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Locally Rotationally Symmetric Bianchi Type II Massive String Cosmological Model For Barotropic Fluid Distribution And Vacuum Energy Density (Λ)

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Abstract

Locally Rotationally Symmetric (LRS) i.e. axis symmetric Bianchi type II massive string cosmological model for barotropic fluid distribution with vacuum energy density (Λ) in context of General Relativity, is investigated. To get the deterministic model of the universe, we have assumed the condition σ/θ =constant and Λ is proportional to H^2 where σ is shear and θ the expansion in the model, H the Hubble parameter. We find that the model represents accelerating and decelerating phases of universe. The vacuum energy density $\Lambda \sim \frac{1}{t^2}$ which matches with the result as obtained by Bertolami (1986). The model starts with a big bang at $t = -b/a$ and the expansion in the model decreases as time increases. The expansion stops at $t = -\pi b/2a$ then the model accelerates. The energy density (ρ) and string tenson density (λ) are initially large but decreases with lapse of time. The model in general represents anisotropic space time. In special case the model isotropizes. The special cases of $\gamma = 1$ (stiff fluid) $\gamma = 0$ (dust filled universe) and $\gamma = 1/3$ (radiation dominated) universe are also discussed. The model has point type singularity at $t = -b/a$.

1 Introduction

Bianchi Type II space-time plays a significant role in constructing cosmological models to describe early stage of evolution of universe. Asseo and Sol (1987) have given the importance of Bianchi Type II space-time for the study of universe. Hajj-Boutros (1989) derived exact solutions of Einstein's field equations for LRS Bianchi Type II space-time using perfect fluid distribution of matter with an equation of state. Coley and Wainwright (1991) studied Bianchi Type II models in two fluid cosmologies. Roy and Banerjee (1995) have investigated LRS Bianchi Type II cosmological models for massive ($\rho = \rho_p + \lambda$) and geometric string ($\rho = \lambda$) where ρ_p is the particle density, λ the string tension density, ρ the energy density. Pradhan et al. (2011) obtained LRS Bianchi Type II string cosmological model in General Relativity. Recently Bali and Singh (2014) investigated LRS Bianchi Type II massive string cosmological model for stiff fluid distribution with decaying vacuum energy density (Λ). In this paper we have investigated locally rotationally symmetric bianchi type II massive string cosmological model for barotropic fluid distribution and vacuum energy density (Λ).

The barotropic fluid is significant in the study of steller interiors or of the interstellar medium. Mathematically it is represented by $p = \gamma\rho$, p the isotropic pressure, ρ the matter density and $0 \leq \gamma \leq 1$ where $\gamma = 0$ leads to dust filled universe, $\gamma = 1$ (stiff fluid universe), $\gamma = 1/3$ radiation dominated universe.

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In Physics, string theory is a theoretical frame work in which the point-like particles of particle physics are replaced by one dimensional objects called strings. The presence of strings in the early universe can be explained using grand unified field theories (GUT) (Kibble (1976), Vilenkin (1961)). In standard big-bang cosmological models, T_c (critical temperature) is exceeded in the very earliest stages of the evolution of the universe. A phase transition as the universe cools below T_c in which a multiple of scalar fields (Higgs field) develops a vacuum structure depending on the structure and topology of the gauge group. These vacuum structures give origin to string in space-time (Higgs field). These strings have stress energy which are coupled with gravitational field. Therefore, it is interesting to study the gravitational effects that arise from strings. The pioneering work in the formulation of energy-momentum tensor for classical massive strings is due to Letelier (1983). He explained that massive strings are formed by geometric strings with particles attached along its extension and used this idea to find some cosmological solutions for massive strings in Bianchi Type I and Kantowski space-time. String cosmological models have been investigated by many authors viz. Banerjee et al. (1990), Tikekar and Patel (1992,1994), Chakraborty and Charkraborty (1992), Barrow and Kunz (1997), Singh and Singh (1999), Bali et al.(2001,2006,2007,2010) using the Letelier (1983) form of energy-momentum tensor.

Einstein theory relates matter and energy as different states of the same thing. As per astronomical observations, universe is composed of roughly 0.03% heavy elements (any things other than hydrogen and helium), 0.3% neutrinos, 0.5% stars, 4% free hydrogen and helium, 25% dark matter and 70% dark energy. Dark matter exist to account for the gravity. It is also responsible for amplifying small fluctuations in the cosmic microwave background back in the early universe to create the large scale structure, we observe in the universe today.

Dark energy which goes by the name of cosmological constant exist due to the rate of expansion we observe for our universe. Not only the universe expanding but this expansion is also accelerating so the unknown anti gravity force is termed as 'dark energy'.

The cosmological constant is the most favoured candidate of dark energy representing energy density of vacuum. The relevance of cosmological constant related with the observation are given by Zeldovich (1968), Krauss and Turner (1995). In 1998, two independent groups led by Riess et al. (1998) and Perlmutter et al (1999) used Type Ia supernovae showed that universe is accelerating. This discovery provided the first direct evidence that Λ is non-zero with $\Lambda \sim 1.7 \times 10^{-121}$ Plank units. Thus $\Lambda \neq 0$ and the present time accelerating behaviour of Universe is due to the dominance of vacuum energy density (Λ). It is now believed that the cosmological constant is a kind of repulsive pressure dubbed as dark energy, is the most suitable candidate to explain the recent observations of universe. Many authors viz. Bertolami (1986), Sahni and Starobinsky (2000), Peebles and Ratra (2003), Chen and Wu (1990), Wang and Meng (2005), Bali and Singh (2008), Bali et al. (2012), Ram and Verma (2010), Abdussattar and Prajapati (2011) have investigated cosmological models with decaying vacuum energy density (Λ). Barrow and Shaw (2011) suggested that cosmological term corresponds to a very small value 10^{-122} when applied to FRW model leading to (Λ) non-zero.

2 Metric and Field Equations

We consider Locally Rotationally Symmetric (LRS) i.e. axisymmetric metric in the form

$$ds^2 = -dt^2 + R^2(dx^2 + dz^2) + S^2(dy + xdz)^2 \tag{1}$$

where R and S are metric potentials and are function of $t-$alone.

We can cast the energy-momentum tensor (T_i^j) for massive string with perfect fluid distribution given by Letelier (1983) as

$$T_i^j = (\rho + p)v_i v^j + p g_i^j - \lambda x_i x^j \tag{2}$$

with

$$\rho = \rho_p + \lambda \tag{3}$$

ρ being the proper energy density, of the cloud of strings with particles attached to them, ρ_p the particle density, λ the string tension density x^i is the space-like 4-vector representing the strings direction which we call the direction of anisotropy satisfying the conditions $\rho \geq 0$, $\lambda \geq 0$ or $\rho \geq 0$, $\lambda < 0$ (Laterlier 1983), we have

$$v_i v^i = -x_i x^i = -1 \tag{4}$$

and

$$v^i x_i = 0 \tag{5}$$

If particles are not attached to the string then $\rho_p = 0$. In this case, we have

$$\rho = \lambda$$

which is called state equation for cloud of geometric strings (Nambu strings) (Letelier 1979, Stachel 1980). We assume the coordinates to be comoving so that

$$v^1 = 0 = v^2 = v^3, \ v^4 = 1. \tag{6}$$

From the non-diagonal components of the energy-momentum tensor, we find at most one non-zero component of x^i i.e. we are considering 1 D string and x^i describes the length of string along x^2 direction. Thus

$$x^2 \neq 0, \ x^1 = x^3 = 0 \tag{7}$$

With the help of metric (1), the Einsteins field equations

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j + \Lambda(t) g_i^j \tag{8}$$

(in geometrized unit $8\pi G = c = 1$) become

$$\frac{S^2}{4R^4} + \frac{\dot{R}\dot{S}}{RS} + \frac{\ddot{R}}{R} + \frac{\ddot{S}}{S} = -p + \Lambda \tag{9}$$

$$-\frac{3S^2}{4R^4} + \frac{\dot{R}^2}{R^2} + \frac{2\ddot{R}}{R} = \lambda x_2 x^2 - p + \Lambda \tag{10}$$

$$\frac{S^2}{4R^4} + \frac{\dot{R}\dot{S}}{RS} + \frac{\ddot{R}}{R} + \frac{\ddot{S}}{S} = -p + \Lambda \tag{11}$$

$$-\frac{S^2}{4R^4} + \frac{2\dot{R}\dot{S}}{RS} + \frac{\dot{R}^2}{R^2} = \rho + \Lambda \tag{12}$$

since x^2 is the only non-vanishing component of x^i. Thus equation (4) leads to

$$x_2 x^2 = 1 \tag{13}$$

Thus, we have

$$g_{22}(x^2)^2 = 1 \tag{14}$$

which leads to

$$x^2 = \frac{1}{S} \ \text{ and } \ x_2 = S \tag{15}$$

3 Solution of Field Equations

We have three equations (9), (10) and (12) in six unknown R, S, ρ, p, λ and Λ. To get the deterministic model, we assume three conditions:

1. the universe is filled with barotropic fluid distribution $p = \gamma\rho$, $0 \leq \gamma \leq 1$,
2. σ (shear) is proportional to expansion (θ),
3. the vacuum energy (Λ) is proportional to H^2, H being the Hubble parameter as assumed by Lima and Carvalho (1994), Wetterich (1995) and Arbab (1997).

Thus

$$\Lambda = \frac{1}{9}\left(\frac{2\dot{R}}{R} + \frac{\dot{S}}{S}\right)^2 \tag{16}$$

where proportionality constant is assumed as unity. We have not used G_3^2 component of the field equation (5) because

$$G_3^2 = x\left(\frac{S^2}{R^4} - \frac{\dot{R}^2}{R^2} - \frac{\ddot{R}}{R} + \frac{\dot{R}\dot{S}}{RS} + \frac{\ddot{S}}{S}\right) \tag{17}$$

and

$$G_3^2 = -(T_3^2 - \Lambda g_3^2) = 0 \tag{18}$$

as

$$T_3^2 = (\rho + p)v^2 v_3 + p g_3^2 - \lambda x_3 x^2 = 0 \tag{19}$$

$$g_3^2 = g^{22}g_{32} + g^{23}g_{33} = \left(\frac{1}{S^2} + \frac{x^2}{R^2}\right)(-xS^2) + \frac{x}{R^2}(R^2 + x^2S^2) = 0 \tag{20}$$

and

$$v^2 = 0 = v^3, \quad x_3 = 0 \tag{21}$$

Also

$$G_3^2 = x(G_3^3 - G_2^2) \tag{22}$$

Therefore, $G_3^2 = 0$ implies $G_2^2 = G_3^3$. Thus the component G_3^2 can be ignored. Conservation equation $T_{i\ ;j}^{j} = 0$ leads to

$$\dot{\rho} + (1+\gamma)\,\rho(\frac{2\dot{R}}{R} + \frac{\dot{S}}{S}) - \lambda\frac{\dot{S}}{S} = -\dot{\Lambda} \tag{23}$$

($\because p = \gamma\rho$). Using the conditions $p = \gamma\rho$ and (16) in equations (9) and (12), we have

$$\left\{\frac{4}{9}(1+\gamma) - \gamma\right\}\frac{\dot{R}^2}{R^2} + (1+\gamma)\frac{\dot{S}^2}{9S^2} + \left\{\frac{4}{9}(1+\gamma) - (1+2\gamma)\right\}\frac{\dot{R}\dot{S}}{RS} - \frac{\ddot{R}}{R} - \frac{\ddot{S}}{S} - \frac{(1-\gamma)S^2}{4R^4} = 0 \tag{24}$$

Now using the condition $\sigma \propto \theta$, we have

$$R = S^n \tag{25}$$

the proportionality constant has been assumed unity for simplicity. The motive behind assuming the condition $\sigma \propto \theta$ is explained as: Referring to Thorne (1967), the observations of the velocity-red shift relation for extra galatic sources suggest that the Hubble expansion of the universe is isotropic today within 30% (Kantowski and Sachs 1966, Kristian and Sachs 1966). More precisely, the red-shift studies place the limit $\frac{\sigma}{H} \leq 0.30$ where σ is shear and H the Hubble constant. Collins et al. (1980)

have pointed out that for spatially homogeneous metric, σ/θ is constant, σ being the expansion of the model. Equations (24) and (25) lead to

$$2\ddot{S}+\frac{2}{9(n+1)}\cdot\{9(n^2-n)-(4n^2+4n+1)(1+\gamma)+9n(1+2\gamma)+9n^2\gamma\}\cdot\frac{\dot{S}^2}{S}=-\frac{(1-\gamma)S^{3-4n}}{18(n+1)} \quad (26)$$

To get the results in terms of cosmic time t, we assume different cases:

3.1 Barotropic fluid model

We assume $\gamma = 1/2$, $n = 1/2$. Thus equation (26) leads to

$$2\ddot{S}+\frac{5\dot{S}^2}{18S}=-\frac{S}{54} \quad (27)$$

To find the solution, we consider $\dot{S}=f(S)$. Thus equation (27) leads to

$$\frac{d}{dS}(f^2)+\frac{5f^2}{18S}=-\frac{S}{54} \quad (28)$$

which leads to

$$S^{41/36}=\sqrt{123\alpha_1}\sin(at+b) \quad (29)$$

where

$$a=\frac{41}{36\sqrt{123}} \quad \text{and } \alpha,\ b \text{ are constants.} \quad (30)$$

Now

$$R=S^{1/2} \quad (\text{as } n=1/2)=(123\alpha_1)^{1/4}\sin^{18/41}(at+b). \quad (31)$$

Therefore, the metric (1) leads to the form

$$ds^2=-dt^2+(123\alpha_1)^{1/2}\sin^{36/41}(at+b)(dx^2+dz^2)+(123\alpha_1)^{72/41}\sin^{18/41}(at+b)(dy+xdz)^2. \quad (32)$$

3.2 Dust fluid distribution ($\gamma = 0$, $n = 1/2$)

Equation (26) in this case leads to

$$2\ddot{S}+\frac{7\dot{S}^2}{27S}=-\frac{S}{27} \quad (33)$$

The solution of equation (33) can be obtained in a similar way and thus equation (33) leads to

$$S=(47\alpha_2)^{27/47}\sin^{54/47}\left(\frac{\sqrt{47}}{54}t+\beta_2\right) \quad (34)$$

and

$$R=S^{1/2}=(47\alpha_2)^{27/94}\sin^{27/47}\left(\frac{\sqrt{47}}{54}t+\beta_2\right) \quad (35)$$

where α_2 and β_2 are constants. The metric (1) reduces to the form

$$ds^2=-dt^2+(47\alpha_2)^{27/47}\sin^{54/47}\left(\frac{\sqrt{47}}{54}t+\beta_2\right)+(47\alpha_2)^{54/27}\sin^{108/47}\left(\frac{\sqrt{47}}{54}t+\beta_2\right)(dy+xdz)^2. \quad (36)$$

3.3 Radiation dominated model ($\gamma = 1/3$, $n = 1/2$)

Equation (26) leads to

$$2\ddot{S} + \frac{8\dot{S}^2}{3S} = -\frac{2S}{81} \tag{37}$$

which leads to

$$S = \left(\frac{189\alpha_3}{2}\right)^{3/14} \sin^{3/7}(\beta_3 t + \beta_4) \tag{38}$$

and

$$R = S^{1/2} = \left(\frac{189\alpha_3}{2}\right)^{3/28} \sin^{3/14}(\beta_3 t + \beta_4) \tag{39}$$

where α_3, β_3, β_4 are constants. The metric (1) reduces to the form

$$ds^2 = -dt^2 + \left(\frac{189\alpha_3}{2}\right)^{3/14} \sin^{3/7}(\beta_3 t + \beta_4)(dx^2 + dz^2) + \left(\frac{189\alpha_3}{2}\right)^{3/7} \sin^{6/7}(\beta_3 t + \beta_4)(dy + xdz)^2. \tag{40}$$

3.4 Stiff fluid model ($\gamma = 1$, $n = 1/2$)

In this case, the equation (26) leads to

$$\ddot{S} = -\frac{11}{27}\dot{S} \tag{41}$$

which leads to

$$S = (\alpha t + \beta)^{27/28} \tag{42}$$

and

$$R = S^{1/2} = (\alpha t + \beta)^{27/56} \tag{43}$$

Therefore the metric (1) leads to the form

$$ds^2 = -dt^2 + (\alpha t + \beta)^{27/28}(dx^2 + dz^2) + (\alpha t + \beta)^{27/56}(dy + xdz)^2 \tag{44}$$

4 Some physical and geometrical aspects

The proper energy density (ρ), the string tension density (λ), the particle density (ρ_p), Hubble parameter (H), the expansion (θ), the shear (σ), the spatial volume (V), the deceleration parameter (q) and vacuum energy density (Λ) for the model (32) are given by

$$\rho = 3H^2 = \frac{4}{369}\cot^2(at + b) \tag{45}$$

where a is the constant given by (30).

$$\lambda = -\frac{53}{4}\cot^2(at + b) - \frac{3}{4} \tag{46}$$

$$p = \gamma\rho = \frac{2}{369}\cot^2(at + b) \tag{47}$$

$$\rho_p = \rho - \lambda = -\frac{19573}{1476}\cot^2(at + b) + \frac{3}{4} \tag{48}$$

$$\theta = \frac{2\dot{R}}{R} + \frac{\dot{S}}{S} = \frac{2\dot{S}}{S} = \frac{2}{\sqrt{123}}\cot(at + b) \tag{49}$$

$$H = \frac{\theta}{3} = \frac{2}{\sqrt{123}}\cot(at+b) \tag{50}$$

$$\Lambda = \frac{4H^2}{1107}\cot(at+b) \tag{51}$$

$$\sigma = \frac{1}{\sqrt{3}}|\frac{\dot{R}}{R} - \frac{\dot{S}}{S}| \tag{52}$$

$$= \frac{36a}{41\sqrt{3}}\cot(at+b) = \frac{1}{\sqrt{369}}\cot(at+b) \tag{53}$$

$$V^3 = (123\alpha_1)^{36/41}\sin 72/41(at+b) \tag{54}$$

$$q = -\frac{\ddot{V}/V}{\dot{V}^2/V^2} = \left\{\frac{24a^2}{41^2}\right\}\{17\cot^2(a+b) - 41\}$$

$$> 0 \ \text{ if } \lfloor 17\cot^2(a+b) - 41 \rfloor > 0 \tag{55}$$

$$< 0 \ \text{ if } \lfloor 17\cot^2(a+b) - 41 \rfloor < 0 \tag{56}$$

Entropy: To study the entropy, we apply the combined form of first and second law of thermodynamics for the system of comoving volume (V) as

$$TdS = d(\rho V) + pdV \tag{57}$$

Equation (57) can be written as

$$TdS = d[(\rho + p)V] - Vdp \tag{58}$$

For integrability condition, it is necessary to define perfect fluid universe as thermodynamic system considered by Myung (2009), Gong et al. (2007), $S = S(T, V)$ for which we have thermodynamic relation

$$\frac{\partial^2 S}{\partial T \partial V} = \frac{\partial^2 S}{\partial V \partial T} \tag{59}$$

which tends to relation between pressure (p) and temperature (T) as

$$dp = (\rho + p)\frac{dT}{T} \tag{60}$$

Using barotropic condition $p = \gamma\rho$ in equation (60), we have

$$\rho^{\gamma/(1+\gamma)} = T \tag{61}$$

which leads to

$$\rho = T^{(1+\gamma)/\gamma} \tag{62}$$

For radiation dominated case,

$$\rho = T^4 \tag{63}$$

Using (60) in (58), we have

$$dS = \frac{1}{T}d[(\rho + p)V] - \frac{(\rho + p)V}{T^2}dT \tag{64}$$

Equation (64) can also be written as

$$dS = d\left[\frac{(\rho+p)V}{T} + k\right] \tag{65}$$

where k is real constant. Hence entropy can be defined as

$$S = \frac{(\rho+p)V}{T} = \frac{(1+\gamma)\rho V}{T} \quad \text{as } p = \gamma\rho \tag{66}$$

Now the entropy density (s) is given by

$$s = \frac{S}{V} = \frac{(1+\gamma)\rho}{T} = \frac{(1+\gamma)T^{(1+\gamma)/\gamma}}{T} \tag{67}$$

For radiation dominated model $(\gamma = 1/3)$, equation (67) leads to $\rho \propto T^4$ and $s \propto T^3$

5 Conclusion

The strong energy condition implies that $\rho \geq 0$ with $\lambda < 0$ are satisfied for the model (32). The proper energy density (ρ) and string tension density (λ) is initially large but decreases with lapse of time. The model (25) starts with a big-bang at $t = -b/a$ and the expansion in the model decreases with time. The model represents decelerating and accelerating phases given by (55) and (56). The decaying vacuum energy density $\Lambda \sim \frac{1}{t^2}$ which matches with the result as obtained by Bertolami (1986). The entropy density (s) is proportional to (absolute temperature)^3. The Hubble parameter $H \sim \frac{1}{t}$ which matches with astronomical observations. Since $\sigma/\theta \neq 0$, hence anisotropy is maintained throughout. However, at $t = -\frac{b\pi}{2a}$, the model isotropizes. The model has point type singularity at $t = -b/a$ (MacCallum 1971).

We find similar type of results for $\gamma = 0$ (dust filled universe), $\gamma = 1/3$ (radiation dominated universe) but our model leads to the model obtained by Bali and Singh (2014) for stiff fluid universe $(\gamma = 1)$.

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