Singularities of the Gravitational Fields of Static Thin Loop and Double Spheres

— Singularity black holes do not exist in nature

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Abstract

In the classical theory of the Newtonian mechanics, the gravity fields of static thin loop and double spheres are two simple but foundational problems. However, in the Einstein's theory of gravity, they are not simple. In fact, we do not know their solutions up to now. Based on the coordinate transformation of the Kerr and the Kerr-Newman solutions of the Einstein's equation of gravity field with axial symmetry, the gravity fields of the static thin loop and double spheres are obtained. The results indicate that, no matter what masses and densities are, there are singularities at the central point of thin loop and the contact point of two spheres. What is more, the singularities are completely exposed in vacuum. Space near the surfaces of thin loop and spheres are highly curved, although the gravity fields are very weak. These results are completely inconsistent with practical experience and are completely impossible. By reasonable analogy, singular black holes in the current cosmology and astrophysics are something illusive and have nothing to do with the real world actually. The only possible explanation is that they are caused by the mathematical description method of curved space-time. If there are black holes in the universe, they can only be the Newtonian black holes, rather than the Einstein's singularity black holes.

Key Words: General relativity; Equation of Gravitational Field; Axially Symmetrical Solutions; Kerr Metric; Kerr-Newman Metric; Black hole; White hole, Quasar; MECO

1. Introduction

According to the Einstein's theory of gravity, the singularities exist at the centers of gravitational fields when material densities are high enough. There exist event horizons in general black holes. The light inside the event horizon can not escape from it. According to the no-hair theorem of black hole, no electromagnetic field can exist in a black hole. The most space inside a black hole is a vacuum for material has collapsed into a singularity point. According to common understanding at present, singularity black holes exist commonly in the universe. For example, huge black holes are considered to hide at the centers of general galaxies and quasars.

However, according to the observations of Rudolf E. Schild and Darryl J. Leiter (Rudolf E. Schild, Darryl J. Leiter, 2006), the centre of Quasar 0957+561 is a close object, called a MECO (Massive Eternally Collapsing Object). Unlike an empty hole, it is surrounded by a strong magnetic field and material. This result challenged traditional astrophysics and cosmology. It implied that the current theory of singular black hole may be wrong. We have reason to ask such a question. Whether or not singularity black holes, predicted by general relativity, really exist in the universe?

Based on the coordinate transformations of the Kerr and the Kerr-Newman solutions of the Einstein's equation of gravity with axial symmetry, the gravitational fields of static thin loop and double spheres are calculated in this paper. The results indicate that, no matter what their masses and density are, the spatial curvatures at the central point of thin loop and the contact point of two spheres are infinite. What is more, the singularities are completely exposed in vacuum. The spaces nearby the surfaces of loop and spheres are highly curved, even though their masses are very small so that the gravitational fields are very weak. The results are completely inconsistent with practical experience. They are very absurd and completely impossible. The only possible explanation is that these singularities are caused by the description method of curved space-time. By logical analogy, so-called singular black holes and white holes as well as wormholes which connect both holes in the current cosmology and astrophysics are something illusive. They have nothing to do with the real world actually. If there are black holes in nature, they can only be the Newtonian black holes, rather than the Einstein's singularity black holes!

2. The Gravitational Field and Singularity of Static Thin Loop

The gravitational field of static thin loop is discussed at first. As shown in Fig.1, a thin loop with mass M and radius b is placed on the $x-y$ plane. The center of ring is located at the origin point of coordinate system. The ring is thin enough so that its cross section can be neglected comparing with its perimeter. It will be seen later that even though the cross section of loop is not zero, the result is also the same essentially. Because the static mass distribution of thin loop has axial symmetry, the metric tensor of gravitational field does not depend on time t and coordinate φ, so the four dimensional linear element can be written as

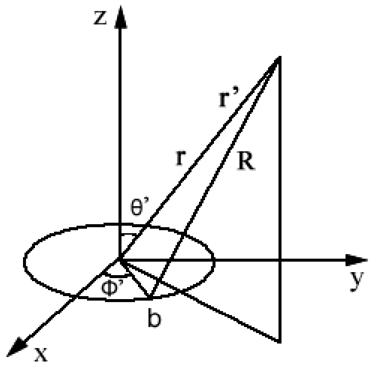
$$ds^2 = g_{00}(r,\theta)dt^2 + g_{11}(r,\theta)dr^2 + g_{22}(r,\theta)r^2 d\theta^2 + g_{33}(r,\theta)r^2 \sin^2\theta d\varphi^2 \tag{1}$$

By this definition, we should have $g_{00}(r,\theta)=1$ and $g_{ii}(r,\theta)=-1$ in flat space-time. By introducing coordinate transformations $t=t'$, $\varphi=\varphi'$, $r \to r' = r'(r,\theta)$, $\theta \to \theta' = \theta'(r,\theta)$, (1) can also be written as

$$ds^2 = g'_{00}(r',\theta')dt'^2 + g'_{11}(r',\theta')dr'^2 + g'_{22}(r',\theta')\ r'^2 d\theta'^2$$

$$+ g'_{33}(r',\theta')\ r'^2 \sin^2\theta' d\varphi'^2 + g_{12}(r',\theta')r'dr'd\theta' \tag{2}$$

The formulas (1) and (2) are with axial symmetry and can be used to describe the gravitational field of thin loop. Using these metrics in the Einstein's equation of gravity, we can obtain the concrete forms of metric tensor in principle. However, it is difficult to solve the equation of gravitational fields directly. On the other hand, we know that there is a ready-made solution of gravitational field's equation with axial symmetry and two independent parameters, i.e., the Kerr solution. If the solution of the Einstein's equation of gravity with the same symmetry is unique, we can obtain the solution of static mass distribution of thin loop by means of the coordinate transformations of the Kerr solution. Besides, we seem to have no other choice. The method is discussed below.



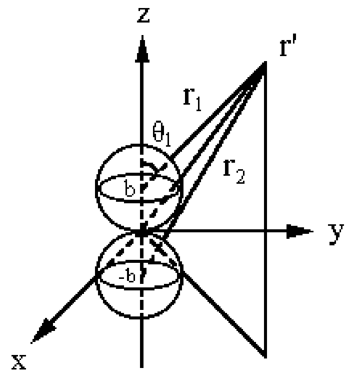


Fig.1 Gravity Field of thin loop **Fig.2 Gravity field and of double spheres**

The Kerr solution with two free parameters (R. P. Kerr, 1963) is

$$ds^2 = \left(1 - \frac{2\alpha\, r}{r^2 + \beta^2 cos^2\theta}\right)dt^2 - \frac{r^2 + \beta^2 cos^2\theta}{r^2 + \beta^2 - 2\alpha\, r}dr^2 - \left(1 + \frac{\beta^2}{r^2}cos^2\theta\right)r^2 d\theta^2$$

$$-\left[1 + \frac{\beta^2}{r^2} + \frac{2\alpha\beta^2 sin^2\theta}{r(r^2 + \beta^2 cos^2\theta)}\right]r^2 sin^2 d\varphi^2 + 2\frac{2\alpha\beta\, sin\theta}{r^2 + \beta^2 cos^2\theta} r\, sin\theta\, dtd\varphi \qquad (3)$$

At present, the Kerr metric is used to describe the gravity field of a rotating sphere, in which parameter $\alpha = GM$, $\beta = J/M$ ($\boldsymbol{c} = 1$). β is considered to be the unit angle momentum. If we use (3) to describe the gravitational field of thin loop, α and β will have different meanings. Because (3) contains a crossing item $dtd\varphi$ which is related to time, the solution is dynamic one. For static mass distribution, this item does not exist and should be canceled. We can remove it by the diagonalization of metric tensors. We set

$$\begin{bmatrix} g_{00} & g_{30} \\ g_{03} & g_{33} \end{bmatrix} = \begin{bmatrix} 1 - \frac{2\alpha r}{r^2 + \beta^2 \cos^2\theta} & \frac{2\alpha\beta\sin\theta}{r^2 + \beta^2\cos^2\theta} \\ \frac{2\alpha\beta\sin\theta}{r^2 + \beta^2\cos^2\theta} & -1 - \frac{\beta^2}{r^2} - \frac{2\alpha\beta^2\sin^2\theta}{r(r^2 + \beta^2\cos^2\theta)} \end{bmatrix} \qquad (4)$$

From the eigen equation

$$\begin{vmatrix} g_{00} - \lambda & g_{03} \\ g_{30} & g_{33} - \lambda \end{vmatrix} = 0 \qquad (5)$$

we get $$\lambda_1 = \frac{1}{2}\left[g_{00} + g_{33} + \sqrt{(g_{00} - g_{33})^2 + 4g_{03}g_{30}}\right]$$

$$=\frac{1}{2}\left[\left(2-\frac{2\alpha r}{r^2+\beta^2\cos^2\theta}+\frac{\beta^2}{r^2}+\frac{2\alpha\beta^2\sin^2\theta}{r(r^2+\beta^2\cos^2\theta)}\right)^2\right.$$

$$\left.+\frac{16\alpha^2\beta^2\sin^2\theta}{(r^2+\beta^2\cos^2\theta)^2}\right]^{\frac{1}{2}}-\frac{1}{2}\left[\frac{2\alpha r}{r^2+\beta\cos^2\theta}+\frac{\alpha^2}{r^2}+\frac{2\alpha\beta^2\sin^2\theta}{r(r^2+\beta^2\cos^2\theta)}\right] \tag{6}$$

$$\lambda_2=\frac{1}{2}\left[g_{00}+g_{33}-\sqrt{(g_{00}-g_{33})^2+4g_{03}g_{30}}\right]$$

$$=-\frac{1}{2}\left[\left(2-\frac{2\alpha r}{r^2+\beta^2\cos^2\theta}+\frac{\beta^2}{r^2}+\frac{2\alpha\beta^2\sin^2\theta}{r(r^2+\beta^2\cos^2\theta)}\right)^2\right.$$

$$\left.+\frac{16\alpha^2\beta^2\sin^2\theta}{(r^2+\beta^2\cos^2\theta)^2}\right]^{\frac{1}{2}}+\frac{1}{2}\left[\frac{2\alpha r}{r^2+\beta\cos^2\theta}+\frac{\alpha^2}{r^2}+\frac{2\alpha\beta^2\sin^2\theta}{r(r^2+\beta^2\cos^2\theta)}\right] \tag{7}$$

The orthogonal transformations of coordinates are

$$\begin{bmatrix} dt \\ r\sin\theta\, d\varphi \end{bmatrix}=\begin{bmatrix} \frac{g_{30}}{\sqrt{g_{03}^2+(g_{00}-\lambda_1)^2}} & \frac{g_{30}}{\sqrt{g_{03}^2+(g_{00}-\lambda_2)^2}} \\ \frac{g_{00}-\lambda_1}{\sqrt{g_{03}^2+(g_{00}-\lambda_1)^2}} & \frac{g_{00}-\lambda_2}{\sqrt{g_{03}^2+(g_{00}-\lambda_2)^2}} \end{bmatrix}\begin{bmatrix} dt' \\ r\sin\theta\, d\varphi' \end{bmatrix}$$

$$\begin{bmatrix} dt' \\ r\sin\theta\, d\varphi' \end{bmatrix}=\begin{bmatrix} \frac{(g_{00}-\lambda_1)\sqrt{g_{03}^2+(g_{00}-\lambda_1)^2}}{g_{03}(\lambda_1-\lambda_2)} & \frac{-\sqrt{g_{03}^2+(g_{00}-\lambda_1)^2}}{(\lambda_1-\lambda_2)} \\ \frac{-(g_{00}-\lambda_2)\sqrt{g_{03}^2+(g_{00}-\lambda_1)^2}}{g_{03}(\lambda_1-\lambda_2)} & \frac{\sqrt{g_{03}^2+(g_{00}-\lambda_2)^2}}{(\lambda_1-\lambda_2)} \end{bmatrix}\begin{bmatrix} dt' \\ r\sin\theta\, d\varphi \end{bmatrix} \tag{8}$$

By the transformation, we can cancel the crossing item containing $dtd\varphi$ and get

$$g_{00}dt^2+g_{33}r^2\sin^2\theta d\varphi^2+2g_{03}r\sin\theta\, dtd\varphi=\lambda_1 dt'^2+\lambda_2 r^2\sin^2\theta d\varphi'^2 \tag{9}$$

Substitute (9) in (3), we can be transformed it into the diagonal form. For the consistency of notations, we set $t'\to t$, $\varphi'\to\varphi$ again and obtain the result

$$ds^2=\frac{1}{2}\left\{\left[\left(2-\frac{2\alpha r}{r^2+\beta^2\cos^2\theta}+\frac{\beta^2}{r^2}+\frac{2\alpha\beta^2\sin^2\theta}{r(r^2+\beta^2\cos^2\theta)}\right)^2\right.\right.$$

$$\left.\left.+\frac{16\alpha^2\beta^2\sin^2\theta}{(r^2+\beta^2\cos^2\theta)^2}\right]^{\frac{1}{2}}-\frac{2\alpha r}{r^2+\beta^2\cos^2\theta}-\frac{\beta^2}{r^2}-\frac{2\alpha\beta^2\sin^2\theta}{r(r^2+\beta^2\cos^2\theta)}\right\}dt^2$$

$$-\frac{r^2+\beta^2 cos^2\theta}{r^2+\beta^2-2\alpha\, r}dr^2-\left(1+\frac{\beta^2}{r^2}cos^2\theta\right)r^2 d\theta^2$$

$$-\frac{1}{2}\left\{\left[\left(2-\frac{2\alpha r}{r^2+\beta^2\cos^2\theta}+\frac{\beta^2}{r^2}+\frac{2\alpha\beta^2\sin^2\theta}{r(r^2+\beta^2\cos^2\theta)}\right)^2+\frac{16\alpha^2\beta^2\sin^2\theta}{(r^2+\beta^2\cos^2\theta)}\right]^{\frac{1}{2}}\right.$$

$$\left.+\frac{2\alpha\, r}{r^2+\beta^2\cos^2\theta}+\frac{\beta^2}{r^2}+\frac{2\alpha\beta^2\sin^2\theta}{r(r^2+\beta^2\cos^2\theta)}\right\}r^2\sin^2\theta\; d\varphi^2 \tag{10}$$

The formula (10) has the form of (1), so we can use it to describe the gravitational field of thin loop.

On the other hand, we know in general relativity that only by comparing with the Newtonian theory in the weak field, the integral constant of the solution of the Einstein's equation of gravity field can be determined. According to this principle, we have relation

$$g_{00}=1+2\psi \tag{11}$$

Here ψ is the Newtonian gravity potential. Now let's discuss the concrete form of ψ for a thin loop. As is shown in Fig.1, suppose that the coordinates of the observation point are $x_0=r'\,sin\theta'\,cos\,\varphi'$, $y_0=r'\,sin\theta'\,sin\varphi'$, $z_0=r'\,cos\,\theta'$. The coordinates of a point on the surface of thin loop are $x=b\,cos\,\varphi$, $y=b\,sin\varphi$, $z=0$. The distance between these two points is

$$R=\sqrt{(x_0-x)^2+(y_0-y)^2+(z_0-z)^2}=\sqrt{r'^2+b^2-2r'b\,sin\theta'\,cos(\varphi-\varphi')} \tag{12}$$

For symmetry and simplicity, we can take $\varphi'=0$, so the Newtonian potential of thin loop is

$$\psi=-\int\frac{GdM}{R}=-\int_0^{\pi}\frac{2G\rho\; bd\varphi}{\sqrt{r'^2+b^2-2r'b\,sin\theta'\,cos\,\varphi}} \tag{13}$$

M , ρ and b are mass, linear density and radius of thin loop individually. Let $\phi=\pi-\varphi'$, $d\phi=-d\phi'$, , $-cos\,\varphi=cos\,\phi'=1-2\,sin^2(\phi'/2)$, and put them into the formula above, we get

$$\psi=\int_{\pi}^{0}\frac{2G\rho\; bd\phi'}{\sqrt{r'^2+b^2+2r'b\,sin\theta-4r'b\,sin\theta'\,sin^2(\phi'/2)}} \tag{14}$$

Then let $\phi''=\phi'/2$ again, the formula above can be written as

$$\psi=-\int_0^{\pi/2}\frac{4G\rho\; bd\phi''}{\sqrt{r'^2+b^2+2r'b\,sin\theta'-4r'b\,sin\theta'\,sin^2\phi''}}$$

$$=-\frac{4G\rho\; b}{\sqrt{r'^2+b^2+2r'b\,sin\theta'}}\int_0^{\pi/2}\frac{d\phi''}{\sqrt{1-k^2\,sin^2\phi''}} \tag{15}$$

In the formula, we have $k^2=4r'b\,sin\theta'/(r'^2+b^2+2r'b\,sin\theta')$. Let

$$K(k^2)=\int_0^{\pi/2}\frac{d\phi''}{\sqrt{1-k^2\sin^2\phi''}} \tag{16}$$

(16) is just the first kind ellipse function. When $r' \to \infty$, we have

$$K(k^2) = \frac{\pi}{2}\left(1 + \frac{1}{4}k^2 + \frac{9}{64}k^4 \cdots\right) = \frac{\pi}{2}\left(1 + \frac{b\sin\theta'}{r'} + \frac{9b^2\sin^2\theta'}{4r'^2}\cdots\right) \quad (17)$$

On the other hand, when $r' >> b$, we have

$$\frac{1}{\sqrt{r'^2 + b^2 + 2r'b\sin\theta'}} = \frac{1}{r'}\left[1 - \frac{b\sin\theta'}{r'} + \frac{b^2(3\sin^2\theta' - 1)}{2r'^2} + \cdots\right] \quad (18)$$

Substitute (16) and (18) in (15) and, because of $2\pi\rho\, b = M$, we get

$$\psi = -\frac{GM}{r'}\left[1 - \frac{b^2\left(2 - 11sin^2\theta'\right)}{4r'^2} + \cdots\right] \quad (19)$$

On the other hand, we can expand $g_{\mu\nu}$ into the power series about $1/r$ and write (10) as

$$ds^2 = \left(1 - \frac{2\alpha}{r} + \frac{2\alpha\beta^2 cos^2\theta}{r^3} + \cdots\right)dt'^2$$

$$-\left(1 + \frac{2\alpha}{r} - \frac{2\alpha + \beta^2 sin^2\theta}{r^2} + \frac{12\alpha\beta^2 - 48\alpha^3}{r^3} + \cdots\right)dr^2$$

$$-\left(1 + \frac{\beta^2}{r^2}cos^2\theta\right)r^2 d\theta^2 - \left(1 + \frac{\beta^2}{r^2} + \frac{2\alpha\beta^2 sin^2\theta}{r^3}\cdots\right)r^2 sin^2\theta\, d\varphi'^2 \quad (20)$$

By considering (11) and comparing the items which contain the order up to r^{-1}, we get

$$1 - \frac{2\alpha}{r} = 1 - \frac{2GM}{r'} \quad (21)$$

Let $\alpha = GM$, we get $r = r'$. However, the formula above is only suitable for the situation when the mass of thin loop is concentrated at the center point of the loop. In order to obtain the more accurate gravity potential of thin loop, we should consider higher order items. There are no items containing r^{-2} order in (19) and the first item of (20). By considering the items to contain the order up to r^{-3}, we have

$$1 - \frac{2GM}{r} + \frac{2GM\beta^2\cos^2\theta}{r^3} = 1 - \frac{2GM}{r'} + \frac{2GMb^2(0.5 - 2.75\sin^2\theta')}{r'^3} \quad (22)$$

We see that the function forms on the two sides of (22) are different. It means that the solution of the Einstein's equation of gravity can not asymptotically coincide with the Newtonian theory of gravity automatically in this case. In order to make them asymptotically consistent, further transformation is needed. Because constant β has the dimension of length, we can take $\beta = b$. Because we always have $cos^2\theta \geq 0$ but may have $0.5 - 2.75\sin^2\theta' < 0$, so we have $0.5 - 2.75\sin^2\theta' \neq \cos^2\theta$ in general. Therefore, we have $r \neq r'$ in (22). However, we can set $\theta = \theta'$ so that (22) becomes

$$\frac{1}{r} - \frac{b^2\cos^2\theta'}{r^3} = \frac{1}{r'} - \frac{b^2(0.5 - 2.75\sin^2\theta')}{r'^3} \quad (23)$$

Let $A = b^2 \cos^2 \theta'$, $B = b^2(0.5 - 2.75\sin^2 \theta')$ and by considering the condition $\boldsymbol{r}' >> \boldsymbol{b}$, the only real number solution of (23) is

$$\frac{1}{r} = \left[\frac{1}{2Ar'^3}(B - r'^2) + \frac{i}{2A^{3/2}r'^3}\sqrt{4r'^6 - A(B - r'^2)^2} \right]^{\frac{1}{3}}$$

$$+ \left[\frac{1}{2Ar'^3}(B - r'^2) - \frac{i}{2A^{3/2}r'^3}\sqrt{4r'^6 - A(B - r'^2)^2} \right]^{\frac{1}{3}}$$

$$= (a + ib)^{1/3} + (a - ib)^{1/3} = 2Q\cos(\delta/3) \qquad (24)$$

Here $a = \frac{B - r'^2}{2Ar'}$ $\quad b = \frac{\sqrt{4r'^6 - A(B - r'^2)^2}}{2A^{3/2}r'^3}$ $\quad Q = (a^2 + b^2)^{1/6}$ $\quad tg\frac{b}{a} = \delta$ (25)

So we can write $r = r(r', \theta')$ and obtain

$$dr = \frac{dr}{dr'}dr' + \frac{dr}{d\theta'}d\theta' = T(r', \theta')dr' + V(r', \theta')d\theta' \qquad (26)$$

The concrete forms of functions $T(r', \theta')$ and $V(r', \theta')$ are unimportant, so we do not write them out here. Now we substitute (26) in (10 and obtain the metric of gravitational equation of thin loop which has the form of (2) with $r = r(r', \theta')$

$$ds^2 = \frac{1}{2}\left\{\left[\left(2 - \frac{2\alpha r}{r^2 + b^2\cos^2\theta'} + \frac{b^2}{r^2} + \frac{2\alpha b^2\sin^2\theta'}{r(r^2 + b^2\cos^2\theta')}\right)^2\right.\right.$$

$$\left.\left. + \frac{16\alpha^2 b^2 \sin^2\theta'}{(r^2 + b^2\cos^2\theta')^2}\right]^{\frac{1}{2}} - \frac{2\alpha r}{r^2 + b^2\cos^2\theta} - \frac{b^2}{r^2} - \frac{2\alpha b^2\sin^2\theta'}{r(r^2 + b^2\cos^2\theta')}\right\}dt'^2$$

$$- \frac{r^2 + b^2\cos^2\theta'}{r^2 + b^2 - 2\alpha r}T^2(r', \theta')dr'^2 - 2\frac{r^2 + b^2\cos^2\theta'}{r^2 + b^2 - 2\alpha r}T(r', \theta')V(r', \theta')dr'd\theta'$$

$$- \left[\left(1 + \frac{b^2}{r^2}\cos^2\theta'\right)\frac{r^2}{r'^2} + \frac{r^2 + b\cos^2\theta'}{r^2 + b^2 - 2\alpha b}\frac{V^2(r', \theta')}{r'^2}\right]r'^2 d\theta'^2$$

$$- \frac{r^2}{2r'^2}\left\{\left[\left(2 - \frac{2\alpha r}{r^2 + b^2\cos^2\theta'} + \frac{b^2}{r^2} + \frac{2ab^2 r\sin^2\theta'}{r(r^2 + b^2\cos^2\theta')^2}\right)^2 + \frac{16\alpha^2 b^2\sin^2\theta'}{(r^2 + b^2\cos^2\theta')^2}\right]^{\frac{1}{2}}\right.$$

$$\left. + \frac{2\alpha r}{r^2 + b^2\cos^2\theta'} + \frac{b^2}{r^2} + \frac{2\alpha b^2\sin^2\theta'}{r(r^2 + b^2\cos^2\theta')}\right\}r'^2\sin^2\theta' \, d\varphi'^2 \qquad (27)$$

As is shown in Fig.1, or by the definition of coordinate systems, we have both $r' = 0$ and $r = 0$ simultaneously for the original points of two coordinate systems. When $r' = 0$, we have $r = 0$ in (27) which leads to $g_{00} \to \infty$, $g_{22} \to \infty$, and $g_{33} \to \infty$. The result shows that a singularity will appear at the centre of thin loop. This singularity is completely exposed in vacuum, no matter how much the mass and the density of thin loop are, even they are very small. The singularity is essential one which can not be removed by coordinate transformation. This result is absurd and unacceptable, for it obviously violate common experience. It does not like the singularity of the Schwarzschild solution which hides in the center of huge mass and is unobservable directly so that physicists can tolerate its existence.

Besides, it can be proved that the space nearby the surface of thin loop is also high curved. Because of $\alpha << b$, we can let $\alpha \to 0$ for approximation. In the nearby region of the surface of thin loop, we take $\theta' = \pi / 2$. In this case, (27) becomes

$$ds^2 = dt'^2 - \frac{r^2}{r^2 + b^2} T^2(r',\theta') dr'^2 - \left[\frac{r^2}{r'^2} + \frac{r^2 V^2(r',\theta')}{r'^2 (r^2 + b^2)} \right] r'^2 d\theta'^2$$

$$- \left(\frac{r^2}{r'^2} + \frac{b^2}{r'^2} \right) r'^2 \sin^2 \theta' \ d\varphi'^2 - \frac{2r^2}{r'(r^2 + b^2)} T(r',\theta') V(r',\theta') r' dr' d\theta' \qquad (28)$$

Take $b = 0.67$, (23) becomes

$$\frac{1}{r} = \frac{1}{r'} + \frac{1}{r'^3} \qquad \text{or} \qquad r = \frac{r'^3}{r'^2 + 1} \qquad (29)$$

So we have

$$T(r',\theta') = \frac{r'^4 + 3r'^2}{(r'^2 + 1)^2} \qquad V(r',\theta') = 0 \qquad (30)$$

Take $r' = b = 0.67$, we have $r = 0.21$, $r = 0.21$. Using these values in (27), we obtain $g'_{11} = -0.10$, $g'_{22} = -0.10$ and $g'_{33} = -1.10$. So the space nearby the surface of thin loop is highly curved. The result does not agree with practical experiences completely. On the surface of thin loop, the gravity is very weak and space should be nearly flat with $g'_{11} = g'_{22} = g'_{33} \approx -1$. Because the curvature of space is a quantity which can be measured directly, the solution (27) is improper for the gravitational field of thin loop. In fact, according to the result (13) of the Newtonian theory, at the center point $\boldsymbol{r}' = 0$, the gravitational potential of loop is a limited constant with

$$\psi = -\int_0^{\pi} 2G\rho \ d\phi = -2\pi \ G\rho = -\frac{GM}{b} \qquad (31)$$

So the gravity at the center point of loop is zero. This agrees with practical experiences. The essential problem is that for such simple and foundational material distribution, if (27) is improper, where is the correct solution for the Einstein's equation of gravity? Can we find another solution? If can, how can we deal with the problem of the uniqueness of theory.

On the other hand, let $r^2 + b^2 - 2\alpha \, r = 0$ in (27), we have $r = \alpha \pm \sqrt{\alpha^2 - b^2}$. By taking $M \sim 1Kg$ and $b = 1m$ for the loop, we have $\alpha = GM / c^2 = 7.41 \times 10^{-28}$ and $\alpha << b$. So if we take $r = \alpha \pm \sqrt{\alpha^2 - b^2}$, r would not be a real number. Therefore, r' would not be a real number too. The second singularity of (27) determined by relation $r = \alpha \pm \sqrt{\alpha^2 - b^2}$ does not exist. In the Kerr solution, $r = \alpha \pm \sqrt{\alpha^2 - b^2}$ describers a surface of elliptical sphere which represents the event horizon of black hole. But for the gravitational field of thin loop, because equation $r^2 + b^2 - 2\alpha \, r = 0$ has no real solution in general situations, the event horizon does not exist.

Next, we discuss the situation when the cross section of thin loop is not zero. In this case, the gravitation field has three independent parameters. The third is the radium of loop's cross section. As we have known that the Kerr-Newman metric (E.T. Newman and A. I., Janis, 1965) is one with axial symmetry and three independent parameters. At present, it is used to describe the external gravitational field of revolving charged sphere. If the solution of the Einstein's equation of gravitational field with three parameters and axial symmetry is unique, by the coordinate transformation, we can also reach the gravitational field of loop with cross section based on the Kerr-Newman metric. By the same method of metric tensor's diagonalization, we can write the Kerr-Newman metric as

$$ds^2 = \frac{1}{2}\left\{\left[\left(2 - \frac{2\alpha\ r - Q^2}{r^2 + \beta^2\cos^2\theta} + \frac{\beta^2}{r^2} + \frac{2\alpha\beta^2\sin^2\theta}{r(r^2+\beta^2\cos^2\theta)}\right)^2\right.\right.$$

$$\left.\left.+\frac{16\alpha^2\beta^2\sin^2\theta}{(r^2+\beta^2\cos^2\theta)^2}\right]^{\frac{1}{2}} - \frac{2\alpha\ r - Q^2}{r^2+\beta^2\cos^2\theta} - \frac{\beta^2}{r^2} - \frac{2\alpha\beta^2\sin^2\theta}{r(r^2+\beta^2\cos^2\theta)}\right\}dt^2$$

$$-\frac{r^2+\beta^2 cos^2\theta}{r^2+\beta^2-2\alpha\ r - Q^2}dr^2 - \left(1+\frac{\beta^2}{r^2}cos^2\theta\right)r^2 d\theta^2$$

$$-\frac{1}{2}\left\{\left[\left(2 - \frac{2\alpha\ r - Q^2}{r^2 + \beta^2\cos^2\theta} + \frac{\beta^2}{r^2} + \frac{2\alpha\beta^2\sin^2\theta}{r(r^2+\beta^2\cos^2\theta)}\right)^2 + \frac{16\alpha^2\beta^2\sin^2\theta}{(r^2+\beta^2\cos^2\theta)^2}\right]^{\frac{1}{2}}\right.$$

$$\left.+\frac{2\alpha\ r - Q^2}{r^2+\beta^2\cos^2\theta} + \frac{\beta^2}{r^2} + \frac{2\alpha\beta^2\sin^2\theta}{r(r^2+\beta^2\cos^2\theta)}\right\}r^2\sin^2\theta\ d\varphi^2 \tag{32}$$

Here constant Q is related to the charge on the sphere. When $r >> \alpha$ and $r >> \beta$, we get following relation from (32)

$$g_{00} = 1 - \frac{2\alpha}{r} + \frac{Q^2}{r^2} + \frac{2\alpha\beta^2 cos^2\theta^2}{r^3} + \cdots \tag{33}$$

When the area of thin loop's cross section is considered, the Newtonian potential of gravity field is very complex. We do not discuss it in detail but can get the same conclusion by the simple estimation. Suppose that the radius of thin loop's cross section is h, when $r >> b$, $r >> h$ and $h \sim b$, due to the axial symmetry, we can always write the Newtonian gravity potential as

$$\psi = -\frac{GM}{r'}\left[1 + \frac{f_1(\theta', b, h)}{r'} + \frac{f_2(\theta', b, h)}{r'^2} + \cdots\right] \tag{34}$$

Similar to the discussion above, when $r \to \infty$, by considering terms up to the order r^{-2}, we can obtain from (11)

$$-\frac{GM}{r} + \frac{Q^2}{2r^2} = -\frac{GM}{r'} + \frac{f_1(\theta', b, h)}{r'^2} \tag{35}$$

Let $x = 1/r$ and $x' = 1/r'$, we can get from (35)

$$x = \frac{GM + \sqrt{(GM)^2 + 2Q^2 (f_1 x'^2 - GMx')}}{Q^2} \tag{36}$$

When $x' \to \infty$, we have also $x \to \infty$. That is when $r' = 0$, we have $r = 0$. Substitute (36) in (32), we can get the metric of loop with cross section. The singularity still exists at the center point of loop which is also exposed in vacuum. Space nearby the surface of loop is also highly curved. The situation is completely the same as that when the area of cross section of thin loop is neglected.

3. The Gravitational Field and Singularity of Static Double Spheres

As shown in Fig.2, the masses and radius of double spheres are M and b. The centers of two spheres are located at the points $\pm b$ on the z axis individually. It is obvious that the gravity field also has axial symmetry and two parameters and can be obtained through the coordinate transformation of the Kerr solution. For this problem, the Newtonian potential is

$$\psi = -GM\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = -GM\left(\frac{1}{\sqrt{r'^2 + b^2 + 2br'\cos\theta}} + \frac{1}{\sqrt{r'^2 + b^2 - 2br'\cos\theta}}\right) \tag{37}$$

When $r' >> b$, we have

$$\psi = -\frac{2GM}{r'}\left(1 - \frac{b^2 - 3b^2 cos^2\theta}{2r'^2}\right) \tag{38}$$

From (11), we get relationship

$$1 - \frac{2\alpha}{r} + \frac{2\alpha\beta^2\cos^2\theta}{r^3} = 1 - \frac{4GM}{r'} + \frac{2GMb^2(1 - 3\cos^2\theta')}{r'^3} \tag{39}$$

Let $\alpha = 2GM$, $\beta = b$, $\theta = \theta'$, we have

$$\frac{1}{r} - \frac{b^2\cos^2\theta'}{r^3} = \frac{1}{r'} - \frac{b^2(1 - 3\cos^2\theta')}{2r'^3} \tag{40}$$

Let $A = b^2 cos^2\theta', B = b^2(1 - 3\cos^2\theta')/2$ in (40), we can obtain the relations similar to (24) and (26). By considering (10), the metrics of static double spheres can be obtained. It also agrees with the form of (27). Further more, it is the same that we have $r = 0$ and $r' = 0$ simultaneously. So there is a singularity at the contact point of double spheres with $g_{00} \to \infty$, $g_{22} \to \infty$, $g_{33} \to \infty$ and $\boldsymbol{g}_{23} \to \infty$. Take $b = \sqrt{2}$ and $\theta' = \pi/2$, (40) becomes

$$\frac{1}{r} = \frac{1}{r'} - \frac{1}{r'^3} \qquad \text{or} \qquad r = \frac{r'^3}{r'^2 - 1} \tag{41}$$

Take $M = 1Kg$. The gravitational field is very weak so that we can let $\alpha \to 0$ in (27) and get the formula similar to (30) with

$$T(r', \theta') = \frac{r'^4 - 3r'^2}{(r'^2 - 1)^2} \qquad V(r', \theta') = 0 \tag{42}$$

Take $r' = 2$, we get $r = 2.67$ and $T = 0.44$. Substitute the values in (28), we obtain $g_{11} = -0.15$,

$g_{22} = -1.78$ and $g_{33} = -2.28$. It means that the space nearby the surfaces of two spheres is also high curved. Of cause, it is impossible. In the weak field, we should have $g_{11} = g_{22} = g_{33} \approx -1$. More serious is that when $r' < 1$, r becomes a negative number according to (41) so that it is meaningless. So (27) is also unsuitable for the gravitational field of static double spheres.

In fact, there are many other axial symmetry distributions of masses with two or three parameters. For example, three spheres which are superposed one by one along a straight line, two cones which are superposed with their cusps meeting together, as well as the hollow column and so on. In principle, all of their gravitational fields can be obtained by means of the coordinate transformations based on the Kerr solution and the Kerr-Newman solutions. To obtain their gravity fields, this method is unique actually. However, we can imagine that same problem will occur in all cases. The singularities exist at some points and are exposed in vacuum, as well as the spaces nearby the surfaces of objects are highly curved under the conditions of weak fields when their masses are very small. All of them can not coincide with practical experiences.

4. Conclusions

According to the singularity theorem (Hawking and Ellis, 1973), space-time singularities exist commonly and unavoidably in the Einstein's theory of gravity. In the universe, the corresponding objects of space-time singularities are black holes just as the Schwarzschild black hole and the Kerr black hole. It is now believed that black holes are created through the collapse of material. Because black holes are considered to be hidden at the centers of super-massive mass with very high density, for example, at the centers of quasars and galaxies so that they can not be observed directly, physicists can tolerate their existence at present. However, if a singularity is exposed in vacuum, the problem will become very serious.

The calculation in this paper proves that the singularities will appear at the center of a thin loop and the contact point of two spheres according to the Einstein's theory of gravity. The singularities are exposed in vacuum completely. The space nearby the surfaces of thin circle and double spheres are high curved. If it is true, the ruler will be bended when it is placed in the center region of a thin loop. Light will bend and the effect of gravitational lenses will occur when then light passes through the central region of circle. These results are obviously unimaginable and absorbed. The conclusion can only be that the singularity is caused by the description method of curved space-time. By the rational analogy, the so-called singular black holes, white holes and wormholes which connect both holes in the current cosmology and astrophysics are something illusive. They have nothing to do with real world. In fact, as the observations of Rudolf E. Schild, the centers of quasars are composed of MECO, rather than singular black holes. There exists magnetic fields and material in the center regions of quasars, rather than vacuum. The observations are consistent with the calculation and analysis in this paper. If there are black holes in nature, they can only be the Newtonian black holes, rather than the Einstein's singularity black holes!

As we know that although the Einstein's theory of gravity is considered to be successful, so far we have only four accurate experiments to verify its validity in the weak gravity field of the solar system. In fact, we only prove that the Schwarzschild solution is effective when following three conditions are satisfied simultaneously. 1. Material is concentrated in the center region; 2. Material is static and distributed with spherical symmetry; 3. Mass and density are relatively small so that the gravitational field is weak. For the general situations in which these three conditions are not satisfied, we need more proofs for the validity of the Einstein's theory of gravity. In the classical Newtonian theory, the gravity fields of thin loop and double spheres are two foundational and simple problems. But in the Einstein's theory, they become very complex. In fact, we do not know the forms of their solution so far. Facing so many forms of material distributions, comparing with so many experiments in the Newtonian theory of gravity, quantum mechanics and special relativity, it is far not enough for the verification of the Einstein's theory of gravity. In fact, according to the calculations in this paper, the Einstein's theory of gravity can not correctly describe the static gravity fields of material contributions with axial symmetry. How can we believe it is effective for other forms of material contributions except spherical symmetry?

As revealed in this paper, singularity in general relativity is caused by the description method of curved space-time actually. The true world excludes infinites. A correct theory of physics can not tolerate the existence of infinites. It is well known that the history of physics is the one to overcome infinites. Modern physics grows up in the process to surmount infinites. Physicists and cosmologists should take cautious and incredulous attitude on the problems of singularity black holes. It is not a scientific attitude to consider singular black holes as objective existence without any question on them. We should think in deep, whether or not our theory has something wrong. When we enjoy so –called beauty and symmetry of the Einstein's theory of gravity, remember that we should not neglect its limitations and possible mistake.

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