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Bianchi Type V Inflationary Universe with Massless Scalar Field And Vacuum Energy Density

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Abstract

Bianchi Type V inflationary cosmological model with massless scalar field and flat potential and decaying vacuum energy density is investigated. To get the deterministic solution in terms of cosmic time t, we assume that the decaying vacuum energy density $\Lambda \sim \frac{\alpha}{R^2}$ where R is scale factor, α a constant, we find that the spatial volume increases exponentially indicating the inflationary scenario in the model. The model represents decelerating and accelerating phases both which matches with the recent astronomical observations. The anisotropy is maintained throughout in the model. However, for large values of time (T), the model isotropizes, where T is rescaling of cosmic time t. The rate of Higgs field decreases slowly with time. The model has Point Type Singularity at T = 0. (MacCallum(1971))

1 Introduction

Bianchi models are significant in the study because these models are homogeneous and anisotropic from which the process of isotropization of the universe is studied through the passage of time. The study of Bianchi Type V cosmological models create more interest in the study because these models are anisotropic generalization of open FRW models and allow arbitrarily small anisotropy levels at any constant of cosmic time. Bianchi Type V models have been studied in detail by number of authors viz. Farnsworth(1967), Collins(1974), Maartens and Nel(1978), Wainwright et al.(1979), Roy and Singh(1983), Banerjee and Sanyal(1988), Coley(1990), Bali and Meena(2004), Bali and Kumawat(2008).

Inflationary universes create more interest in the study because these universes play a significant role in solving number of outstanding problems in cosmology like homogeneity, the isotropy, the horizons, flatness and primordial monopole problem in grand unified field theories. Cosmic inflation was first pioneered by starobinsky (1979). Guth(1981) introduced the concept of inflation while investing the problem of why we do not see magnetic monopole today. He found that a positive-energy false vacuum generates an exponential expansion of space in general relativity. In particular, our universe is homogeneous and isotropic to a very high degree of precision. Such a universe is described by FRW space-time. Several version of inflationary scenario are studied by number of authors viz. Linde(1982), Wald(1983), Barrow(1987), Burd and Barrow(1988), La and Steinhardt(1989) in FRW space-time. Rothman and Ellis(1986) have pointed out that we can have a solution of isotropic problem if we work with anisotropic metric and these metrics can be isotropized under very general circumstances. Stein-Schabes(1987) has shown that inflation will take place if effective potential $V(\phi)$ has flat region while Higgs field evolves slowly but the universe expands in an exponential way due to vacuum field energy. Therefore, it is interesting to investigate inflationary scenario in anisotropic metric which isotropizes at late time or in a very general circumstances. Keeping such type of investigations, Bali and

Jain(2002) investigated inflationary scenario in LRS Bianchi Type I space-time in the presence of massless scalar field with flat potential. Recently Bali(2011) investigated inflationary scenario in Bianchi Type I space-time with flat potential considering the scale factor $a \sim e^{3Ht}$.

The cosmological constant (Λ) was introduced by Einstein to find the solution of static universes because at that time universe was supposed to be static. But after the discovery of Hubble constant, it was realized that universe is expanding. Also FRW obtained an expanding dust filled homogeneous and isotropic model in which there was no need to introduce the cosmological constant (Λ) into the Einstein's field equations. Einstein rejected the introduction of Λ term into his field equations after the realization that universe is expanding. A wide range of observations suggest that the cosmological constant Λ is the most favourable candidate of dark energy representing energy density of vacuum. The dark energy driven accelerating universe comsology with a small cosmological constant was put forward by Siddharth (1998). After that, two independent groups led by Riess et al.(1998) and Perlmutter et al.(1999) used Type Ia Supernovae and showed that universe is accelerating. This discovery provided the first direct evidence that Λ is non-zero with $\Lambda \sim 1.7 \times 10^{\&121}$ Planck units. It is now commonly believed by Scientific community that via the cosmological constant, a kind of repulsive pressure dubbed as dark energy, is the most suitable candidate to explain recent observations that universe appears to be expanding and accelerating. According to the first year data not of Supernovae Legacy Survey (SNLS), dark energy behaves like the cosmological constant to a precision of 10% (Astier et al.(2007)). ΛCDM models agree closely with almost all the established cosmological abbreviations. Obviously, this extremely small value of cosmological constant, indicates that vacuum energy density (Λ) is not a strict constant but decays as the universe expands. Corda (2009) pointed out that accelerating universe cosmology can be explained by extended theories of gravity. Recently Barrow and Shaw(2011) suggested that cosmological constant term corresponds to a very small value of the order of $10^{\&122}$ when applied to Friedmann universe. A number of cosmological .models in which Λ decays with time have been investigated by several authors viz. Bertolami(1986), Ram(1990), Berman(1991), Beesham(1993), Sahni and Starobinski(2000), Bronnikov et al.(2004), Singh and Chaubey(2006), Singh et al.(2007), Bali and Singh(2008), Ram and Verma(2010), Bali et al.(2012) and Saha(2013).

In this paper, we have investigated inflationary scenario in Bianchi Type V space-time with flat potential and decaying vacuum energy density (Λ). We find that spatial volume increases exponentially indicating inflationary scenario in the model. The vacuum energy density Λ decreases with time. The model describes a unified expansion history of the universe indicating decelerating and accelerating phases both. The anisotropy is maintained throughout. However, if the constant L = 0 then the model isotropizes. The constant L appears due to integration of equation (19). The rate of Higgs field decreases slowly with time but universe expands.

2. Metric and Field Equations

We consider Bianchi Type V line-element given by Ryan and Shepley (1975) in orthogonal form as

$$ds^2 = -dt^2 + A^2(t)\,dx^2 + e^{2x}\,(B^2(t)dy^2 + C^2(t)\,dz^2) \tag{1}$$

where A, B, C are metric potentials and functions of t- alone.

We assume the coordinates to be comoving so that $v^1 = 0 = v^2 = v^3, v^4 = 1$.

The Einstein's field equations (in gravitational units 8πG = c = 1) with time varying cosmological term Λ(t) are given by Kramer et al. (1980) as

$$R_{ij} - \frac{1}{2}R\,g_{ij} + \Lambda\,g_{ij} = -\,T_{ij} \tag{2}$$

with

$$T_{ij} = \partial_i\phi\,\partial_j\phi - \left[\frac{1}{2}\partial_\rho\phi\,\partial^\rho\phi + V(\phi)\right]g_{ij} \tag{3}$$

and

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\,\partial^\mu\phi) = -\frac{dV}{d\phi} \tag{4}$$

where ϕ is Higgs field.

The field equations (2) for the line-element (1) lead to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{1}{A^2} + \Lambda(t) = -\frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{5}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} + \Lambda(t) = -\frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} + \Lambda(t) = -\frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{7}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{3}{A^2} + \Lambda(t) = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{8}$$

$$\frac{2\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0 \tag{9}$$

The equation for scalar field (4) leads to

$$\ddot{\phi}+\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\dot{\phi}=\frac{dV}{d\phi} \tag{10}$$

3. Solution of Field Equations

We are interested in inflationary solution so flat region is considered. Thus $V(\phi)$ is constant. Now equation (10) leads to

$$\ddot{\phi}+\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\dot{\phi} = 0 \tag{11}$$

From equation (11), we have

$$\dot{\phi} = \frac{\ell}{ABC} \tag{12}$$

where ℓ is constant of integration.

The scale factor R is given by

$$R^3 = \sqrt{-g} = ABC = A^3 \tag{13}$$

as $BC = A^2$ from equation (9). Equations (5) and (8) lead to

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}-\frac{4}{A^2}+2\Lambda = 2k \tag{14}$$

where $V(\phi)$ = constant = k. To get the deterministic solution, we assume that

$\Lambda = \dfrac{\alpha}{R^2}$ as considered by Chen and Wu (1990).

For the sake of simplicity, we take

$$\alpha = 2 \tag{15}$$

Thus equations (14) and (15) lead to

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{2\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}=2k \tag{16}$$

Using equation (9) in equation (16), we have

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{1}{2}\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)^2+\frac{2\dot{B}\dot{C}}{BC}=2k \tag{17}$$

Equations (6) and (7) lead to

$$\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}+\frac{\dot{A}}{A}\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)=0 \tag{18}$$

which again leads to

$$\frac{(C\dot{B}-B\dot{C})_4}{(C\dot{B}-B\dot{C})}=-\frac{\dot{A}}{A}=-\frac{1}{2}\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right) \tag{19}$$

using equation (9)

To find the solution, we assume $BC=\mu$ and $\frac{B}{C}=\nu$, thus from equation (19), we have

$$C^2\frac{d}{dt}\left(\frac{B}{C}\right)=\frac{L}{\mu^{1/2}} \tag{20}$$

where L is constant of integration. To find the solution of equation (17) and (20), we put that $BC=\mu$ and $\frac{B}{C}=\nu$. Thus equations (17) and (20) lead to

$$2\ddot{\mu}+\frac{1}{\mu}\dot{\mu}^2=4k\,\mu \tag{21}$$

and

$$\frac{\dot{\nu}}{\nu}=\frac{L}{\mu^{3/2}} \tag{22}$$

To find the solution of equation(21), we assume that $\dot{\mu} = f(\mu)$. Thus

$$\ddot{\mu} = \frac{d\dot{\mu}}{dt} = \frac{d}{d\mu} f \frac{d\mu}{dt} = f' f .$$

Now equation (21) leads to

$$\frac{d}{d\mu}(f^2) + \frac{1}{\mu} f^2 = 4k\mu$$

which leads to

$$f^2 = \frac{4}{3} k \mu^2 + \frac{\gamma}{\mu} \tag{23}$$

Equation (23) leads to

$$\frac{\sqrt{\mu}\, du}{\sqrt{(\mu^{3/2})^2 + \gamma/\beta^2}} = \beta\, dt \tag{24}$$

where $\frac{4k}{3} = \beta^2$ and $\gamma_1^2 = \frac{\gamma}{\beta^2}$. From equation (24), we have

$$\mu^{3/2} = \gamma_1 \sinh\left(\frac{3\beta}{2} t + \gamma_2\right) \tag{25}$$

γ_2 being constant of integration. Now equations (22) and (25) lead to

$$\nu = \gamma_3 \tan h^{\frac{2L}{3\beta\gamma_1}}\left(\frac{T}{2}\right) \tag{26}$$

where $\frac{3\beta}{2} t + \gamma_2 = T$ and γ_3 being constant of integration and T is mere rescaling of t.

$$A^2 = BC = \mu = \gamma_1^{2/3} \sinh^{2/3} T \tag{27}$$

$$B^2 = \mu\nu = \gamma_3 (2\gamma_1)^{2/3} \sinh^{\left(\frac{2}{3}+\frac{2L}{3\beta\gamma_1}\right)}\left(\frac{T}{2}\right)\cosh^{\left(\frac{2}{3}-\frac{2L}{3\beta\gamma_1}\right)}\left(\frac{T}{2}\right) \tag{28}$$

$$C^2 = \frac{\mu}{\nu} = \frac{(2\gamma_1)^{2/3}}{\gamma_3}\sinh^{\left(\frac{2}{3}-\frac{2L}{3\beta\gamma_1}\right)}\left(\frac{T}{2}\right)\cosh^{\left(\frac{2}{3}+\frac{2L}{2\beta\gamma_1}\right)}\left(\frac{T}{2}\right) \tag{29}$$

After suitable transformation of coordinates, the metric (1) leads to the form

$$ds^2 = -\frac{4}{9\beta^2}dT^2 + \sinh^{2/3}T\,dX^2 + e^{\frac{2X}{\gamma_1^{1/3}}}\left[\sinh^{\left(\frac{2}{3}+\frac{2L}{3\beta\gamma_1}\right)}\frac{T}{2}\cosh^{\left(\frac{2}{3}-\frac{2L}{3\beta\gamma_1}\right)}\frac{T}{2}\,dY^2\right.$$

$$\left. + \sinh^{\left(\frac{2}{3}-\frac{2L}{2\beta\gamma_1}\right)}\frac{T}{2}\cosh^{\left(\frac{2}{3}+\frac{2L}{3\beta\gamma_1}\right)}\frac{T}{2}\,dZ^2\right] \tag{30}$$

where

$$\gamma_1^{1/3}x = X,$$

$$\gamma_3^{1/2}(2\gamma_1)^{1/3}y = Y$$

$$\frac{(2\gamma_1)^{1/3}}{\gamma_3^{1/2}}z = Z$$

4 Some Physical and Geometrical Aspects

The vacuum energy density (Λ) is given by

$$\Lambda = \frac{2}{R^2} = \frac{2}{A^2} = \frac{2}{\gamma_1^{2/3}\sinh^{2/3}T} \tag{31}$$

The rate of Higgs field (ϕ) is given by equation (12) as

$$\dot{\phi} = \frac{\ell}{A^3} = \frac{\ell}{\gamma_1 \sinh T} \tag{32}$$

which leads to

$$\phi = \frac{2\ell}{3\beta\gamma_1} \log \tanh \frac{T}{2} + N \tag{33}$$

where N is constant of integration.

The spatial volume (R^3) for the model (30) is given by

$$R^3 = \gamma_1 \sinh T \tag{34}$$

The expansion (θ), shear (σ), the deceleration parameter (q) are given by

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} = \frac{3}{2}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) = \frac{3}{2}\frac{\mu_4}{\mu}$$

$$= \frac{3\beta}{2} \coth T = \frac{3\beta}{2}\left(\frac{e^T + e^{-T}}{e^T - e^{-T}}\right) \tag{35}$$

$$\sigma = \frac{1}{2}\left(\frac{B_4}{B} - \frac{C_4}{C}\right) = \frac{1}{2}\frac{\nu_4}{\nu} = \frac{L}{2\gamma_1 \sinh T} = \frac{L}{\gamma_1(e^T - e^{-T})} \tag{36}$$

$$q = -\frac{\ddot{R}/R}{\dot{R}^2/R^2} = -3 \tan h^2 T + 2 \tag{37}$$

5 Conclusion

We observe that expansion (θ), shear (σ) and vacuum energy density (Λ) all diverge at T = 0. The model (30) starts expanding with a big-bang from its singular state at T = 0 and tends to a finite limit at late time. The spatial volume (R^3) increases exponentially as T increases. Thus the model represents inflationary scenario. The model has Point Type singularity at T = 0 (Maccallum (1971)) For large values of T, $\frac{\sigma}{\theta} \to 0$ which implies that the model approaches isotropy at late times. When $T \rightarrow 0$ then $\frac{\sigma}{\theta}$ is finite. Hence the model represents anisotropic space-time initially and isotropizes at late times. The model describes a unified expansion history of the universe which starts with decelerating expansion and the expansion accelerates at late time. The decelerating expansion at initial epoch provides obvious provision for the formation of large structure in the universe. The formation of structure is better supported by decelerating expansion. Thus the model is astrophysically relevant. Also late time acceleration is in agreement with the observations of 16 type Ia supernovae made by Hubble space Telescope (HST) (Riess et al.(2004)). The vacuum energy density (Λ) is initially large but decreases with time. This result matches with astronomical observations. The rate of Higgs field decreases slowly.

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