Marginal evidence for cosmic acceleration from Type Ia supernovae

Jeppe Trøst Nielsen[1], Alberto Guffanti[1], and Subir Sarkar[1,2]
[1] Niels Bohr International Academy, Blegdamsvej 17, Copenhagen 2100, Denmark and
[2] Rudolf Peierls Centre for Theoretical Physics, 1 Keble Road, Oxford OX1 3NP, UK
(Dated: June 10, 2015)

The 'standard' model of cosmology is founded on the basis that the expansion rate of the universe is accelerating at present — as was inferred originally from the Hubble diagram of Type Ia supernovae. There exists now a much bigger database of supernovae so we can perform rigorous statistical tests to check whether these 'standardisable candles' indeed indicate cosmic acceleration. Taking account of the empirical procedure by which corrections are made to their absolute magnitudes to allow for the varying shape of the light curve and extinction by dust, we find, rather surprisingly, that the data are still quite consistent with a constant rate of expansion.

According to HGD cosmology, there should be NO accelerated expansion rate. CHG

I. INTRODUCTION

Not true from HGD.

In the 1990's, studies of Type Ia supernovae (SN Ia) showed that the expansion rate of the universe appears to be accelerating as if dominated by a cosmological constant [1, 2]. Since then supernova cosmology has developed rapidly as an important probe of 'dark energy'. Empirical corrections are made to reduce the scatter in the observed magnitudes by exploiting the observed (anti)correlation between the peak luminosity and the light curve width [3]. Other such correlations have since been found e.g. with the host galaxy mass [4] and metallicity [5]. Cosmological parameters are then fitted, along with the parameters determining the light curves, by simple χ^2 minimisation [1, 6–8]. This method has a number of pitfalls as has been emphasised earlier [9, 10].

With ever increasing precision and size of SN Ia datasets, it is important to also improve the statistical analysis of the data. To accomodate model comparison, previous work [11–13] has introduced likelihood maximisation. In this work we present an improved maximum likelihood analysis, finding rather different results.

II. SUPERNOVA COSMOLOGY

There are several approaches to making SN Ia 'standardiseable candles'. The different philosophies lead to mildly different results but the overall picture seems consistent [14]. In this paper we adopt the transparent approach of SALT2 [15] wherein the SN Ia are standardised by fitting their light curve to an empirical template, and the parameters of this fit are used in the cosmological analysis. Every SN Ia is assigned three parameters, one being m_B^*, the apparent magnitude at maximum (in the rest frame 'B-band'), while the other two describe the light curve shape and colour corrections: x_1 and c. The distance modulus is then assumed to be:

$$\mu_{\rm SN} = m_B^* - M + \alpha x_1 - \beta c, \qquad (1)$$

where M is the absolute magnitude, and α, β are assumed to be constants for *all* SN Ia. These global constants are fitted along with the cosmological parameters. The physical mechanism(s) which give rise to the correlations that underlie these corrections remain uncertain [16]. The SN Ia magnitude as a function of its redshift z is then compared to the expectation in the standard ΛCDM cosmological model:

$$\begin{aligned}
&\mu \equiv 25 + 5\log_{10}(d_{\rm L}/{\rm Mpc}), \quad \text{where:}\\
&d_{\rm L} = (1+z)\frac{d_{\rm H}}{\sqrt{\Omega_k}}\,{\rm sinn}\left(\sqrt{\Omega_k}\int_0^z \frac{H_0 {\rm d}z'}{H(z')}\right),\\
&d_{\rm H} = c/H_0, \quad H_0 \equiv 100h\ {\rm km\,s^{-1}Mpc^{-1}},\\
&H = H_0\sqrt{\Omega_{\rm m}(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda}, \qquad (2)
\end{aligned}$$

where $d_{\rm L}, d_{\rm H}, H$ are the luminosity distance, Hubble distance and Hubble parameter respectively, and $\Omega_{\rm m}, \Omega_\Lambda, \Omega_k$ are the matter, cosmological constant and curvature density in units of the critical density, with sinn $\to$ sinh for $\Omega_k > 0$ and sinn $\to$ sin for $\Omega_k < 0$ [2]. There is a degeneracy between H_0 and M_0 so we fix the value of the Hubble parameter today to $h = 0.7$ which is consistent with independent measurements.

III. MAXIMUM LIKELIHOOD ESTIMATORS

To find the maximum likelihood estimator (MLE) from the data, we must define the appropriate likelihood:

$$\mathcal{L} = \text{probability density(data|model)},$$

i.e. we have to first specify our model of the data. For a given SN Ia, the true data (m_B^*, x_1, c) is drawn from some global distribution. These values are contaminated by various sources of noise, yielding the observed values $(\hat{m}_B^*, \hat{x}_1, \hat{c})$. Assuming the SALT2 model is correct, only the true values obey the relation (1). However when the experimental uncertainty is of the same order as the intrinsic variance as in the present case, the observed value is *not* a good estimate of the true value. Parametrising the cosmological model by θ, the likelihood function can be written as

$$\begin{aligned}
\mathcal{L} &= p[(\hat{m}_B^*, \hat{x}_1, \hat{c})|\theta] \qquad (3)\\
&= \int p[(\hat{m}_B^*, \hat{x}_1, \hat{c})|(M, x_1, c), \theta], p[(M, x_1, c)|\theta]{\rm d}M{\rm d}x_1{\rm d}c,
\end{aligned}$$

showing explicitly where the experimental uncertainties enter (first term) and where the variances of the intrinsic distributions enter (second term).

Having a theoretically well-motivated distribution for the light curve parameters would be helpful, however this is not available. For simplicity we adopt global, independent gaussian distributions for all parameters, M, x_1 and c, i.e. model their probability density as:

$$
\begin{aligned}
&p[(M,x_1,c)|\theta] = p(M|\theta)p(x_1|\theta)p(c|\theta), \quad \text{where:}\\
&p(M|\theta) = (2\pi\sigma_{M_0}^2)^{-1/2}\exp\left\{-\left[(M-M_0)/\sigma_{M_0}\right]^2/2\right\},\\
&p(x_1|\theta) = (2\pi\sigma_{x_0}^2)^{-1/2}\exp\left\{-\left[(x_1-x_0)/\sigma_{x_0}\right]^2/2\right\},\\
&p(c|\theta) = (2\pi\sigma_{c_0}^2)^{-1/2}\exp\left\{-\left[(c-c_0)/\sigma_{c_0}\right]^2/2\right\}. \qquad (4)
\end{aligned}
$$

All 6 free parameters $\{M_0, \sigma_{M_0}, x_0, \sigma_{x_0}, c_0, \sigma_{c_0}\}$ are fitted along with the cosmological parameters. Introducing the vectors $Y = \{M_1, x_{11}, c_1, \dots M_N, x_{1N}, c_N\}$, the zero-points Y_0, and the matrix $\Sigma_l = \mathrm{diag}(\sigma_{M_0}^2, \sigma_{x0}^2, \sigma_{c0}^2, \dots)$, the probability density of the true parameters writes:

$$
p(Y|\theta) = |2\pi\Sigma_l|^{-1/2}\exp\left[-(Y-Y_0)\Sigma_l^{-1}(Y-Y_0)^{\mathrm{T}}/2\right]. \qquad (5)
$$

What remains is to specify the model of uncertainties on the data. Introducing another set of vectors $X = \{m_{B1}^*, x_{11}, c_1, \dots\}$, the observed $\hat{X}$, and the estimated experimental covariance matrix Σ_{d} (including both statistical and systematic errors), the probability density of the data given some set of true parameters is:

$$
p(\hat{X}|X,\theta) = |2\pi\Sigma_{\mathrm{d}}|^{-1/2}\exp\left[-(\hat{X}-X)\Sigma_{\mathrm{d}}^{-1}(\hat{X}-X)^{\mathrm{T}}/2\right]. \qquad (6)
$$

To combine the exponentials we introduce the vector $\hat{Z} = \{\hat{m}_{B1}^* - \mu_{C1}, \hat{x}_{11}, \hat{c}_1, \dots\}$ and the block diagonal matrix

$$
A = \begin{pmatrix} 1 & 0 & 0 & \\ -\alpha & 1 & 0 & 0 \\ \beta & 0 & 1 & \\ & 0 & & \ddots \end{pmatrix}. \qquad (7)
$$

With these, we have $\hat{X} - X = (\hat{Z}A^{-1} - Y)A$ and so $P(\hat{X}|X,\theta) = P(\hat{Z}|Y,\theta)$. The likelihood is then

$$
\begin{aligned}
\mathcal{L} &= \int P(\hat{Z}|Y,\theta)P(Y|Y_0,\theta)\mathrm{d}Y \qquad (8)\\
&= |2\pi\Sigma_{\mathrm{d}}|^{-1/2}|2\pi\Sigma_l|^{-1/2}\int \mathrm{d}Y\\
&\times\exp\left(-(Y-Y_0)\Sigma_l^{-1}(Y-Y_0)^{\mathrm{T}}/2\right)\\
&\times\exp\left(-(Y-\hat{Z}A^{-1})A\Sigma_d^{-1}A^{\mathrm{T}}(Y-\hat{Z}A^{-1})^{\mathrm{T}}/2\right),
\end{aligned}
$$

which can be integrated analytically to obtain:

$$
\begin{aligned}
\mathcal{L} &= |2\pi(\Sigma_{\mathrm{d}} + A^{\mathrm{T}}\Sigma_l A)|^{-1/2} \qquad (9)\\
&\times\exp\left[-(\hat{Z}-Y_0A)(\Sigma_{\mathrm{d}} + A^{\mathrm{T}}\Sigma_l A)^{-1}(\hat{Z}-Y_0A)^{\mathrm{T}}/2\right].
\end{aligned}
$$

This is the likelihood (3) for the simple model (4), and the quantity which we seek to maximise in order to derive confidence limits. The 10 parameters we fit are $\{\Omega_{\mathrm{m}}, \Omega_\Lambda, \alpha, x_0, \sigma_{x0}, \beta, c_0, \sigma_{c0}, M_0, \sigma_{M_0}\}$. We stress that it is necessary to consider all of these *together* and Ω_{m} and Ω_Λ have no special status in this regard. The major advantage of our method is that we get a goodness-of-fit statistic in the likelihood which can be used to compare models or judge whether a particular model is a good fit. Note that the model is not just the cosmology, but includes modelling the distributions of x_1 and c.

With this MLE, we can construct a confidence region in the 10-dimensional parameter space by defining its boundary as one of constant $\mathcal{L}$. So long as we do not cross a boundary in parameter space, this volume will asymptotically have the coverage probability

$$
p_{\mathrm{cov}} = \int_0^{-2\log\mathcal{L}/\mathcal{L}_{\max}} \chi^2(x;\nu)\mathrm{d}x, \qquad (10)
$$

where $\chi^2(x;\nu)$ is a chi-squared distribution with ν degrees of freedom [17], and $\mathcal{L}_{\max}$ is the maximum likelihood. With 10 parameters in the present model, the values $p_{\mathrm{cov}} = 0.68$ (“1σ”), 0.95 (“2σ”) give $-2\log\mathcal{L}/\mathcal{L}_{\max} \simeq 11.54, 18.61$ respectively.

To eliminate the so-called ‘nuisance parameters’, we set similar bounds on the profile likelihood. Writing the interesting parameters as θ and nuisance parameters as ϕ, the profile likelihood is defined as

$$
\mathcal{L}_{\mathrm{p}}(\theta) = \max_\phi \mathcal{L}(\theta,\phi). \qquad (11)
$$

We substitute $\mathcal{L}$ by $\mathcal{L}_{\mathrm{p}}$ in Eq.(10) in order to construct confidence regions in this lower dimensional space; ν is now the dimension of the remaining parameter space. Looking at the $\Omega_{\mathrm{m}} - \Omega_\Lambda$ plane, we have for $p_{\mathrm{cov}} = \{0.68$ (“1σ”), 0.95 (“2σ”), 0.997 (“3σ”)$\}$ the likelihood values $-2\log\mathcal{L}_{\mathrm{p}}/\mathcal{L}_{\max} \approx \{2.30, 6.18, 11.8\}$ respectively.

A. Comparison to other methods

It is illuminating to relate our work to previously used methods in SN Ia analyses. One method [11] minimises a likelihood, which is written in the case of uncorrelated magnitudes as

$$
\tilde{\mathcal{L}} \propto \prod (2\pi\sigma_{\mathrm{tot}}^2)^{-1/2}\exp\left(-\Delta\mu^2/2\sigma_{\mathrm{tot}}^2\right). \qquad (12)
$$

so it integrates over μ_{SN} to unity and can be used for model comparison. From Eq. (3) we see that this corresponds to assuming *flat* distributions for x_1 and c. However the actual distributions of $\hat{x}_1$ and $\hat{c}$ are close to gaussian, as seen in Fig. 1. Moreover although this likelihood apparently integrates to unity, it accounts for only the m_B^* data. Integration over the x_1, c data demands compact support for the flat distributions so the normalisation of the likelihood becomes arbitrary, making model comparison tricky.

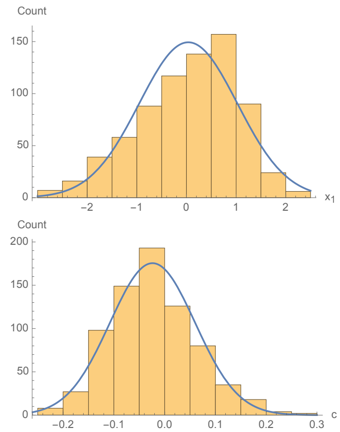


FIG. 1. Distribution of the stretch (x_1) and colour (c) corrections in the JLA sample, with gaussians superimposed.

More commonly used is the 'constrained χ^2' [1, 6]

$$\chi^2 = \sum \Delta\mu^2/(\sigma_\mu^2 + \sigma_{\rm int}^2), \tag{13}$$

but this cannot be used to compare models, since it is *tuned* to be 1 per degree of freedom for the ΛCDM model by adjusting an arbitrary error $\sigma_{\rm int}$ added to each data point. This has been criticised earlier [9, 10], nevertheless the method continues to be widely used and the results presented without emphasising that it is suitable only for parameter estimation for the *assumed* (ΛCDM) model, rather than determining if this is indeed the best model.

IV. ANALYSIS OF JLA CATALOGUE

We focus [18] on the Joint Lightcurve Analysis (JLA) catalogue [8]. Maximisation of the likelihood under specific constraints is summarised in Tab. I. Profile likelihood contours in the $\Omega_{\rm m} - \Omega_\Lambda$ plane are shown in Fig. 2. The Hubble diagram and the residuals with respect to the Milne universe are shown in Figs. 3 and 4 respectively.

To assess how well our model describes the data, we present in Fig. 5 the 'pull' distribution. While the gaussian model is not perfect, it appears to be an adequate first step towards understanding SN Ia standardisation.

To check the validity of our method and approximations, we do a Monte Carlo simulation of experimental outcomes from a model with parameters matching our best fit (see Tab. I). Fig. 6 shows the distribution of $-2\log[\mathcal{L}_{\rm true}/\mathcal{L}_{\rm max}]$, which is just as is expected, showing that our constructed confidence regions can be trusted.

V. DISCUSSION

That the SN Ia Hubble diagram appears consistent with an uniform rate of expansion has been noted earlier

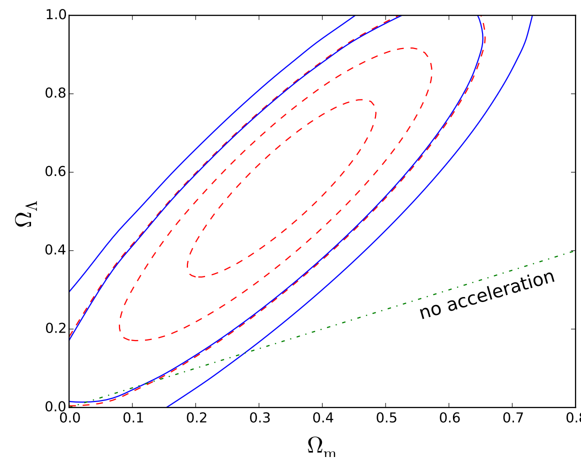


FIG. 2. Contour plot of the profile likelihood in the $\Omega_{\rm m} - \Omega_\Lambda$ plane. 1, 2 and 3σ contours, regarding all other parameters as nuisance parameters, are shown as red dashed lines, while the blue lines are 1 and 2σ contours from the 10-dimensional parameter space projected on to the plane.

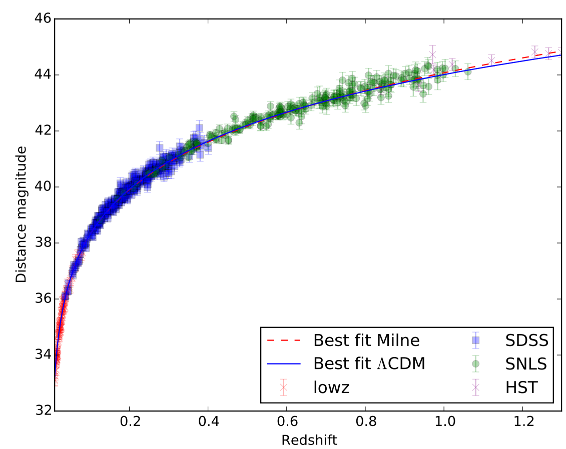


FIG. 3. Comparison of the measured distance magnitudes, $\mu_{\hat{\rm SN}} = \hat{m}_B^* - M_0 + \alpha\hat{x}_1 - \beta\hat{c}$ with the expected value in two cosmological models — 'ΛCDM' is the best fit accelerating universe, and 'Milne' is an empty universe expanding with constant velocity. The error bars are the square root of the diagonal elements of $\Sigma_l + A^{\rm T-1}\Sigma_{\rm d}A^{-1}$, and include both experimental uncertainties and intrinsic dispersion.

[13, 19]. We have confirmed this by a rigorous statistical analysis, using the JLA catalogue of 740 SN Ia processed by the SALT2 method. We find marginal ($< 3\sigma$) evidence for the widely accepted claim that the expansion of the universe is presently accelerating [2].

The Bayesian equivalent of our method (a "Bayesian Hierarchical Model") has been presented in Ref.[10]. In another development, a Bayesian consistency test [20] has

This falsifies both dark energy and LCDMHC cosmology, and supports HGD cosmology of Schild and Gibson (1996).

TABLE I. Maximum likelihood parameters under specific (boldface) constraints. ($-2\log\mathcal{L}_{\max} = -214.97$)

Constraint	$-2\log\mathcal{L}/\mathcal{L}_{\max}$	Ω_m	Ω_Λ	α	x_0	σ_{x_0}	β	c_0	σ_{c_0}	M_0	σ_{M_0}
None (best fit)	**0**	0.341	0.569	0.134	0.038	0.932	3.059	-0.016	0.071	-19.052	0.108
Flat geometry	0.147	0.376	**0.624**	0.135	0.039	0.932	3.060	-0.016	0.071	-19.055	0.108
Empty universe	11.9	**0.000**	**0.000**	0.133	0.034	0.932	3.051	-0.015	0.071	-19.014	0.109
Non-accelerating	11.0	0.068	**0.034**	0.132	0.033	0.931	3.045	-0.013	0.071	-19.006	0.109
Matter-less universe	10.4	**0.000**	0.094	0.134	0.036	0.932	3.059	-0.017	0.071	-19.032	0.109
Einsten-deSitter	221.97	**1.000**	**0.000**	0.123	0.014	0.927	3.039	0.009	0.072	-18.839	0.125

See attached figure.

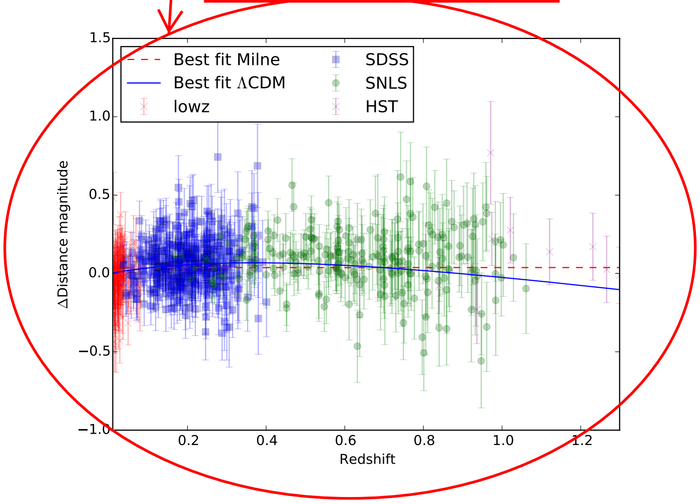


FIG. 4. Residuals relative to the Milne model for Fig. 3.

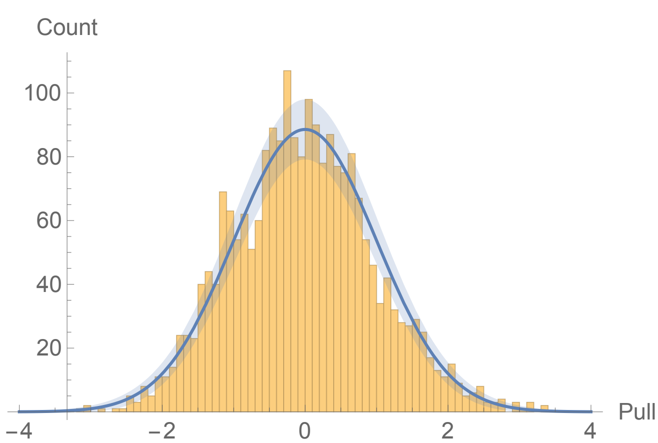


FIG. 5. Distribution of pulls for the best-fit model compared to a normal distribution. The pulls are defined as $(\hat{Z} - Y_0 A)U^{-1}$, where U is the (upper triangular) Cholesky factor of the covariance matrix $\Sigma_d + A^T\Sigma_l A$.

been applied (albeit using the flawed 'likelihood' (12) and 'constrained χ^2' (13) methods) to determine the consistency between the SN Ia data sets acquired with different telescopes [21]. These authors do find inconsistencies in the UNION2 catalogue but none in JLA. This test had been applied earlier to the UNION2.1 compilation finding no contamination, but those authors [22] *fixed* the light curve fit 'nuisance' parameters, so their result is inconclusive. A recent Markov Chain Monte Carlo approach to light curve fitting in the JLA catalogue has revealed that 11 of the SN Ia ought to be discarded [23]. However we find little impact on our results when this is done.

ACKNOWLEDGMENTS

We thank the JLA collaboration for making their data and software public and Marc Betoule for making the corrections we suggested to the catalogue. Adam Riess kindly pointed out that the error bars in Figs. 3 and 4 had been incorrectly plotted. This work was supported by the Danish National Research Foundation through the Discovery Center at the Niels Bohr Institute and the award of a Niels Bohr Professorship to SS.

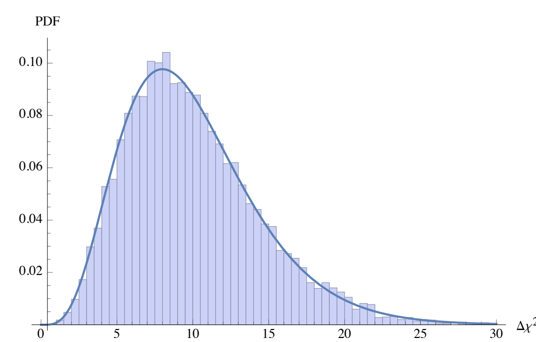


FIG. 6. The distribution of the likelihood ratio from Monte Carlo, with a χ^2 distribution with 10 d.o.f. superimposed.

[1] S. Perlmutter *et al.* (Supernova Cosmology Project), Astrophys.J. **517**, 565 (1999), arXiv:astro-ph/9812133 [astro-ph]; A. G. Riess *et al.* (Supernova Search Team), Astron.J. **116**, 1009 (1998), arXiv:astro-ph/9805201 [astro-ph].
[2] A. Goobar and B. Leibundgut, Ann.Rev.Nucl.Part.Sci. **61**, 251 (2011), arXiv:1102.1431 [astro-ph.CO].
[3] M. Phillips, Astrophys.J. **413**, L105 (1993); R. Tripp,

From HGD cosmology, dark energy is a systematic dimming error of the SNeIa brightness from neglect of refraction effects of the dark matter planets at the Oort cavity boundary surrounding stars in formation within PGC clumps of a trillion earth mass dark matter planets. CHG

Astron.Astrophys. **331**, 815 (1998).
[4] P. L. Kelly, M. Hicken, D. L. Burke, K. S. Mandel, and R. P. Kirshner, Astrophys.J. **715**, 743 (2010), arXiv:0912.0929 [astro-ph.CO].
[5] B. T. Hayden, R. R. Gupta, P. M. Garnavich, F. Mannucci, R. C. Nichol, *et al.*, Astrophys.J. **764**, 191 (2013), arXiv:1212.4848 [astro-ph.CO].
[6] P. Astier *et al.* (SNLS), Astron.Astrophys. **447**, 31 (2006), arXiv:astro-ph/0510447 [astro-ph].
[7] A. J. Conley *et al.* (Supernova Cosmology Project), Astrophys.J. **644**, 1 (2006), arXiv:astro-ph/0602411 [astro-ph]; M. Kowalski *et al.* (Supernova Cosmology Project), Astrophys.J. **686**, 749 (2008), arXiv:0804.4142 [astro-ph].
[8] M. Betoule *et al.* (SDSS Collaboration), Astron.Astrophys. **568**, A22 (2014), arXiv:1401.4064 [astro-ph.CO].
[9] R. G. Vishwakarma and J. V. Narlikar, Res.Astron.Astrophys. **10**, 1195 (2010), arXiv:1010.5272 [physics.gen-ph].
[10] M. March, R. Trotta, P. Berkes, G. Starkman, and P. Vaudrevange, Mon.Not.Roy.Astron.Soc. **418**, 2308 (2011), arXiv:1102.3237 [astro-ph.CO].
[11] A. Kim, Publ.Astron.Soc.Pac. **123**, 230 (2011), arXiv:1101.3513 [astro-ph.CO].
[12] B. Lago, M. Calvao, S. Joras, R. Reis, I. Waga, *et al.*, Astron.Astrophys. **541**, A110 (2012), arXiv:1104.2874 [astro-ph.CO].
[13] J.-J. Wei, X.-F. Wu, F. Melia, and R. S. Maier, Astron.J. **149**, 102 (2015), arXiv:1501.02838 [astro-ph.CO].
[14] M. Hicken, W. M. Wood-Vasey, S. Blondin, P. Challis, S. Jha, *et al.*, Astrophys.J. **700**, 1097 (2009), arXiv:0901.4804 [astro-ph.CO].
[15] J. Guy, P. Astier, S. Nobili, N. Regnault, and R. Pain (SNLS), Astron.Astrophys. **443**, 781 (2005), arXiv:astro-ph/0506583 [astro-ph]; J. Guy *et al.* (SNLS), Astron.Astrophys. **466**, 11 (2007), arXiv:astro-ph/0701828 [ASTRO-PH].
[16] P. Hoflich, A. Khokhlov, J. Wheeler, M. Hamuy, M. Phillips, *et al.*, Astrophys.J. **472**, L81 (1996), arXiv:astro-ph/9609070 [astro-ph]; D. Kasen and S. Woosley, Astrophys.J. **656**, 661 (2007), arXiv:astro-ph/0609540 [astro-ph].
[17] The upper integration limit is given in terms of χ^2s: $-2\log\mathcal{L}/\mathcal{L}_{\max} \to \Delta\chi^2 = \chi^2 - \chi^2_{\min}$. We will use the two interchangeably, but always refer to the full expression.
[18] All data available on `http://supernovae.in2p3.fr/sdss_snls_jla/ReadMe.html`. We use the `covmat_v6`.
[19] F. J. Farley, Proc.Roy.Soc.Lond. **A466**, 3089 (2010), arXiv:1005.5052 [astro-ph.CO]; F. Melia, Astron.J. **144**, 110 (2012), arXiv:1206.6289 [astro-ph.CO]; F. Melia and R. S. Maier, Mon.Not.Roy.Astron.Soc. **432**, 2669 (2013), arXiv:1304.1802 [astro-ph.CO].
[20] P. Marshall, N. Rajguru, and A. Slosar, Phys.Rev. **D73**, 067302 (2006), arXiv:astro-ph/0412535 [astro-ph].
[21] N. Karpenka, F. Feroz, and M. Hobson, Mon.Not.Roy.Astron.Soc. **449**, 2405 (2015), arXiv:1407.5496 [astro-ph.IM].
[22] L. Amendola, V. Marra, and M. Quartin, Mon.Not.Roy.Astron.Soc. **430**, 1867 (2013), arXiv:1209.1897 [astro-ph.CO]; C. Heneka, V. Marra, and L. Amendola, Mon.Not.Roy.Astron.Soc. **439**, 1855 (2014), arXiv:1310.8435 [astro-ph.CO].
[23] M. Dai and Y. Wang, (2015), arXiv:1505.05086 [astro-ph.CO].

Supplementary material

Appendix A: Confidence ellipsoids

The confidence ellipsoid is the collection of points $x = \{\Omega_{\rm m}, \Omega_\Lambda, \alpha, x_0, \sigma^2_{x_0}, \beta, c_0, \sigma^2_{c_0}, M_0, \sigma^2_{M_0}\}$, which obey

$$[x - x_{\rm MLE}]\mathcal{F}[x - x_{\rm MLE}]^{\rm T} \le \Delta\chi^2, \tag{A1}$$

where $\mathcal{F}$ is a symmetric matrix and $x_{\rm MLE}$ is the MLE. The enclosed volume is a confidence region with coverage probability corresponding with high precision to the value obtained from Eq.(10). The eigenvectors of $\mathcal{F}$ are then the principal axes of the ellipsoid, and the eigenvalues are the inverse squares of the lengths of the principal axes. We approximate this matrix with the sample covariance from the MC of Sect. IV as $\mathcal{F} = \mathrm{cov}(x,x)^{-1}$.

To make reading the matrix of eigenvectors easier, we round all numbers to 0.1. Thus, we get the following approximate eigenvectors of $\mathcal{F}$, in columns

$$\begin{array}{c} \Omega_{\rm m} \\ \Omega_\Lambda \\ \alpha \\ x_0 \\ \sigma^2_{x_0} \\ \beta \\ c_0 \\ \sigma^2_{c_0} \\ M_0 \\ \sigma^2_{M_0} \end{array}
\begin{pmatrix}
0.5 & 0 & 0 & 0.8 & 0.1 & -0.2 & 0 & 0 & 0 & 0 \\
0.8 & 0 & 0 & -0.5 & -0.1 & 0.2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
-0.1 & 0 & 0 & 0.3 & 0.1 & 1 & 0 & 0.1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{pmatrix} \tag{A2}$$

with respective lengths of semi-axes

$$10^{-3}\{172, 85.1, 49.8, 43.9, 38.1, \\ 9.89, 5.93, 4.24, 1.01, 0.304\} \tag{A3}$$

We also list the rounded correlation matrix,

$$\begin{pmatrix}
\Omega_{\rm m} & & & & & & & & & \\
0.9 & \Omega_\Lambda & & & & & & & & \\
0 & 0 & \alpha & & & & & & & \\
0 & 0 & 0 & x_0 & & & & & & \\
0 & 0 & -0.1 & 0 & \sigma^2_{x_0} & & & & & \\
0 & 0 & 0 & 0 & 0 & \beta & & & & \\
0.1 & -0.1 & 0 & 0 & 0 & 0 & c_0 & & & \\
0 & 0 & 0 & 0 & 0 & -0.3 & 0 & \sigma^2_{c_0} & & \\
-0.2 & -0.6 & 0 & 0 & 0 & 0.1 & 0.2 & 0 & M_0 & \\
0 & 0 & -0.1 & 0 & 0 & -0.3 & 0 & 0 & 0 & \sigma^2_{M_0}
\end{pmatrix} \tag{A4}$$

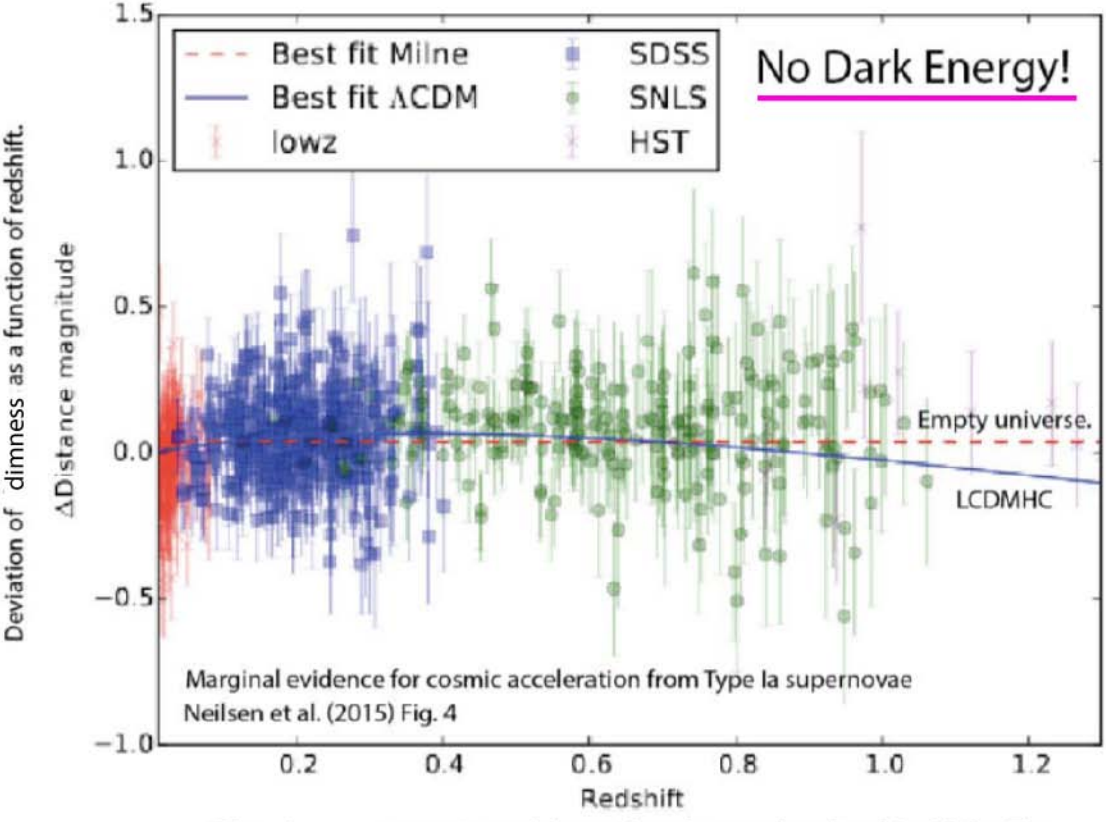
We see that the only pronounced correlations are between $\Omega_{\rm m}, \Omega_\Lambda$ and M_0. This is also apparent from Tab. I.

Appendix B: Code release

The code/data used in the analysis are available at: `http://nbia.dk/astroparticle/SNMLE/`

Paper that discusses the reduction of transparency of the Universe by the refractive effects of Ly-alpha clouds is Schild, R. and Dekker, M. 2006, Astron. Nachr. **327**, 729, THE TRANSPARENCY OF THE UNIVERSE LIMITED BY Ly-alpha CLOUDS. Also available as astro-ph/0512.236, CHG.

Large scatter in SuperNovaIa dimness is due to random amounts of dark matter planet refraction at the Oort cavity boundary rather than dark energy and accelerated expansion of the universe



The dimness scatter is much larger than the error bars for all Redshifts. The explanation is NOT dark energy and an acceleratied expansion rate of the universe, as Neilsen et al. (2015) discover.