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The fluid mechanics of dark matter formation

Why does Jeans's (1902 & 1929) theory fail?

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Abstract. Jeans's (1902 & 1929) gravitational instability criterion gives truly spectacular errors in its predictions of cosmological structure formation according to Gibson's (1996) new theory. It is suggested that the linear perturbation stability analysis leading to the acoustic Jeans length scale criterion $L \geq L_J \equiv V_s/(\rho G)^{1/2}$ is quite irrelevant to gravitational condensation, which is intrinsically nonlinear and nonacoustic. Instead, condensation is limited by viscous or turbulent forces, or by diffusivity, at Schwarz length scales L_{SV}, L_{ST}, or L_{SD}, respectively, whichever is larger, independent of L_J and the sound speed V_s. By these new criteria, cosmological structure formation begins in the plasma epoch soon after matter dominates energy, at $L \approx L_{SV} \equiv (\gamma\nu/\rho G)^{1/2}$ scales corresponding to protosuperclusters, decreasing to protogalaxies at the plasma-gas transition, where γ is the rate-of-strain of the expanding universe, ν is the kinematic viscosity, ρ is the density, and G is Newton's gravitational constant. Condensation of the primordial gas occurs at mass scales a trillion times less than the Jeans mass to form a 'fog' of micro-brown-dwarf (MBD) particles that persist as the galactic baryonic dark matter, as reported by Schild (1996) from quasar-microlensing studies. Nonbaryonic dark matter condensation is prevented by its enormous diffusivity at scales smaller than $L_{SD} \equiv (D^2/\rho G)^{1/2}$, where D is the diffusivity, so it forms outer halos of galaxies, cluster-halos of galaxy clusters, and supercluster-halos.

Key words: Cosmology:theory – dark matter – early Universe – gravitational lensing – large-scale structure formation of the Universe

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1. Introduction

Jeans's theory fails because it is linear. Linear theories typically fail when applied to nonlinear processes; for example, applications of the linearized Navier-Stokes equations to problems of fluid mechanics give laminar solutions, but observations show that actual flows are turbulent when the Reynolds number is large.

Cosmology (e.g.; Peebles 1993, Padmanabhan 1993) relies exclusively on Jeans's theory in modelling the formation of structure by gravitational instability. Because L_J for baryonic (ordinary) matter in the hot plasma epoch following the Big Bang is larger than the Hubble scale of causality $L_H \equiv ct$, where c is the speed of light and t is the time, no baryonic structures can form until the cooling plasma forms neutral gas. Star and galaxy formation models invented to accommodate both Jeans's theory and the observations of early structure formation have employed a variety of innovative maneuvers and concepts. Nonbaryonic "cold dark matter" was invented to permit nonbaryonic condensations in the plasma epoch with gravitational potential wells that could guide the early formation of baryonic galaxy masses. Fragmentation theories were proposed to produce $M_{\odot}$-stars rather than Jeans-superstars at the $10^5 M_{\odot}$ proto-globular-cluster Jeans mass of the hot primordial gas.

Both of these concepts have severe fluid mechanical difficulties according to the Gibson 1996 theory. Cold dark matter cannot condense at the galactic scales needed because its nonbaryonic, virtually collisionless, nature requires it to have an enormous diffusivity with supergalactic L_{SD} length scales. Fragmentation theories (e.g., Low and Lynden-Bell 1976) are based on a faulty condensation premise that implies large velocities and a powerful turbulence regime that would produce a first generation of large stars with minimum mass determined by the turbulent Schwarz scale $L_{ST} \equiv \varepsilon^{1/2}/(\rho G)^{3/4}$, where ε is the viscous dissipation rate of the turbulence, with a flurry of starbursts, supernovas, and metal production that is not observed in globular star clusters. The population of small, long-lived, metal-free, globular cluster stars observed is strong evidence of a quiet, weakly turbulent formation regime.

2. Jeans's acoustic theory

Jeans considered the problem of gravitational condensation in a large body of nearly constant density, nearly motionless gas. Viscosity and diffusivity were ignored. The density and momentum conservation equations were linearized by dropping second order terms after substituting mean plus fluctuating values for the density, pressure, gravitational potential, and velocity. Details of the derivation are given in many cosmological texts (e.g.;

Kolb and Turner 1994, p342) so they need not be repeated here. The mean gravitational force $\nabla\phi$ is assumed to be zero, violating the Poisson equation

$$\nabla^2\phi = 4\pi G\rho, \tag{1}$$

where ϕ is the gravitational potential, in what is known as the Jeans swindle. Cross-differentiating the linearized perturbation equations produces a single, second order differential equation satisfied by Fourier modes propagating at the speed of sound V_s. From the dispersion equation

$$\omega^2 = V_s^2 k^2 - 4\pi G\rho, \tag{2}$$

where ω is the frequency and k is the wavenumber, a critical wavenumber $k_J = (4\pi G\rho/V_s^2)^{1/2}$ exists, called the *Jeans wavenumber.* For k less than k_J, ω is imaginary and the mode grows exponentially with time. For k larger than k_J, the mode is a propagating sound wave. Density was assumed to be a function only of pressure (the barotropic assumption).

Either the barotropic assumption or the linearization of the momentum and density equations are sufficient to reduce the problem to one of acoustics. Physically, sound waves provide density nuclei at wavecrests that can trigger gravitational condensation if their time of propagation λ/V_s for wavelength λ is longer than the *gravitational free fall time* $\tau_g \equiv (\rho G)^{-1/2}$. Setting the two times equal gives the Jeans gravitational instability criterion: gravitational condensation occurs only for $\lambda \geq L_J$.

Jeans's analysis fails to account for the effects of gravity, diffusivity, or fluid mechanical forces upon nonacoustic density maxima and density minima; that is, points surrounded on all sides by either lower or higher density. These move approximately with the fluid velocity, not V_s, (Gibson 1968). The evolution of such *zero gradient points* and associated *minimal gradient surfaces* is critical to turbulent mixing theory (Gibson et al. 1988). Turbulence scrambles passive scalar fields such as temperature, chemical species concentration and density to produce nonacoustic extrema, saddle points, doublets, saddle lines and minimal gradient surfaces. A quasi-equilibrium develops between convection and diffusion at such zero gradient points and minimal gradient surfaces that is the basis of a universal similarity theory of turbulent mixing (Gibson 1991) analogous to the universal similarity theory of Kolmogorov for turbulence. Just as turbulent velocity fields are damped by viscosity at the Kolmogorov length scale $L_K \equiv (\nu/\gamma)^{1/2}$, where ν is the kinematic viscosity and γ is the rate-of-strain, scalar fields like temperature are damped by diffusivity at the Batchelor length scale $L_B \equiv (D/\gamma)^{1/2}$, where D is the molecular diffusivity. This prediction has been confirmed by laboratory experiments and numerical simulations (Gibson et al. 1988) for the range $10^{-2} \leq Pr \leq 10^5$, where the Prandtl number $Pr \equiv \nu/D$.

On cosmological length scales, density fields scrambled by turbulence are not necessarily dynamically passive but may respond to gravitational forces. In the density conservation equation

$$\partial\rho/\partial t + v_i(\partial\rho/\partial x_i) = D_{\text{eff}}\partial^2\rho/\partial x_j\partial x_j \tag{3}$$

the effective diffusivity of density $D_{\text{eff}} \equiv D - L^2/\tau_g$ is affected by gravitation in the vicinity of minimal density gradient features, and reverses its sign to negative if the feature size L is larger than the diffusive Schwarz scale L_{SD} (Gibson and Schild 1998a). $L_{SD} \equiv (D^2/\rho G)^{1/4}$ is derived by setting the diffusive velocity $v_D \approx D/L$ of an isodensity surface a distance L from a minimal gradient configuration equal to the gravitational velocity $v_g \approx L/\tau_g$. Thus, nonacoustic density maxima in a quiescent, otherwise homogeneous, fluid are absolutely unstable to gravitational condensation, and nonacoustic density minima are absolutely unstable to void formation, on scales larger than L_{SD}.

Jeans believed from his analysis (Jeans 1929) that sound waves with $\lambda \geq L_J$ would grow in amplitude indefinitely, producing unlimited kinetic energy from his gravitational instability. This is clearly incorrect, since any wavecrest that collects a finite quantity of mass from the ambient fluid will also collect its zero momentum and become a nonacoustic density nucleus. From the enormous Jeans mass values indicated at high temperature, he believed he had proved his speculation that the cores of galaxies consisted of hot gas (emerging from other Universes!) and not stars, which could only form in the cooler (smaller L_J) spiral arms, thrown into cold outer space by centrifugal forces of the spinning core. The concepts of *pressure support* and *thermal support* often used to justify Jeans's theory are good examples of bad dimensional analysis, lacking any proper physical basis.

3. Fluid mechanical theory

Gravitational condensation on a nonacoustic density maximum is limited by either diffusion or by viscous, magnetic or turbulent forces at diffusive, viscous, magnetic, or turbulent Schwarz scales L_{SX}, whichever is largest, where X is D, V, M, T, respectively (Gibson 1996, Gibson and Schild 1998a). Magnetic forces are assumed to be unimportant for the cosmological conditions of interest. Gravitational forces $F_g \approx \rho^2 G L^4$ equal viscous forces $F_V \approx \rho\nu\gamma L^2$ at $L_{SV} \equiv (\nu\gamma/\rho G)^{1/2}$, and turbulent forces $F_T \approx \rho(\varepsilon)^{2/3}L^{8/3}$ at $L_{ST} \equiv \varepsilon^{1/2}/(\rho G)^{3/4}$. Kolmogorov's theory is used to estimate the turbulent forces as a function of length scale L.

The criterion (Gibson 1996, 1997a, 1997b; Gibson and Schild 1998a, 1998b) for gravitational condensation or void formation at scale L is therefore

$$L \geq (L_{SX})_{\max}\,; X = D, V, M, T. \tag{4}$$

4. Structures in the plasma epoch

Without the Jeans constraint, structure formation begins in the early stages of the hot plasma epoch after the Big Bang when decreasing viscous forces first permit gravitational decelerations and sufficient time has elapsed for the information about density variations to propagate; that is, the decreasing viscous Schwarz scale L_{SV} becomes smaller than the increasing Hubble scale $L_H \equiv ct$, where c is the velocity of light. Low levels of temperature fluctuations of the primordial gas indicated by the COsmic microwave Background Experiment (COBE) satellite ($\delta T/T \approx 10^{-5}$) constrain the velocity fluctuations $\delta v/c \ll 10^{-5}$ to levels of very weak turbulence. Setting the observed mass of superclusters $\approx 10^{46}$ kg equal to the Hubble mass ρL_H^3 computed from Einstein's equations (Weinberg 1972, Table 15.4) indicates the time of first structure was $\approx 10^{12}$ s, or 30 000 y (Gibson 1997b).

Setting $L_H \approx 3\,10^{20}$ m (10 kpc) $= L_{SV}$ gives $\nu \approx 6\,10^{27}\ \mathrm{m^2\,s^{-1}}$ with $\rho \approx 10^{-15}\ \mathrm{kg\,m^{-3}}$ and $\gamma \approx 1/t = 10^{-12}\ \mathrm{rad\,s^{-1}}$. Such a large viscosity suggests a neutrino-electron-proton coupling mechanism, presumably through the Mikheyev-Smirnov-Wolfenstein (MSW) effect (Bahcall 1997), supporting the Neutrino-98 claim that neutrinos have mass.

The viscous condensation mass ρL_{SV}^3 decreases to about 10^{42} kg (Gibson 1996) as the Universe expands and cools to the plasma-gas transition at $t \approx 10^{13}$ s, or 300 000 y, based on Einstein's equations to determine T and ρ and assuming the usual dependence of viscosity ν on temperature T (Weinberg 1972). Assuming gravitational decelerations that are possible always occur, we see that protosupercluster, protocluster, and protogalaxy structure formation should be well underway before the emergence of the primordial gas.

5. Primordial fog formation

The first condensation scales of the primordial gas mixture of hydrogen and helium are the maximum size Schwarz scale, and an initial length scale $L_{IC} \equiv (RT/\rho G)^{1/2}$ equal to the Jeans scale L_J but independent of Jeans's linear perturbation stability analysis, and acoustics, where R is the gas constant of the mixture. From the ideal gas law $p/\rho = RT$ we see that density increases can be compensated by pressure increases with no change in temperature in a uniform temperature gas, and that gravitational forces $F_g \approx \rho^2 G L^4$ will dominate the resulting pressure gradient forces $F_p \approx pL^2 = \rho RTL^2$ for length scales $L \geq L_{IC}$. Taking $R \approx 5\,000\ \mathrm{m^2\,s^{-2}\,K^{-1}}$, $\rho \approx 10^{-18}\ \mathrm{kg\,m^{-3}}$ m (Weinberg 1972) and $T \approx 3\,000$ K gives a condensation mass $\rho L_{IC}^3 \approx 10^5 M_\odot$, the mass of a globular cluster of stars. Because the temperature of the primordial gas was observed to be quite uniform by COBE, we can expect the protogalaxy masses of primordial gas emerging from the plasma epoch to immediately fragment into proto-globular-cluster (PGC) gas objects on $L_{IC} \approx 3\,10^{17}$ m (10 pc) scales, with subfragments at $(L_{SX})_{\max}$.

The kinematic viscosity ν of the primordial gas mixture decreased by a factor of about a trillion from plasma values at transition, to $\nu \approx 2.4\,10^{12}$ $\mathrm{m^2\,s^{-1}}$ assuming the density within the PGC objects are about 10^{-17} $\mathrm{kg\,m^{-3}}$. Therefore, the viscous Schwarz scale $L_{SV} \approx (2.4\,10^{12}\,10^{-13}/10^{-17}\,6.7\,10^{-11})^{1/2} = 1.9\,10^{13}$ m, so the viscous Schwarz mass $M_{SV} \approx L_{SV}^3 \rho = 6.8\,10^{22}$ kg, or $M_{SV} = 6.8\,10^{24}$ kg using $\rho = 10^{-18}$. The turbulent Schwarz mass $M_{ST} \approx 8.8\,10^{22}$ kg assuming 10% of the COBE temperature fluctuations are due to turbulent red shifts ($[(\delta v/c)/(\delta T/T)] = 10^{-1}$) as a best estimate.

We see that the entire universe of primordial H-He gas turned to fog soon after the plasma-gas transition, with primordial fog particle (PFP) mass values in the range 10^{23} to 10^{25} kg depending on the estimated density and turbulence levels of the gas. The time required to form a PFP is set by the time required for void regions to grow from minimum density points and maximum density saddle points to surround and isolate the condensing PFP objects (Gibson and Schild 1998a). Voids grow as rarefaction waves with a limiting maximum velocity V_s set by the second law of thermodynamics, so the minimum PFP formation time is $\tau_{PFP} \leq (L_{SX})_{\max}/V_s$, or about 10^3 y. Full condensation of the PFP to form a dense core near hydrostatic equilibrium requires a much longer time, near the gravitational free fall time $\tau_g \approx 2\,10^6$ y.

Radiation heat transport during the PFP condensation period before the creation of dust should have permitted cooling to temperatures near those of the expanding universe. After about a billion years hydrogen dew point and freezing point temperatures (20-13 K) would be reached, forming the micro-brown-dwarf conditions expected for these widely separated ($10^3 - 10^4$ AU) small planetary objects that comprise most of the baryonic dark matter of the present universe and the materials of construction for the stars and heavy elements. Because such frozen objects occupy an angle of less than a micro-arcsecond viewed from their average separation distance, they are invisible to most observations except by gravitational microlensing, or if a nearby hot star brings these volatile comets out of cold storage.

6. Observations

A variety of observations confirm the new theory that fluid mechanical forces and diffusion limit gravitational condensation (Gibson 1996), and confute Jeans's (1902 & 1929) acoustic criterion:

- quasar microlensing at micro-brown-dwarf frequencies (Schild 1996),
- tomography of dense galaxy clusters indicating diffuse (nonbaryonic) superhalo dark matter at L_{SD} scales with $D_{nb} \approx 10^{28}\mathrm{m^2\,s^{-1}}$ (Tyson and Fischer 1995),
- the Gunn-Peterson missing gas sequestered as PFPs,
- the dissipation of 'gas clouds' in the $Ly-\alpha$ forest,

– extreme scattering events, cometary globules, FLIERS, ansae, Herbig-Haro 'chunks', etc.

Evidence that the dark matter of galaxies is dominated by small planetary mass objects has been accumulating from reports of many observers that the multiple images of lensed quasars twinkle at corresponding high frequencies. After several years spent resolving a controversy about the time delay between images Q0957+561A,B, to permit correction for any intrinsic fluctuations of the light intensity of the source by subtraction of the properly phased images, Schild 1996 announced that the lensing galaxy mass comprises $\approx 10^{-6} M_{\odot}$ "rogue planets" that are "likely to be the missing mass."

Star-microlensing studies from the Large Magellanic Cloud have failed to detect lensing at small planetary mass frequencies, thus excluding this quasar-microlensing population as the Galaxy halo missing mass (Alcock et al. 1998, Renault et al. 1998). However, the exclusion is based on the unlikely assumption that the number density of such small objects is uniform. The population must have mostly primordial gas composition since no cosmological model predicts this much baryonic mass of any other material, and must be primordial since it constitutes the material of construction, and an important stage, in the condensation of the gas to form stars. Gravitational aggregation is a nonlinear, self-similar, cascade process likely to produce an extremely intermittent lognormal spatial distribution of the PFP number density, with mode value orders of magnitude smaller than the mean. Since star-microlensing from a small solid angle produces a small number of independent samples, the observations estimate the mode rather than the mean, resolving the observational conflict (Gibson and Schild 1998b).

7. Conclusions

1. Jeans's gravitational instability criterion $L \geq L_J$ is irrelevant to gravitational structure formation in cosmology and astrophysics, and is egregiously misleading in all of its applications.
2. The correct criterion for gravitational structure formation is that L must be larger than the largest Schwarz scale; that is, $L \geq (L_{SX})_{\max}$, where X is D, V, M, T, depending on whether diffusion or viscous, magnetic or turbulent forces limit the gravitational effects.
3. Structure formation began in the plasma epoch with protosupercluster to protogalaxy decelerations.
4. Gravitational condensations began soon after the plasma-gas transition, forming micro-brown-dwarfs, clustered in PGCs, that persist as the dominant dark matter component of inner galactic halos (50 kpc).

5. The present fluid mechanical theory and its cosmological consequences regarding the forms of baryonic and nonbaryonic dark matter (Gibson 1996) is well supported by observations, especially the quasar-microlensing of Schild 1996 and his inference that the lens galaxy mass of Q0957+561A,B is dominated by small rogue planets (interpreted here as PFPs).
6. Star-microlensing studies that rule out MBDs as the Galaxy missing mass (Alcock et al. 1998, Renault et al. 1998), contrary to the quasar-microlensing evidence and the present theory, are subject to extreme undersampling errors from their unwarranted assumption of a uniform number density distribution, rather than extremely intermittent lognormal distributions expected from nonlinear aggregational cascades of such small objects as they form nested clusters and stars (Gibson and Schild 1998b).

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