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Helicity and the Inverse Turbulence Cascade

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Vortex line stretching manifested by helicity[d] is crucial to understanding transport processes of turbulence in natural fluids, from Planck scales of the big bang[e] to submerged turbulence in the ocean and atmosphere[f].

In 1941, A. N. Kolmogorov proposed two universal similarity hypotheses describing the statistical properties of homogeneous, isotropic turbulence[1]. The first hypothesis, or similarity law, requires statistical parameters of turbulence at all scales to depend on the viscous dissipation rate ε and the kinematic viscosity ν. The second hypothesis is that for turbulent lengths larger than the Kolmogorov length scale $L_K = (\nu^3/\varepsilon)^{1/4}$ and for times larger than the Kolmogorov time scale $T_K = (\nu/\varepsilon)^{1/2}$ the parameters of turbulence should be independent of ν. Although some Western oceanographers did not immediately adopt them, these laws and their extensions to turbulent mixing have been verified over the years, mostly by comparisons of spectral data to the universal forms required after normalization to Kolmogorov space and time. Immediately after they were proposed, Landau criticized Kolmogorov's laws on the basis of the fact that the term ε was a random variable and he (Kolmogorov) did not introduce statistics for ε that were dependent on the Reynolds Number. This oversight was subsequently corrected by Kolmogorov in 1962[2] in a refinement termed the Kolmogorov third law, under which it is assumed that the logarithm of the dissipation rate averaged over different length scales is normally distributed. While statistical hypotheses are correct, they rely on a faulty physical model of the cascade[g].

As a footnote to his first paper, Kolmogorov noted that "For very large R (Reynolds Number) the turbulent flow may be thought of in the following way: on the flow are superimposed the 'pulsation of the first order', consisting in disorderly displacements of separate fluid volumes, one with respect to another, of diameters of the order $l^{(1)} = l$ (where l is the Prandtl mixing length). The order of magnitude of these velocities we denote by $v^{(1)}$. The pulsations of the first order are for very large R, and on them are superposed pulsations of the second order with mixing length $l^{(2)} < l^{(1)}$ and relative

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[d] Helicity is the integral scalar product of velocity with vorticity.
[e] See Gibson (2004,2005). The Planck scales depend only on c, h, G and k because the big bang represents the first turbulent combustion, the first fossil turbulence and the first fossil turbulence waves. Negative turbulence stresses > 10^{113} Pa extract mass energy from the vacuum to produce the universe by vortex line stretching.
[f] See Volume 21 of the Journal of Cosmology, devoted to stratified turbulent mixing.
[g] Kolmogorov 1941ab accepted the G.I. Taylor-L. F. Richardson assumption that turbulent kinetic energy cascades from large scales to small. It never does. Turbulent kinetic energy always begins at small inertial-viscous (Kolmogorov) scales and cascades to larger scales by inertial vortex forces. This is the basis of hydro-gravitational-dynamics (HGD) cosmology, Gibson (1996) and Schild (1996). HGD cosmology falsifies ΛCDMHC cosmology.

velocities $v^{(2)} < v^{(1)}$; such a process of successive refinement of turbulent pulsation may be carried for the pulsations of some sufficiently large order n, until the Reynolds Number

$$R^{(n)} = \frac{l^{(n)}v^{(n)}}{\upsilon}$$

becomes so small that the effect of viscosity on pulsations of order n finally prevents the formation of pulsations of the order n+1. From the energetic point of view, it is natural to imagine the process of turbulent mixing in the following way: the pulsations of the first order absorb the energy of motion and pass it on successively to pulsations of higher order. The energy of the finest pulsations is dispersed in the energy of heat due to viscosity." This later comment became known as the (direct) energy cascade.

It is instructive also to note Kolmogorov's comments on his laws: that they could not be rigorously proved, but that he would elaborate on them pointing out their various consequences that could then be verified experimentally. These consequences were developed in the following way:

The first example of the consequences of his laws resulted from consideration of his second law:

In this case, if the Reynolds Number is sufficiently high, the energy spectrum in the subrange satisfying the condition $k_e << k << k_d$ is independent of v and is solely determined by one parameter, ε. In this subrange, inertia is the dominating factor, so the range was defined as the inertial subrange. Various investigators have shown that the relationship $E(k,\varepsilon) = A\varepsilon^{\frac{2}{3}}k^{-\frac{5}{3}}$ holds for Kolmogorov's second law[3].

Experimentalists then began to look for a verification of this dependency, often called the "Kolmogorov spectrum law" or "the 5/3 law". The problem with the experiments was that to get several decades of spectral slope required a large experimental volume and microstructure probes. Early work at GALCIT produced a spectral slope close to the Kolmogorov slope[4]; however, some closure schemes of Kraichnan[5] produced a spectral slope of -3/2, close to -5/3, and the scatter in the data did not permit discrimination between these two slopes[6]. But in 1962, deploying towed hot film anemometers, Grant, Stewart and Moilliet produced more than two decades of -5/3 slope, operating in a very high Reynolds Number tidal current in Seymour Narrows, off Vancouver Island in Canada. This settled the matter, and years later, an analysis of the CMB (cosmic microwave background) revealed many decades of a -11/3 slope, providing a cosmological verification of Kolmogorov's laws[7]. The difference in the two exponents (-5/3 and -11/3) arises from the fact that Grant et al. were measuring exactly the point property Kolmogorov studied in his work which led to the -5/3 slope, but studies of the cmb spectra are looking into a three dimensional cosmos, not a point, which led to a -11/3 slope for the Kolmogorov power law.

Summarizing the verifications:

First two Laws: Grant, Stewart and Moilliet[8], Gibson and Schwarz[9].

Third Law; Baker and Gibson[10], Bershadskii and Sreenivasan (CMB spectrum)[11].

While Kolmogorov's Laws have had thorough and definitive verification, the same is not true for the energy cascade. Following Kolmogorov's definition cited previously, Batchelor also expressed the idea[12]. Current texts also express it[13], although in at least one case, somewhat equivocally[14]. Historically, one of the first to express the idea was L. F. Richardson in the 1920s, who composed a catchy short rhyme describing the effect[15]. On the other hand, some investigators have proposed an inverse cascade, as will be discussed later. It should be pointed out that Kolmogorov's statement was a heuristic assertion. In spite of this, that assertion has been generally accepted as inviolate.

So, by the year 2000, Kolmogorov's Laws had been thoroughly confirmed by various investigators. However, in a series of oceanographic experiments carried out in Hawaii in 2002-2004, powerful qualitative evidence was discovered for a number of nonlinear processes associated with ocean turbulence[16]. In these experiments, turbulent flow fields were generated below the surface, but instead of decaying down in time and space, as suggested by Gregg and others[17], a spatial Fourier analysis revealed a strong correlation of larger scale anomalies in the surface glint pattern, and the subsurface turbulence. These repeated and correlated observations indicate the presence of the so-called reverse (or inverse) cascade, in which small-scale events reorganize themselves into large-scale artifacts. This is a very significant set of data if it proves to be evidence of a reverse cascade. These measurements were augmented by a Russian team with comprehensive studies of ocean conditions in Hawaiian coastal waters, thereby eliminating environmental artifacts as possible sources of spurious effects that could have been misinterpreted in the satellite imagery.[18]

One of the key elements in these nonlinear processes would have to be the presence of fossil turbulence, a concept introduced by Gamow[19], and then studied by Woods[20]. For the definition of turbulence given as "an eddy-like state of fluid motion where the inertial vortex forces of the eddies are larger than any of the other forces which tend to damp the eddies out," the definition for fossil turbulence follows as "a fluctuation in some hydrophysical field of the fluid caused by turbulence that persists after the flow ceases to be turbulent at the scale of the fluctuations."[21] Fossil turbulence is one of the key players in the set of processes involved in the transport of energy and momentum throughout the stratified structure of the ocean. Fossil turbulence is created as a natural phase of the evolution of turbulence, and can eventually produce fossil turbulence waves; these are a unique class of internal waves emitted by turbulence in a stratified medium after the turbulence fossilizes at the Ozmidov scale $L_{Ro} = \left(\frac{\varepsilon_0}{N^3}\right)^{\frac{1}{2}}$.

The frequency N is defined by $N^2 = -g\left(\frac{d\rho}{dz}\right)/\rho_0$ and is called the buoyancy frequency. A thorough discussion is given by Thorpe[22].

The evolution of wake turbulence into fossil turbulence has recently been calculated in numerical solutions of the Navier-Stokes equation[23]. Another phenomenon, first proposed by Yamazaki, is so-called Zombie Turbulence[24]. Zombie turbulence consists of decaying turbulent eddies that can be activated by shear and other external energy sources. Also, there is strong evidence of internal waves tilting fossil turbulence patches, creating secondary turbulence by the baroclinic torque produced along the strong density gradients of the fossil patches [25]. The secondary turbulence again fossilizes and radiates internal waves that propagate at near vertical angles. These latter waves originate from the submerged turbulence and from the internal waves that do the tilting and then propagate to the sea surface where they perturb the glint pattern and can be detected remotely.

Fossil turbulence is still not broadly accepted in the West, although it was defined in a standard text recently[26]. However, the concept has been in wide use in Russia and the other countries of the Former Soviet Union (FSU)[27]. Most recently, a striking verification of the existence of fossil turbulence was found in the behavior of the Weddell seals in hunting down their underwater prey in complete darkness. These seals do not migrate out of the Antarctic region of complete darkness during their winter season. They not only hunt down their prey by tracking their wakes, but then return to their original breathing hole, using their vibrating whiskers to follow fossil vorticity turbulence information preserved in their own wake[28].

The experimental evidence obtained in Hawaii speaks strongly to the presence of an inverse energy cascade[29, 30, 31]. But there have been previous proposals of the inverse cascade. The traditional perception of the cascade process (large-scale to small-scale) was first suggested by Richardson, along with the catchy short rhyme describing the process. Then followed Kolmogorov's footnote, cited previously. Most current texts address the traditional perception of the process cascade as the accepted mode of wake decay. A very recent example is in Physics Today[32]. However, in 1961, Gilles Corcos suggested that an inverse cascade (small-scale to large-scale) could be possible[33]. Shortly thereafter, Cecil E. Leith argued that turbulence originating in a two-dimensional configuration would exhibit the properties of the inverse cascade[34] and that this was, in fact, the cause of clear air turbulence. Unfortunately, Leith dropped this promising avenue of research to focus on the computer simulation of macroscale atmospheric circulation. However, Leith was certainly influenced by the work of Victor Starr, who invoked the "negative viscosity" to describe this and other phenomena[35]. His work on the accuracy of weather prediction certainly strongly influenced Leith, who worked at NCAR in this area.

Confronted by numerous examples of what appeared to be the inverse cascade, Meunier & Spedding[36] devised an extremely simplistic "model" and experiment. He proposed that, when two counterrotating eddies encounter one another in a turbulent flow of two-dimensional eddies, the eddies merge to form a larger eddy if their osculating boundaries are moving in the same direction. But when their osculating boundaries are flowing in opposite directions, the eddies repel each other. He demonstrated this by introducing eddies in two parallel channels, then having them flow past a point where the separating barrier ended and the two flows merged. The two eddies behaved as predicted.

Perhaps the theory could be applied to a random assembly of eddies representing a given turbulence spectrum. An excellent review of the two possibilities is given in this reference [Meunier and Spedding, 2004]. A recent text also did an interesting analysis of the phenomenon, even including a discussion of the relevance of helicity[37].

More recently, Linden and Sutherland have developed a nonlinear formalism for producing spectrally low wave numbers from large wave numbers[38]. In a striking laboratory demonstration, Sutherland and his students showed that a small-scale, grid-produced turbulence field radiated coherent internal waves into a thermocline directly adjacent to this field[39]. Recent astrophysical visual imagery also indicates the possibility of an inverse cascade.[40] So, there is both experimental and theoretical evidence that an inverse cascade exists, as well as hydrodynamic modeling, which produces a transformation of smaller to larger waves in the turbulence energy spectrum.

Because of the importance of this issue in remote sensing and surveillance, it is important to ask the following questions: Is the inverse cascade an anomaly or is it a natural feature of naturally occurring turbulence? Are there properties of the overall flow field that can reverse the cascade, or are there conditions, which when varied, can produce either cascade?

Although there have been numerous publications on fossil turbulence, and more recently on related concepts, including the inverse cascade concept,[41] views on the inverse cascade were implicit in some earlier work. With respect to the energy cascade, a special study of wake hydrodynamics provided the following as the "conventional" view[42].

> *"Turbulence always starts at large scales and cascades to small; for example, by the gravitational collapse of KH [Kelvin-Helmholtz] billows which form with small or no embedded small-scale turbulence."*

This, of course, is but one example of the general statement of the normal cascade given previously, but one that is opposed by another version, namely

> *"Turbulence always starts at small scales and cascades to large. Consider jets, wakes, boundary layers and mixing layers. Tile tube KH billows and wing tip vortices are optical illusions of large scale turbulent eddies that have formed prior to the formation of small scale eddies. Both of these structures contain rolled up turbulent boundary layers temporarily inhibited in the radial directions of the rolls by Coriolis forces. They don't collapse."*

Both of these arguments are semi-quantitative, and are based largely on the observation of the production of larger vortices by the paring of smaller vortices in turbulent flow, similar to the concepts cited previously. This latter view has been controversial among some Western observers, since it lacked a rigorous mathematical basis and is contradictory to the conventional view of turbulence in vogue since the work of Kolmogorov became known in the West after World War II. Most of this viewpoint rests with oceanographers. But in atmospheric research, the investigators who follow Leith and others argue that the inverse cascade is most significant, while others[43] have tried to show that the traditional cascade can occur as well.

For this discussion, the very nature of the ocean itself must be taken into consideration. Wunsch[44] has pointed out that when addressing ocean turbulence, we must realize that there are some vastly different physical ranges involved. At very small scales where stratification is unimportant, Kolmogorov-like arguments apply. At a larger vertical scale where stratification is important (which includes the range where internal waves occur), the so-called “wave turbulence” may occur[45]. Then, there are balanced scales where rotation is important, both with and without stratification, and, of course, there can be intermediate ranges.

Helicity, as it relates to present conventions in the treatment of turbulence, was first examined by Betchov[46]. More recent work by Moiseev *et al* [47,48,49] has provided a wider perspective on the issue and has supplied the mathematical basis for the inverse cascade mechanism. The key insight in the theory of Moiseev *et al* is the interaction between the *helicity* of the turbulent flow and some mechanism of *symmetry breaking* supplied by the environment. When both helicity and symmetry breaking are present in a flow, the inverse cascade is operative, and small turbulent structures can lead to larger structures. Helicity alone or symmetry breaking alone will not produce that result, and the conventional view of turbulence then governs. In particular, homogeneous isotropic turbulence follows the traditional cascade from large structures to small structures to thermal dissipation.

However, the ocean and atmosphere have multiple mechanisms for generating helical flows and for breaking symmetry, and so the conditions can be met in the real world, as it were, for supporting an inverse cascade. In fact, initial attempts have been made to directly measure helicity in the atmosphere, but the experimental sensors required are sophisticated, and difficult to field[50]. It is not clear how similar measurements could be carried out in the ocean.

The method of Moiseev *et al.* begins with the Navier-Stokes equations and the equation of continuity. Unlike the common Reynolds averaging approach that partitions velocity and density fields into steady and fluctuating components, Moiseev *et al.* partition the velocity and density fields into *three* components: a large-scale mean field, a small-scale turbulent field, and a large-scale fluctuating field. The small-scale turbulent field is assumed to be specified statistically by spatio-temporal correlation functions. The Navier-Stokes equations are then used to derive separate differential equations for the large-scale mean field and the large-scale fluctuating field by averaging over the small scales. Statistical moments of the turbulent field, as well as statistical moments of products of the turbulent field and the large-scale fluctuating field, appear as terms in these equations for the large-scale fields.

Closure of these sets of equations involves two steps: (1) Ensemble averaged products of the turbulent field and the large-scale fluctuating field can be related to the statistical correlation functions of the turbulent field alone through a procedure due to Furutsu;[51] and (2) The statistical correlation functions of the turbulent field can be expressed in terms of ensemble averaged characteristics of the turbulent field.

A critical feature of this approach is the recognition by Moiseev *et al.* that the turbulent field is not necessarily invariant under reflection of coordinates. In particular, a turbulent velocity field with non-zero helicity has properties that are absent in homogeneous isotropic turbulence. The two-point spatial correlation function for the turbulent velocity field can be written as

$$\langle v_i(r_1)v_j(r_2)\rangle \equiv K_{ij}(r_1, r_2) = C(r)\,\delta_{ij} + B(r)\,r_i r_j + G(r)\,\varepsilon_{ijk}\,r_k,$$

where $r = |r_1 - r_2|$ and ε_{ijk} is the Levi-Cevita anti-symmetric tensor. C(r) and B(r) are scalar functions of r, whereas G(r) is a pseudo-scalar function, expressing the lack of reflection symmetry of the turbulent velocity field. In particular, G(0) is proportional to the ensemble average helicity density of the field.

$$G(0) \sim \langle \mathbf{v}(\mathbf{r}) \,.\, \nabla \times \mathbf{v}(\mathbf{r})\rangle.$$

Despite the presence of the helicity term in the correlation function, Krause & Rudiger[52] showed that there is no net effect on the averaged equations of large-scale motion if the turbulent motion is both incompressible and isotropic. However, if either condition is dropped, i.e., if the fluid is compressible or the isotropic symmetry is broken, the equations of large-scale motion must be modified. Moiseev and his associates recognized the importance of these conditions, and have analyzed several examples of the effects of compressibility and of symmetry breaking due to Coriolis forces, thermal stratification, and density stratification. Stratification is a normally encountered phenomenon in the ocean.

When symmetry is broken, the equation for mean vorticity, $\mathbf{\Omega}$, of the large-scale flow can be written

$$\partial\mathbf{\Omega}/\partial t - \alpha\, \nabla \times \mathbf{\Omega} = \nu\, \nabla^2\, \mathbf{\Omega},$$

where $\alpha \sim \langle \mathbf{v}(\mathbf{r}) \,.\, \nabla \times \mathbf{v}(\mathbf{r})\rangle$ is the helicity coefficient and $\nu \sim \langle \mathbf{v}(\mathbf{r})\, \mathbf{v}(\mathbf{r})\rangle$ is the turbulent viscosity coefficient. The presence of the term with $\alpha \neq 0$ differentiates this equation from the conventional diffusion equation for vorticity. The spatial Fourier components of the vorticity obey a related equation:

$$d\mathbf{\Omega}/dt - i\,\alpha\,\mathbf{k}\times\mathbf{\Omega} = -\,\nu\,k^2\,\mathbf{\Omega},$$

where k is the wavenumber.

Expressing the time dependence of the vorticity components as $\mathbf{\Omega} = \mathbf{\Omega}_0\, e^{\gamma t}$ leads to a cubic eigenvalue equation for the growth rate γ. The three roots of the eigenvalue equation are

$$\gamma_1 = -\,\nu\,k^2$$

$$\gamma_2 = -\,\nu\,k^2 - \alpha\,k$$

$$\gamma_3 = -\,\nu\,k^2 + \alpha\,k.$$

For real positive values of k, the first two eigenvalues are always negative, corresponding to stable, damped solutions. However, for some combinations of k, α, and ν, γ_3 can be positive, corresponding to an unstable, exponentially growing solution.

For fixed α and ν, the system is unstable for

$$0 < k < \alpha/\nu = \langle \mathbf{v}(\mathbf{r})\,.\,\nabla\times\mathbf{v}(\mathbf{r})\rangle / \langle \mathbf{v}(\mathbf{r})\,\mathbf{v}(\mathbf{r})\rangle.$$

Since $k = 2\pi/L$, this criterion shows that a large-scale disturbance with characteristic scale length L will grow exponentially when

$$L > 2\pi\nu/\alpha.$$

The maximum growth rate

$$\gamma_{max} = \alpha^2/4\nu$$

occurs for large-scale disturbances with scale length

$$L^* = 4\pi\nu/\alpha.$$

The fact that the maximum growth rate is proportional to the square of α emphasizes the importance of the helicity in determining the growth of this type of instability.

The results of Moiseev *et al.* have implications for at least two phenomena proposed by Gibson and others; zombie turbulence and the swirl wake.

The theory of fossil turbulence and its resurrected offshoot, zombie turbulence, was developed by Gibson and others over a number of years. Fossil turbulence is a ubiquitous phenomenon in the oceans, and zombie turbulence is thought to provide a mechanism whereby information from late jets and late wakes in the deep ocean can be transmitted efficiently to the ocean surface. The existence of the inverse cascade from small-scale turbulence to large-scale turbulence is an integral component of that theory.

It should be noted that in some earlier papers, "Zombie turbulence" is described without giving it that particular name. The name of "Zombie Turbulence" was assigned during a review written by H. Yamazaki. The editor was displeased with Yamazaki's review of that particular paper, and did not allow the term to be used in the review; but from that time on, the term has been used. (with the consent of Yamazaki)

The work of Moiseev provides a sound mathematical basis for the inverse cascade. Contrary to the previous assertion that: "Turbulence *always* starts at large scales and cascades down to small scales", Moiseev *et al.* find that the turbulence cascade can go either way, and they specify the conditions under which the inverse cascade exists. They show that exponential growth of large-scale disturbances, driven by small-scale turbulence, can occur when:

- There is a non-zero value of mean helicity in the turbulent flow; *and*

- A symmetry-breaking mechanism is present in the flow or in the environment.

- The mean turbulent helicity, the mean turbulent viscosity, and the wavenumber of the large-scale disturbance satisfy a specific quantitative relationship.

Under other conditions, large-scale structures are damped and the traditional cascade from large-scale to small-scale applies.

If nothing else, understanding this distinction and the mathematics behind it should assuage some critics who may have felt that the traditional viewpoint had to be universally correct. In the real environment, there are almost always enough sources of helicity in geophysical fluids and enough symmetry-breaking mechanisms to make the inverse cascade an almost universal phenomenon in the ocean, atmosphere and space[53] as well.

It is important to recognize that these effects suggested by the results of Moiseev *et al.* may not be the only consequences, or even the principal consequences, of these mechanisms. However, good scientific practice suggests that these effects be investigated individually before considering more complex interactions of the hydrodynamic flow with other components or with the oceanic environment. Nevertheless, there are now persuasive theoretical arguments for the inverse cascade.

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