrt{\gamma}$.
The damping grows approximately quadratically with y.
The damping of the acoustic modes is $\approx (0.2 N_T+ 0.5 N)y^2$ while the
non-pressure mode damping rate is $\approx 0.6 N_T y^2$.
Your length scale L_{GIV} corresponds to $N y^2 = 1$, while L_{SD}
corresponds to $N_T y^2 = 1$.
Thus all that happens at your scales is that the damping becomes
faster than free fall so $\Re(z) > 1$.

\end{document}