THE ASTROPHYSICAL JOURNAL, Vol 154, December 1968

HGD cosmology argues that globular star clusters originated AFTER the galaxies formed at 10^12 seconds: as Jeans mass fragments at 10^13 seconds, simultaneous to fragmentation of Earth-mass dark matter planets.

ORIGIN OF THE GLOBULAR STAR CLUSTERS*

P. J. E. PEEBLES AND R. H. DICKE
Palmer Physical Laboratory, Princeton, New Jersey
Received March 8, 1968; revised June 12, 1968

ABSTRACT

We argue that the globular clusters may have originated as gravitationally bound gas clouds before the galaxies formed. This idea follows from the primeval-fireball picture, which suggests that the first bound systems to have formed in the expanding Universe were gas clouds with mass and shape quite similar to the globular star clusters. We present also a picture for the evolution from these assumed proto-globular gas clouds to globular star clusters

I. INTRODUCTION

a) Statement of the Problem

If other physical sciences are any guide to how one should attempt to proceed in cosmology, it is evident that one should pay a good deal of attention to those phenomona that happen to be simple and reproducible. On this basis, the globular clusters are prime candidates for attention. Although one must bear in mind that individual globular clusters do show marked individual variations, one must also consider it a striking fact that the luminosities of these systems vary so little. In our own Galaxy, half the globular clusters are contained in a range of a factor of just 3 in absolute luminosity (Arp 1965). There is no manifest correlation of absolute luminosity with distance from the center of the Galaxy. Still more remarkable, the luminosities of the five globular clusters associated with the Fornax dwarf galaxy are consistent with the distribution of luminosities of those in the Galaxy (Hodge 1965), as are the globular clusters associated with the Magellanic Clouds (Bok 1966). We are led therefore to ask the following question:

1. Why are the globular clusters associated with such wildly different systems so very similar?

In answer to this question, we suggest that the globular clusters formed (as gas clouds) before the galaxies appeared, by a universal process independent of the peculiar conditions in which the globular clusters now find themselves. This idea is based on the primeval-fireball picture for the evolution of the Universe (Gamow 1948; Alpher 1948; Dicke *et al.* 1965). In this picture, radiation drag on the matter prevents the formation of non-relativistic bound systems until the Universe has expanded and cooled to a temperature of about 4000° K (and density $\sim 10^4$ atoms cm^{-3}) and the plasma recombines and decouples from the radiation-drag force (Peebles 1965). When this happens, the only part of the density irregularity that can grow with time has wavelength (in the sense of a Fourier decomposition) in excess of the classical critical Jeans length (Gamow 1948; Peebles 1965):

$$\lambda > \lambda_J = (\pi k T / G \rho m)^{1/2} = 5 \text{ pc} . \quad (1)$$

Here m is the mass of a hydrogen atom. We have evaluated the equation at $T = 4000°$ K, assuming present radiation temperature $T_0 = 2.7°$ K (Stokes, Partridge, and Wilkinson 1967) and present mass density $\rho_0 = 1.8 \times 10^{-29}$ g cm^{-3}.

Equation (1) is a lower bound on the radius (and hence mass) of the first generation of bound systems, but of course the actual masses depend on the assumed initial values

* This research was supported in part by the National Science Foundation and by the Office of Naval Research of the United States Navy.

at the epoch of recombination. However, we show in § II that for a fairly broad range of possible assumptions, namely, where the power spectrum of the irregularities varies less rapidly than λ^2, the first generation of bound systems would have had characteristic dimension fixed by the Jeans length, and the mass of these clouds would be about $10^5 M_\odot$.

There are three important reasons for the tentative identification of these first-generation gas clouds with protoglobular clusters.

a) The gas clouds would have about the same mass as the globular clusters and would have ended up as star clusters with about the right radius.

b) The gas clouds would have the observed globular shape, with low rotation velocity, because the material should be substantially free of eddies, having just decoupled from the very strong radiation-drag force.

c) The gas clouds would have formed when the developing Galaxy amounted to a few per cent fluctuation in the mean density (Peebles 1967*a*). We can therefore understand the universal nature of the globular clusters, whether they are found now in the central part of the Galaxy, in the halo, or around a system like the Fornax galaxy.

The idea that the earliest systems formed at the Jeans limit (mass $\sim 10^5 M_\odot$) has been considered also by Doroshkevich, Zel'dovich, and Novikov (1967). It should be noted, however, that their theory is quite different from the present one. As described below, we assume that each cloud fragments into stars of conventional size rather than becoming a single superstar as assumed by Doroshkevich *et al.* Furthermore, as described in § II, we do not invoke thermal instabilities in the formation of protogalaxies.

In abandoning the conventional picture, that the globular clusters formed along with the halo field-population stars during the initial collapse of the protogalaxy (von Weizsäcker 1955), we generate some new problems concerning the globular clusters:

2. Why is the heavy-element content of globular-cluster stars correlated with the position of the cluster in the Galaxy?

An associated question arises in any theory in which the globular clusters are supposed to have formed from primeval material (which contains negligible abundances of the elements heavier than helium):

3. Why are the stars in globular clusters always polluted with at least a trace of heavy elements?

Finally, we are faced with a possible embarrassment of riches:

4. Why did such a small fraction (perhaps 10^{-3}) of the original globular-sized gas clouds end up as globular clusters?

b) *Evolution of the Protoglobular Gas Clouds*

The answers to these questions may lie in the evolution of the protoglobular gas clouds. The cloud would form as a bound system with a central temperature of perhaps 1000° K. The material in the cloud, atomic hydrogen and perhaps helium, is a poor radiator, but some molecular hydrogen can form, mainly by way of negative hydrogen ions (McDowell 1961). Energy loss due to radiation by the molecular hydrogen causes the cloud to contract until stars form and redress the energy balance.

We argue in § III that only a small fraction of the total cloud mass would fragment into stars. This is because the molecular-hydrogen production rate and radiation rate both increase toward the center of the cloud. The result is the formation of a small dense core which collapses well ahead of the main system. According to this picture, the cloud finds itself in a new situation in which a considerable fraction of the matter still is atomic hydrogen in the original cloud, and the cloud has local heat sources due to the massive young stars.

In this second phase of the evolution the energy balance shifts, the cloud gains more energy from the stars than it can radiate via molecular hydrogen, and the cloud puffs up somewhat. When the hot stars die, the energy balance shifts once more, and the cloud can

Contrary to the Peebles, Dicke (1968) scenario, protogalaxies fragmented at viscous gravitational scales early during the plasma epoch, retaining their density and baryonic mass to the present time in protoglobularstarcluster (PGC) clumps of a trillion Earth-mass dark matter planets in metastable equilibrium, as observed by Schild (1996) and predicted by Gibson (1996).

start to contract. Presumably, the system could cycle through this phase a number of times. In Figure 1 we have plotted a possible graph showing the time variation of the central density in an evolving gas cloud.

This second phase could be terminated in several different ways. If the system happened not to find itself in a galaxy, the system could remain in the second phase until the debris from evolved stars in the cloud became so great that the gas could not absorb stellar radiation as fast as it could radiate energy (§ IV). Only the first stellar generation would have formed from unpolluted material. Question 3 therefore has a natural answer in the present model—even extragalactic globular clusters, which never passed through a galaxy, would be polluted to some extent, because only a small fraction of the cloud mass could be deposited in the first generation.

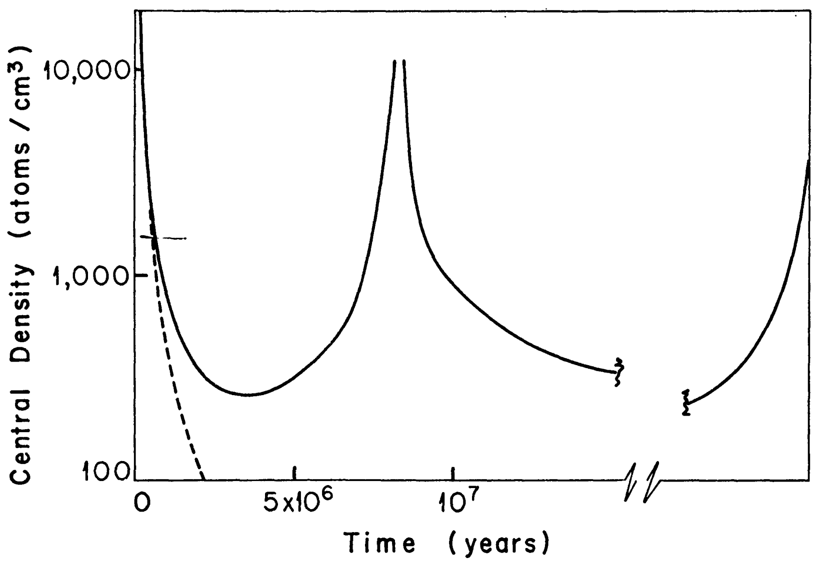

FIG 1.—Evolution of a globular cluster. The solid line is a sketch of the expected time variation of the central density in a developing globular cluster. The first falling curve is a zero-pressure solution for a spherically symmetric cloud. For comparison, we have included a dashed line showing the time variation of the mean mass density in the Universe (eq. [27]). Past the point of maximum expansion (minimum density), the central parts of the cloud contract back in, giving rise to a burst of star formation. The hot young stars provide a new heat source, causing the gas cloud to expand again Finally, when the hot stars die the cloud can contract back in. This cycle could be repeated.

If the initial perturbation had a power spectrum flat enough to produce protoglobular gas clouds, most of the material in the Universe would have ended up gravitationally bound in these clouds. However, this need not conflict with the fourth question, because the continued existence of the clouds is precarious. Only small numbers of massive stars are needed to hold up the cloud, so even statistical fluctuations in the number created could have pronounced effects. In particular, we show in § IV that if the first stellar generation happened to contain too many massive stars it could blow the remaining gas cloud apart. Also, clouds would be destroyed in the collapse of the protogalaxy.

c) Globular Clusters in the Collapsing Protogalaxy

In the present model the galaxies would have formed as clusters of these clouds (plus debris from clouds). This is not a separate assumption but a consequence of the basic idea that the initial irregularities have a more or less white power spectrum, with power at the characteristic wavelengths of protogalaxies as well as protoglobular clusters.

In the initial collapse of the protogalaxy the cloud velocities would have been substantially higher than the internal velocity of the cloud. This means that collisions between clouds, or between clouds and the generally broadcast debris, are energetic enough to break up the clouds. A gas cloud could survive the initial collapse of the protogalaxy if the cloud were induced to fragment out into stars before it suffered high-velocity collisions. This star formation would be seeded by the rain of debris from the rapidly evolving massive stars in the first stellar generations to have formed in the collapsing protogalaxy. If the gas cloud does manage to fragment out more than about half its mass before it passes through the galaxy, it should survive the loss of its remaining gas in the pass through the galaxy (von Weizsäcker 1955). Such a globular star cluster would end up with an abnormally high heavy-element content. This might account for the observed correlation of heavy-element abundance and motion of the globular clusters in the Galaxy.

II. FORMATION OF THE GAS CLOUDS

a) Assumptions

The analysis of the formation of the first bound systems in the expanding Universe necessarily involves a number of basic assumptions and simplifying approximations. To keep these assumptions and approximations in the proper perspective, we list the important ones here.

The cosmological model is based on general relativity without the cosmological constant. For the most part, we use the closed, or cosmologically flat, model, with acceleration parameter $q_0 = \frac{1}{2}$, present Hubble constant $H_0^{-1} = 1 \times 10^{10}$ years and present fireball temperature $T_0 = 2.7°$ K (Stokes *et al.* 1967). We consider also the effect of a scalar-tensor theory (Brans and Dicke 1961) and also the open cosmological model, with present mass density 4.5×10^{-31} g cm^{-3}. We assume that the primeval material was pure hydrogen. The results would be little changed by the addition of primeval helium.

When the plasma recombines and decouples from the radiation drag, the mass density in non-relativistic matter is a factor of about 20 greater than the mass density in radiation. This means that we can in a reasonable approximation use the matter-dominated cosmological model, for which the cosmic time t is related to the mean mass density $\rho(t)$ by the equation

$$t = [6\pi G\rho_0(t)]^{-1/2} . \tag{2}$$

The growing gas clouds are much smaller than the Hubble radius ct, so that, to an excellent approximation, we can use the weak-field limit of general relativity, which amounts to ordinary Newtonian theory when $P \ll \rho c^2$ (McCrea and Milne 1934; Callan, Dicke, and Peebles 1965).

The initial values for the perturbations to the distribution and motion of the matter will be specified at some initial time t_1. We take it that the perturbations to the density and to the Cartesian components of the velocity of the matter are specified by a Gaussian random process (in the form of a white power-spectrum cut off at some suitably high wavenumber). The main conclusions regarding the gas clouds would be little affected by coloration of the power spectrum, so long as the power spectrum does not vary faster than k^{-2}.

The time t_1 at which the perturbations are specified might conveniently be taken to be the time at which the plasma recombines. This is not to say that the perturbations originated at that epoch—we suspect, rather, that the irregularities are remnants from earlier irregularities in the plasma (Peebles 1967*a*). The ultimate origin of the irregularities is a controversial question (cf. Layzer 1964 and references contained therein; Peebles 1967*b*). However, in view of the existence of galaxies, it does not seem too daring an assumption to suppose that the Universe was in fact somewhat irregular at the time the plasma recombined.

For some time after the plasma recombination the Jeans length varies nearly in direct proportion to the expansion radius of the Universe, and the characteristic cloud mass defined by the Jeans length varies little with the expansion. We assume that the perturbations are strong enough so that the Jeans instability defines the cloud mass before the characteristic Jeans mass starts to decrease appreciably.

In the following discussion, we introduce the further simplifying approximation that the matter pressure is unimportant when the wavelength exceeds the Jeans length at the epoch of recombination and that all waves with wavelength shortward of the Jeans length are dissipated away. In effect, we treat the development of a zero-pressure fluid with irregularities having a flat-power-spectrum cut off at the fixed Jeans length.

The condensation of galaxies is treated in much the same way, the individual isolated gas clouds serving as the molecules of a new fluid. It will be noted that in this new "fluid" the molecules are so large that the fluid approximation applies only when the wavelength contains many times $10^5 M_\odot$. Thus the approach should apply to condensations ending up as galaxies, or as clusters of galaxies.

b) Linear Perturbations

The following development is a natural continuation of several earlier papers (Peebles 1965, 1967*a*, *b*) on a simple theory of small perturbations to the Friedman model. We write the density and matter velocity in the form

$$\rho = \rho_0(t)[1 + \delta(\boldsymbol{x},t)] , \qquad \boldsymbol{v} = \frac{1}{a}\frac{da}{dt}\boldsymbol{r} + \boldsymbol{u}(\boldsymbol{x},t) . \tag{3}$$

The computations are carried out to terms of first order in δ and $\boldsymbol{u}$. In the second equation, $\boldsymbol{v}$ is the velocity of the matter as measured in a locally Minkowski coordinate system, and $\boldsymbol{u}$ can be considered the irregularity in the velocity as measured by an observer comoving in the unperturbed cosmological model. These comoving observers have fixed spatial coordinates $\boldsymbol{x}$ defined by the equation

$$\boldsymbol{x} = \boldsymbol{r}/a(t) . \tag{4}$$

The equations for δ and $\boldsymbol{u}$ are most conveniently expressed in the "comoving" coordinates $\boldsymbol{x}$ and t (eq. [4]). In these coordinates the first-order part of Euler's equation of fluid dynamics is (Peebles 1965)

$$\frac{\partial \boldsymbol{u}}{\partial t} + \frac{\boldsymbol{u}}{a}\frac{da}{dt} = -\frac{\nabla\psi}{a} - \frac{kT}{m}\frac{\nabla\delta}{a} , \tag{5}$$

where ψ is the perturbation to the Newtonian gravitational potential,

$$\nabla^2\psi = 4\pi G\rho_0\delta a^2 . \tag{6}$$

In these equations the spatial derivatives are taken with respect to the comoving coordinates $\boldsymbol{x}$.

The last term on the right-hand side of equation (5) represents the effect of the pressure-gradient force. We have assumed that the perturbations are isothermal; if they were adiabatic, this term would have to be multiplied by the factor $\frac{5}{3}$. The isothermal assumption is the better approximation as long as the residual ionization of the matter is high enough to assure approximate thermal equilibrium between matter and radiation.

The first-order part of the mass conservation equation is

$$\frac{\partial\delta}{\partial t} + \frac{\nabla\cdot\boldsymbol{u}}{a} = 0 . \tag{7}$$

The equation for δ is obtained by taking the divergence of equation (5) and then using equations (6) and (7). This equation is (Peebles 1965)

$$\frac{\partial^2\delta}{\partial t^2}+\frac{2}{a}\frac{da}{dt}\frac{\partial\delta}{\partial t}=4\pi G\rho_0\delta+\frac{kT}{m}\frac{\nabla^2\delta}{a^2}\,. \tag{8}$$

The Jeans criterion (eq. [1]) is just the condition that the right-hand side of equation (8) shall be positive. When the wavelength is less than the Jeans length, the perturbation acts like a propagating wave, and the amplitude is adiabatically damped by the expansion of the Universe.

As mentioned above, as long as the matter and radiation temperatures are equal, the Jeans length measured in the coordinates $\boldsymbol{x}$ (eq. [4]) is a constant. The correction for the temperature difference between matter and radiation is not large. In the closed model, when the radiation temperature has fallen to 1000° K the Jeans length measured in co-moving units falls to 83 per cent of the value at recombination, and when the radiation temperature is 500° K the Jeans length is 63 per cent of the value at recombination (Peebles 1968).

We have adopted the approximation that, when equation (1) is satisfied, we neglect the second term on the right-hand side of equation (8). The desired solution for $\delta(\boldsymbol{x},t)$ may be fitted to the initial values δ_1 and $\boldsymbol{u}_1$ at time t_1 with the help of equation (7). The result is (Peebles 1967*a*)

$$\begin{aligned}\delta(\boldsymbol{x},t)&=A(\boldsymbol{x})t^{2/3}+B(\boldsymbol{x})/t\,,\\ A(\boldsymbol{x})&=\tfrac{3}{5}[\delta_1(\boldsymbol{x})-t_1\nabla\cdot\boldsymbol{u}_1(\boldsymbol{x})/a_1]/t_1^{2/3}\,,\\ B(\boldsymbol{x})&=[\tfrac{2}{5}\delta_1+\tfrac{3}{5}t_1\nabla\cdot\boldsymbol{u}_1/a_1]t_1\,.\end{aligned} \tag{9}$$

In solving equation (5) for the velocity perturbation $\boldsymbol{u}$, it is convenient to introduce the Fourier resolution

$$\delta(\boldsymbol{x})=\sum_{\boldsymbol{k}}\delta_{\boldsymbol{k}}e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\,, \tag{10}$$

and similarly for the Cartesian components $u^a(\boldsymbol{x})$, $a=1, 2, 3$. It will be noted that the propagation vector $\boldsymbol{k}$ is measured in comoving coordinates, so the wavelength as measured in locally Minkowski coordinates varies directly as $a(t)$. In terms of the Fourier amplitudes, the solution to equations (5) and (6) is

$$\begin{aligned}u^a{}_{\boldsymbol{k}}(t)&=\frac{ik_a a(t)}{k^2}\left(\frac{2}{3}\frac{A_{\boldsymbol{k}}}{t^{1/3}}-\frac{B_{\boldsymbol{k}}}{t^2}\right)\\ &\quad+\left(\frac{t_1}{t}\right)^{2/3}\left(u_1{}^a{}_{\boldsymbol{k}}-\frac{k_a k_\beta}{k^2}u_1{}^\beta{}_{\boldsymbol{k}}\right),\end{aligned} \tag{11}$$

where $A_{\boldsymbol{k}}$ and $B_{\boldsymbol{k}}$ are the Fourier components of the initial-value functions in equation (9).

The behavior of the solutions (9) and (11) is complicated by the existence of four free initial functions, $\delta_1(\boldsymbol{x})$ and $u_1{}^a(\boldsymbol{x})$. It will be noted, however, that the solutions simplify when $t\gg t_1$. If we neglect the unlikely chance that the initial density and velocity irregularities just cancel in the growing term A of equation (9), we conclude that when $t\gg t_1$ the solutions (9) and (11) take the form

$$\delta_{\boldsymbol{k}}=A_{\boldsymbol{k}}t^{2/3}\,,\qquad u^a{}_{\boldsymbol{k}}=\tfrac{2}{3}i\frac{k_a}{k^2}\frac{a(t)}{t^{1/3}}A_{\boldsymbol{k}}\,. \tag{12}$$

In this limit the density and velocity fields are specified by the single scalar function $A(\mathbf{x})$ (with Fourier components $A_{\mathbf{k}}$).

We have assumed that $A(\mathbf{x})$ has a flat-Fourier-spectrum cut off at the Jeans length, the components $A_{\mathbf{k}}$ having random phase. Now we have to ask, with this characterization of the initial values, what does the distribution $\delta(\mathbf{x})$ look like? And what sort of gravitationally bound systems will the distribution develop into?

We note first that the variance of $\delta(\mathbf{x})$ is given by the formula

$$\langle\delta(\mathbf{x})^2\rangle = \sum_{\mathbf{k}} |\delta_{\mathbf{k}}|^2 = \frac{V}{6\pi^2} k_J{}^3 |\delta_{\mathbf{k}}|^2 , \quad (13)$$

where the Fourier amplitudes (eq. [10]) vary with time according to equation (12). We have fixed on a box of volume V, with periodic boundary conditions. The cutoff k_J is 2π divided by the critical Jeans length (1) measured in comoving coordinates.

The autocovariance function for δ is

$$C(y) = \langle\delta(\mathbf{x})\delta(\mathbf{x}+\mathbf{y})\rangle = 3\langle\delta(\mathbf{x})^2\rangle\left[\frac{\sin k_J y}{(k_J y)^3} - \frac{\cos k_J y}{(k_J y)^2}\right] . \quad (14)$$

This equation shows that $\delta(\mathbf{x})$ does not change appreciably over distances much shorter than the characteristic length $k_J{}^{-1}$ and that there is little correlation between the values of δ at points separated by distances much greater than $k_J{}^{-1}$. An excellent analogy to the fluctuation of $\delta(\mathbf{x})$ is provided by the observed speckled pattern (scintillation) in the light from a laser beam scattered off a piece of cardboard (see Fig. 2 [Pl. 5]). Here, too, the power spectrum of the intensity pattern is approximately flat longward of the cutoff. The cutoff is provided here by the pupil diameter of the eye (or, in Fig. 2, the iris opening of the camera lens). Although the power spectrum is flat, the prominent features in the spatial distribution appear at the characteristic length defined by the cutoff, because phase space increases rapidly with increasing wavenumber k. For the same reason, our conclusions are little affected by changes in the coloration of the power spectrum.

To obtain a more definite picture for the typical shape of one of the patches (speckles) in the distribution, let us consider the following question: Given that the value of the distribution is equal to δ_0 at the point $\mathbf{x}$, what is the average value of δ at the point $(\mathbf{x}+\mathbf{y})$? Since the distribution $\delta(\mathbf{x})$ is supposed to be statistically uniform, we can choose $\mathbf{x}$ to be the origin of coordinates and then seek the ensemble average,

$$f(\mathbf{y}) = (\delta(\mathbf{y}))_{\text{av}} = \sum_{\mathbf{k}} (\delta_{\mathbf{k}})_{\text{av}} e^{i\mathbf{k}\cdot\mathbf{y}} , \quad (15)$$

where the ensemble average is to be taken subject to the constraint

$$\sum_{\mathbf{k}} \delta_{\mathbf{k}} = \delta_0 \quad (16)$$

within a small range of uncertainty. This problem involves the statistical distribution of the amplitudes $\delta_{\mathbf{k}}$. It will be noted that a large number of independent Fourier amplitudes $\delta_{\mathbf{k}}$ is involved and that, by virtue of the assumption of white noise, each amplitude is supposed to be chosen from the same statistical distribution. We expect, therefore, that we do not appreciably alter the final result if we simplify the statistical distribution of the individual $\delta_{\mathbf{k}}$. The problem is very much simplified if we permit each of the $\delta_{\mathbf{k}}$ to assume only the values $\pm K$, where K is a constant. Then it is readily seen by counting that, with the constraint (16), the ensemble average of the amplitude $\delta_{\mathbf{k}}$ is

$$(\delta_{\mathbf{k}})_{\text{av}} = \delta_0 / \sum_{\mathbf{k}} 1 . \quad (17)$$

PLATE 5

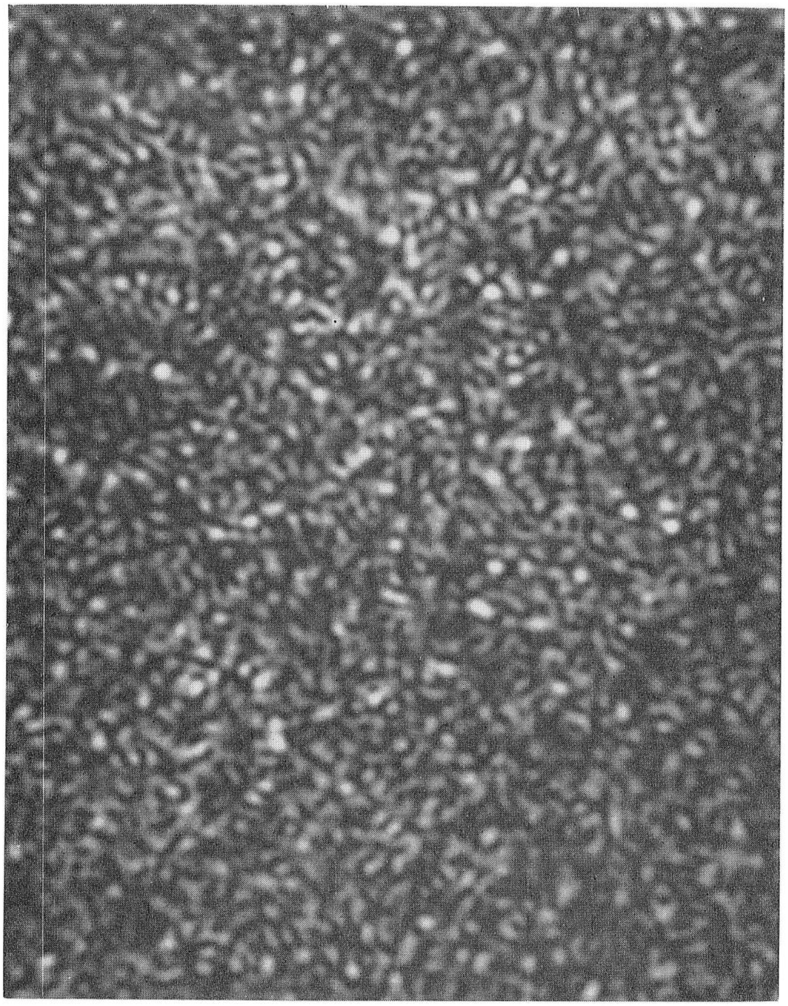

FIG. 2.—An early phase in the formation of globular clusters reconstructed by means of an analogue device. Pictured is a laser scintillation pattern, a two-dimensional random distribution of fluctuations characterizing a flat power spectrum with a short-wave cutoff.

PEEBLES AND DICKE (*see* page 897)

The ensemble average (15) is then

$$f(y) = \delta_0 \sum_k e^{i\boldsymbol{k}\cdot\boldsymbol{y}} \Big/ \sum_k 1 = 3\delta_0 \left[\frac{\sin k_J y}{(k_J y)^3} - \frac{\cos k_J y}{(k_J y)^2} \right] . \tag{18}$$

This is the same as the autocovariance function (14).

We conclude that if we choose δ_0 positive and relatively large, perhaps equal to the square root of the variance (13), we will have chosen a spot within a coherent patch with density higher than the average, and equation (18) gives the ensemble-average shape of the patch around the chosen spot. Equation (18) might then be adopted as the shape of a "typical" patch.

c) Non-linear Regime

The following considerations are applied to gain some insight into the problem of the development of the small perturbation into a stationary bound system. The discussion is based in part on energy conservation. In these considerations, we ignore the internal (thermal) energy of the cloud. This is consistent with the approximation that, the Jeans length having impressed a shoulder on the power spectrum, pressure forces otherwise play an unimportant role in the formation of the cloud.

We suppose first that space is divided up into small comoving cells. If the cell size is small enough, we are entitled to consider separately the total internal energy of a cell, kinetic plus gravitational potential. The total energy of the matter would be the sum of this internal energy and the kinetic energy of translation and the gravitational energy of interaction with matter outside the cell. It is readily seen that, in the linear approximation, and using equation (2) and the solution (12) for the velocity, the internal energy in a sphere of radius a about the point $\boldsymbol{x}$ is

$$E(x) = -\frac{8\pi}{27} \rho_0 \frac{a^5 x^5}{t^2} \delta(\boldsymbol{x}) . \tag{19}$$

This equation shows that, within a coherent patch, where the density is higher than the average, the material is everywhere self-gravitating. According to equation (18), the typical radius of a patch out to the point at which $\delta(\boldsymbol{x})$ vanishes is

$$x_1 \cong 4.5/k_J , \tag{20}$$

or, in proper coordinates,

$$r_1 \cong 0.72\lambda_J , \tag{21}$$

where λ_J is the Jeans length. All the material out to this radius is self-gravitating, in the sense that the internal energy is negative, so all this material should end up in the same cloud. On this basis, and using the Jeans length (1), we would conclude that the typical cloud mass is at least

$$M_1 = \frac{4\pi}{3} \rho r_1^3 = 2.0 \times 10^5 M_\odot . \tag{22}$$

The mass (22) accounts for half the total. If all the intercloud gas fell back on the clouds, the value (22) should be doubled.

As a second means of estimating the typical mass of a gas cloud, we count the number density of maxima of the distribution $\delta(\boldsymbol{x})$. At each extremum $\boldsymbol{x}_e$ of $\delta(\boldsymbol{x})$ we have the three equations

$$0 = \sum_k k_a \delta_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot\boldsymbol{x}_e} . \tag{23}$$

When the Fourier waves are chosen with periodic boundary conditions in a box of volume V, there are

$$N = \frac{V}{6\pi^2} k_J^3 \tag{24}$$

independent amplitudes and phases up to the cutoff, and these can belong to $N/3$ chosen extrema $\boldsymbol{x}_e$. Of these extrema, 1 in 9 is a maximum. If all the matter ends up bound to the gas cloud forming around each maximum, the average cloud mass is

$$M_2 = 8.2 \times 10^5 \, M_\odot \,, \tag{25}$$

in rough agreement with twice the mass (22).

It is concluded that, in the adopted cosmological model, the mean mass of the clouds would be about $6 \times 10^5 \, M_\odot$. If the Universe were open, with mass density $\rho_0 = 4.5 \times 10^{-31}$ g cm^{-3}, the mass estimate would be larger by a factor of 6.3 (eq. [1]).

It is of interest to consider also the scalar-tensor theory of gravitation (Brans and Dicke 1961; Dicke 1968). Under the Brans-Dicke form of the theory, the gravitational constant G varies as ϕ^{-1}. For the cosmologically flat space ($\rho_0 = 2 \times 10^{-29}$ g cm^{-3}) ϕ varies as $a^{1/(1+\omega)}$, where a is the expansion radius of the Universe and ω is a constant $\sim$5. The radiation temperature varies as a^{-1}; hence 4000° K is reached when a^{-1} is 1500 times its present value. Assuming that $\omega = 5$, G is 3.4 times its present value. From

TABLE 1

GLOBULAR-CLUSTER MASSES FOR TWO POSSIBLE VALUES OF THE PRESENT MEAN MASS DENSITY IN THE UNIVERSE AND FOR TWO THEORIES OF GRAVITATION

Present Mass Density (g cm^{-3})	General Relativity ($M_\odot$)	Scalar-Tensor ($M_\odot$)
2×10^{-29} . . .	6×10^5	1×10^5
4.5×10^{-31} . . .	4×10^6	4×10^6

equation (1), λ_J is decreased by a factor of $3.4^{-1/2}$, and the mass of the cluster is decreased by a factor $3.4^{3/2} = 6.2$. Under this theory, if the present mean mass density were 4.5×10^{-31} g cm^{-3}, the expected change in the scalar field ϕ would be small enough to be neglected.

The four values discussed above for the mass of the cloud are tabulated in Table 1. Since so few globular-cluster masses are known, it would be difficult to judge which of these possibilities is to be preferred. It is noteworthy, however, that the two mass estimates for the closed Universe do agree in order of magnitude with known globular-cluster masses (the available evidence on masses is discussed by Arp 1965, p. 424).

Some properties of the "typical" cloud are defined by equation (18). The velocity of the material is given by equation (12). For the spherically symmetric distribution (18) the velocity is radial,

$$u_r = -2 \frac{\delta_0(t)}{k_J{}^3} \frac{a(t)}{t} \frac{1}{x^2} \left(\int_0^x \frac{\sin k_J x'}{x'} dx' - \sin k_J x \right) \tag{26}$$

when $k_J x \ll 1$, this reduces to

$$u_r = -\tfrac{2}{9} \delta_0(t) a(t) x/t \,. \tag{27}$$

In these equations $\delta_0(t)$ is the density perturbation at the center of the developing cloud. The total energy, kinetic plus Newtonian gravitational potential, contained within a sphere of radius x_m centered on the distribution (18) is

$$E(x_m) = -\frac{80\pi^2 G \rho_0{}^2 a^5 \delta_0(t)}{k_J{}^3} \int_0^{x_m} x dx \left(\int_0^x \frac{\sin k_J x_1}{x_1} dx_1 - \sin k_J x \right) . \tag{28}$$

When $x_m \ll k_J^{-1}$, this reduces to

$$E(x_m) = -GM_m^2\delta_0(t)/a(t)x_m , \tag{29}$$

where M_m is the constant mass within x_m. This result agrees with equation (19).

When the central part of the cloud reaches the point of maximum expansion, the kinetic energy vanishes and the potential energy of the above central mass M_m is

$$\phi = -\tfrac{3}{5}GM_m^2/R_{\max} . \tag{30}$$

We conclude from equations (29) and (30) that, if the irregularity started out with central amplitude $\delta_1(g)$ in the growing term of the perturbation (coefficient A in eq. [9]), the material will have expanded by a factor

$$R_{\max}/R_1 = 3/5\delta_1(g) \tag{31}$$

in radius to the point of maximum expansion. The corresponding time $t_{\max}$ is given by the expression (Partridge and Peebles 1967)

$$t_m/t_1 = 6\pi[3/10\delta_1(g)]^{3/2} . \tag{32}$$

d) Protogalaxies

The above general considerations apply to the formation of galaxies and of groups of galaxies. In the present picture, these systems would have originated as gravitationally bound clusters of the fundamental gas clouds. We have no provision in this theory for a characteristic scale of length or mass for the galaxies. The only two natural scales are the Jeans length and the Hubble length ct. We have already assigned the former to the globular clusters. At the epoch of recombination, the second length already contains $3 \times 10^{17}\ M_\odot$, which exceeds even the masses of the great clusters. We must assume, therefore, that the characteristics of galaxies, the decision whether a collection of gas clouds ended up as a single galaxy or as a group of galaxies, rests ultimately on the detailed interaction among the building blocks.

In discussing the formation of bound systems of clouds, it is useful to consider the growing irregularities in the total mass integrated over a chosen volume V. That is, we consider the smoothed density function

$$\rho_s(x,V) = \int d^3y\rho(y)g(x-y,V) , \tag{33}$$

where the smoothing function g is normalized to unity and vanishes outside the volume size of interest. As before, we subtract the mean density $\rho_0(t)$ from equation (33), to obtain the smoothed dimensionless perturbation $\delta_s(x,V) = (\rho_s/\rho_0 - 1)$ (cf. eq. [3]). The power spectrum of the smoothed perturbation δ_s is just the power spectrum of δ multiplied by the power spectrum of the smoothing function g, so we have again a flat-power-spectrum cut off in the neighborhood of the new wavenumber $k_s = 2\pi/x_s$, where x_s is the comoving radius of the smoothing volume. The variance of δ_s therefore is reduced from the variance of δ by the factor (eq. [13])

$$\langle\delta_s^2\rangle/\langle\delta^2\rangle \cong (\lambda_J/x_s)^3 . \tag{34}$$

If $\lambda_J \ll x_s$, the above linear dynamical considerations will apply to the smoothed distribution $\delta_s(x)$ considered as a coarse-grained "gas" of the clouds, even after the clouds themselves have formed out as stationary bound systems. It follows, then, from equations (12), (32), and (34) that, at the time t_m when the fundamental gas clouds have just stopped expanding, the variance of the smoothed perturbation δ_s is of the order of $(\lambda_J/x_s)^3$. By equation (24), this is of the order of the number of clouds within the smoothing volume V, so that, in effect, matter now is concentrated in randomly distributed clouds. As described above, the irregularities in this distribution continue to grow into bound systems like galaxies and clusters of galaxies.

III. INITIAL CONTRACTION OF THE CLOUD

The purpose of this section is go give some model results substantiating the assumption that only a small fraction of the cloud mass would have entered the first stellar generation.

The cloud loses energy and is forced to contract because some molecular hydrogen can form in the cloud. The molecular hydrogen originates mainly through negative-hydrogen-ion formation (McDowell 1961),

$$H + e^- \rightarrow H^- + \gamma , \quad (35)$$

followed by the reaction

$$H^- + H \rightarrow H_2 + e^- . \quad (36)$$

When the electron energy is less than 0.75 eV, the cross-section is given by the threshold formula (fitted to the integrations of Chandrasekhar 1958)

$$\sigma = 7.9 \times 10^{-23} E_{eV}^{1/2} \text{ cm}^2 . \quad (37)$$

According to equation (37), the rate coefficient for the reaction (35) is given by the formula

$$a_1 = 6.06 \times 10^{-15} T_4 \text{ cm}^3 \text{ sec}^{-1} , \quad (38)$$

where T_4 is the electron temperature measured in units of 10000° K. This formula is a satisfactory approximation in the temperature range in the model, $T \lesssim 3000°$ K. Also, in the model the temperature is low enough so that we can neglect collisional dissociation of the molecular hydrogen. The rate of the reaction (36) is much greater than the rate (38) (Schmeltekopf, Fehsenfeld, and Ferguson 1967), so that we may take it that the formation of each negative hydrogen ion is followed promptly by the conversion to a hydrogen molecule. The rate for the formation of H_2^+ by radiative capture of protons by atomic hydrogen is smaller than the rate (38) by a factor of 100 at 1000° K (Bates 1951). Thus the process is unimportant here.

The rate of energy radiation by molecular hydrogen at fairly low density has been computed by Takayanagi and Nishimura (1960). Since the contemplated densities exceed the range of values in these computations, we have interpolated by hand between the numerical results and the formula valid in the limit of high density,

$$\Lambda = 2.73 \times 10^{-18} T_4^3 \text{ ergs sec}^{-1} (H_2 \text{ molecule})^{-1} . \quad (39)$$

This formula is obtained from two assumptions: (1) that the density is high enough so that the rotational levels are populated according to the Boltzmann distribution and (2) that the temperature is high enough ($\gtrsim 300°$ K) so that the sums over angular momenta can be approximated as integrals. The transition rates were taken from Spitzer (1949).

For the free electrons and protons, the rate for radiative recombinations to excited states of hydrogen is given by Boardman (1964). We use the approximate formula

$$a_r \cong \frac{2.84 \times 10^{-13}}{T_4^{1/2}} \text{ cm}^3 \text{ sec}^{-1} . \quad (40)$$

Again, in the range of the numerical integration, collisional ionization may be ignored.

In the model, the initial state is an adiabatic polytrope (in hydrostatic equilibrium) with mass equal to $10^5 M_\odot$ and central temperature equal to 1000° K (Emden 1907), This yields a density-temperature combination such that the central parts of the cloud can collapse at almost the free-fall rate. The initial fractional ionization is 3×10^{-5}. This is about the expected residual ionization following recombination of the primeval plasma (Peebles 1968). The initial molecular-hydrogen abundance is equal to zero.

The results of numerical integration of the dynamic contraction of the cloud are shown in Table 2. The numbers in a given column are evaluated at the radius which contains a fixed fraction of the cloud mass. The mass fractions are given at the heads of the columns. At the last time shown in the table, 1.41×10^{14} sec after the start of contraction, the molecular-hydrogen abundance at the cloud center has risen to 3 parts in 1000 by number. At the mass fraction 0.5, the molecular-hydrogen abundance is 7 parts in 100000. The residual ionization at the center has fallen to 1×10^{-6}, while in the outer layers the ionization is very nearly the initial value.

TABLE 2

CONTRACTION OF THE MODEL GAS CLOUD

TIME*	0		0 006		0 02		0 21		0 93	
	n†	T‡	n	T	n	T	n	T	n	T
0	0 26	1000	0 25	980	0 24	950	0 17	750	0 018	170
5	0 32	440	0 31	450	0 29	450	.17	500	018	170
10	1 04	380	0 90	390	0 75	390	.24	400	018	160
12 .	3 3	380	2 6	380	1 9	390	34	400	017	160
13	10	370	6 8	380	4 3	380	45	410	017	160
14	160	350	65	370	25	380	68	430	016	160
14 1.	430	350	130	370	42	390	0 74	440	0 016	160

* Time units 10^{13} sec

† Density unit 1000 atoms cm^{-3}.

‡ Temperature in ° K.

For the adopted initial conditions, the free-fall collapse time for the cloud center would be 1.0×10^{14} sec. The collapse of the center thus is nearly a free fall. On the other hand, the outer layers of the cloud have responded very little. The large increase in density toward the very center of the cloud becomes still more pronounced when the integration is carried beyond the last point shown in the table. Ultimately, the central temperature rises again, but by now the central density is so high that this does not slow the collapse of the center until the matter becomes opaque.

It is not in fact very realistic to carry the integration much beyond the point at which the center achieves free fall, because the free fall surely generates turbulence. Therefore, we do not attempt to estimate the fraction of the cloud mass that ended up in the first stellar generation. We do conclude that, in the model, a small fraction of the cloud mass finds itself in a position to collapse to stars well before the main bulk of the cloud is able to respond to the collapse.

IV. EVOLUTION OF A PROTOGLOBULAR CLUSTER

Our purpose in this section is to discuss the energy balance in an isolated protoglobular cluster in the second phase of its evolution. To obtain definite results, we adopt a model for the cloud. We suppose that the stars all are concentrated at the cloud center. We take the gas to be distributed according to an adiabatic polytrope, with a mass of $10^5 M_\odot$ and a central temperature of 3000° K. Then the central density in the cloud is 7.1×10^3 atoms cm^{-3}, and the cloud radius is 9.4 pc (Emden 1907).

The rates of energy loss and deposition in the cloud depend on the free-electron abundance x_e in nearly the same way; hence the stability conclusions are little affected by the choice of this parameter. We adopt the value

$$x_e = 3 \times 10^{-5} . \tag{41}$$

This is the ionization if the characteristic recombination time is 10^{13} sec.

The rate coefficient for the reaction (36) is of the order of 10^{-9} cm^3 sec^{-1} (Schmeltekopf *et al.* 1967), so that the characteristic time for the reaction (36) in the cloud is 10^5 sec. By contrast, if the cloud luminosity were one solar luminosity per solar mass, the photon flux 10 pc from the cloud center would amount to about 10^{10} photons cm^{-2} sec^{-1} and the photodissociation time would be greater than 3×10^6 sec (Chandrasekhar 1958). That is, we expect that the rate of formation of molecular hydrogen is fixed by equation (38).

The rate of photodissociation of molecular hydrogen has been given by Stecher and Williams (1967). For purposes of discussion, we specify the rate β in terms of the equivalent number $\mathfrak{N}$ of B0 stars in the cloud. A distance R pc from the cloud center β is

$$\beta = 5.5 \times 10^{-9} R^{-2} \mathfrak{N} \text{ sec}^{-1} . \tag{42}$$

The minimum dynamic evolution time is the free-fall time, $\sim 10^{13}$ sec; hence β is high enough (when $\mathfrak{N} \geq 1$) so that the molecular-hydrogen abundance is given by the equilibrium formula,

$$x_2 = \alpha_1 n x_e / \beta . \tag{43}$$

Here x_2 is the abundance of molecular hydrogen by number relative to atomic hydrogen.

On using the values (41) and (43), and using the molecular-hydrogen radiation rate estimated as in the previous section, we find that the rate at which the cloud loses energy due to the molecular hydrogen is

$$J_-(H_2) = \frac{1.8 \times 10^{-4}}{\mathfrak{N}} \frac{\mathfrak{L}_\odot}{M_\odot} . \tag{44}$$

We have expressed this result as a luminosity per unit mass, in units of solar luminosities per solar mass. The numerical coefficient in equation (44) is the result of numerical integration over the polytrope model.

The gas absorbs energy in the cycle of molecular-hydrogen formation (eqs. [35] and [36]) followed by photodissociation, the energy released in the cycle being of the order of $B \simeq 5$ eV. In the polytrope model the total rate of energy deposition divided by the mass of the cloud is

$$J_+ = 2.6 \times 10^{-4} \mathfrak{L}_\odot / M_\odot . \tag{45}$$

It is concluded from equations (44) and (45) that a single B0 star in the cloud would provide sufficient energy deposition to overbalance the loss of energy due to molecular hydrogen. Since the cloud mass is $10^5 M_\odot$, the net absorption rate (45) amounts to only 30 $\mathfrak{L}_\odot$.

When the rate of energy absorption (45) exceeds the rate of loss, the cloud expands. In the polytrope model, the total energy of the gas cloud is

$$-E = -3.9 \times 10^{49} \text{ ergs} . \tag{46}$$

The characteristic expansion time τ for the cloud is the ratio of this energy to the absorption rate (45),

$$\tau = \frac{E}{J_+ M} = 4 \times 10^{14} \text{ sec} . \tag{47}$$

The energy E varies in direct proportion to the central temperature T_0, while the rate of energy absorption J_+ varies directly with T_0 and inversely as the cube of the cloud radius R (eq. [38]), so that the expansion time scale τ varies as R^3. This means that the cloud would have to expand only by a factor of 10 in radius before τ exceeded the age of the Universe.

Under the scalar-tensor theory of gravitation we would expect some of the above numerical values to differ somewhat, but not enough to change the over-all picture.

We obtain a rough estimate of the efficiency of heat transfer from the H II region and the star to the cloud in the following way. For a B0 star at the center of the cloud, the radius of the H II region would be 0.07 pc (Allen 1963). If the star velocity were equal to the mean velocity of a particle in the cloud model, 8×10^5 cm sec^{-1}, the H II region would be sweeping through the gas at the rate of 8×10^{44} atoms sec^{-1}. On multiplying this rate by the energy transfer per atom, $1.5k\Delta T$, we find that the rate of energy transfer to the cloud from the one star is of the order of $5 \times 10^{-6}\ \mathfrak{L}_\odot/M_\odot$. On comparing this result with equation (44), we conclude that the stability of the cloud is not affected if $\mathfrak{N} \sim 1$ but that if $\mathfrak{N} \gtrsim 30$ the cloud apparently might heat up and blow apart.

Massive stars with the high emission temperatures needed to ionize hydrogen and hold up the cloud have lifetimes in the range 10^7–10^8 years in the general-relativity cosmology. Under the Brans-Dicke (1961) cosmology the luminosity and emission temperature of a star increase with the larger gravitational constant in the past, the

TABLE 3

ENERGY BALANCE IN THE CLOUD

Temperature (° K)	$J_-(z)/J_+$
3000	$100z/z_\odot$
1000	$370z/z_\odot$
500	$550z/z_\odot$

former varying with G and M as G^4M^3 and the latter as $G^{3/4}M^{3/8}$. These results are obtained from simple homology arguments. For fixed emission temperature, the necessary stellar mass M varies as G^{-2}, the luminosity varies as G^{-2}, and the stellar lifetime (for a star of a chosen emission temperature) is therefore independent of G. Under the scalar-tensor theory, a larger fraction of the stellar population is blue enough to support the gas cloud. Since $G^2 \sim a^{-1/3} \sim t^{-2/9}$, we have $(G/G_0)^2 = (t_0/t)^{2/9}$. On taking $t_0 = 6.6 \times 10^9$ years to be the present age of the Universe, and $t = 10^6$ years, we have $(G/G_0)^2 = 7$. For $t = 10^7$ and $t = 10^8$ years the values of $(G/G_0)^2$ would be 4 and 2.6, respectively.

When the massive stars die, there may be supernovae in the cloud. The energy release by a supernova exceeds the cloud energy (46), but it is difficult to believe that this energy could be applied efficiently to the destruction of the cloud. The time for a shock wave to travel across the cloud must be in excess of the light travel time at least 30 years, which is well above the time required for radiative loss of the energy. We expect, therefore, that, when the massive stars have died, the cloud contracts again, until a new generation of stars is formed.

The evolved massive stars would dump heavy elements into the cloud. It appears that the same massive stars that serve to photodissociate the molecular hydrogen would be capable of destroying other molecules as well and would also be capable of singly ionizing some of the elements. We therefore adopt Seaton's computation of the radiation rate of ions due to electron collisional excitation (Seaton 1955). The rate of energy loss due to the ions and the rate of energy gain due to the photodissociation of molecular hydrogen both vary as the density and as the electron abundance x_e. Therefore, the ratio of these two rates, energy loss to energy gain, depends only on the temperature of the gas and on the heavy-element abundance. This ratio is evaluated in Table 3. The heavy elements would not contribute appreciably to the opacity of the cloud.

The ratio of energy loss to energy gain in Table 3 increases with decreasing temperature. Therefore, when heavy elements are an important factor, the energy loss is greatest toward the cloud surface and the cloud is convecting. When the heavy-element pollution rises above 1 per cent of the solar abundance, the energy deposition by the stars no longer can hold up the cloud, and the second phase in the evolution of the cloud is terminated. We conclude that, in the present model, the heavy-element abundance in extragalactic globular clusters is about 1 per cent of the solar abundance.

It is not clear how long it would have taken for the heavy-element abundance in the cloud to reach this limiting value. We do know that, by the time the Galaxy had collapsed to the disk, the interstellar heavy-element abundance had become comparable to the abundance in the Sun. Furthermore, one imagines that by this time the bulk of the material would have been cycled through stars about once. Therefore, it may be reasonable to assume that the heavy-element abundance in the cloud reached 1 per cent of the solar abundance when about 1 per cent of the matter had been cycled through stars. The fractional amount of material deposited in stars of low mass in these early star generations depends on the stellar creation function and on the cosmology. If one adopted the creation function deduced from young Population I star clusters, and if one assumed general relativity to be valid, one would conclude that for every B0 star about 300 $M_\odot$ were deposited in stars of lower mass. Under the scalar-tensor cosmology, a large fraction of the stars would be blue and would be capable of supporting the cloud. At 10^8 years the star mass for a given emission temperature (and given lifetime) is reduced by a factor of 2.6. For the Population I creation function, about 40 $M_\odot$ would be deposited in small stars for every B0 star. These are minimum requirements. However, we have noted that if the requirements were too amply met the heat contact might be great enough to blow the cloud apart.

The above numbers suggest that, under general relativity, the cloud might last through three cycles of star formation before the heavy-element pollution triggered the general collapse. Under the scalar-tensor theory a larger number of cycles is indicated. This result is quite uncertain, however, because the stellar creation function for the gas in the cloud could have been quite different from the creation function in present interstellar material (because the opacity in the primeval material is so much lower). In any case, it appears possible to assume that the clouds would have lasted in the gaseous second phase until well after the Galaxy formed, at perhaps 2×10^8 years (Partridge and Peebles 1967).

There remains the question of the final radius of the star cluster. It will be noted that the cloud can hope to contract without turbulence at best when the temperature is less than 10000° K. Once the rate of energy loss is high enough to permit free fall, the collapse will be turbulent, and the turbulent velocities will preserve a memory of the initial radius of the system. For the adiabatic polytrope model this would imply a minimum final radius of a few parsecs. It may be, however, that the velocity dispersion imparted by the turbulence has a long Boltzmann-like tail, in which case the final radius would be substantially larger. In the absence of a better theory for the collapse, we can conclude only that the expected cluster radius in the model is at about the right order of magnitude.

V. INTERMEDIATE AND PECULIAR CLUSTERS

We have emphasized the very distinctive nature of the globular clusters as opposed, for example, to open star clusters. There are, however, a few clusters which might be considered intermediate between the globular and open clusters, and there are also some peculiar outlying globular clusters. One prototype in the Galaxy for possibly transitional or intermediate clusters is NGC 2158 (Arp and Cuffey 1962). Gascoigne (1966) has tentatively identified five clusters of this type associated with the Small Magellanic Cloud. A most dramatic feature of these clusters is the absence of a horizontal branch in the color-magnitude diagram. The age of NGC 2158 is estimated to be about 10^9 years (Arp and Cuffey 1962). Also, Arp and Thackeray (1967) have discussed the star cluster NGC 1866 near the Large Magellanic Cloud. This cluster presents a globular appearance, but the stars in the cluster apparently are quite young.

Another question is provided by the existence of outlying clusters of our Galaxy, clusters undoubtedly globular in form but seemingly requiring two parameters to characterize the color-magnitude diagrams (Woolf 1965; Sandage and Wildey 1967; van den Bergh 1967). A theory of globular clusters might be expected to explain the properties of these outlying systems.

If there truly are young globular clusters, their existence could be a serious complication for our picture. However, one must be cautious about this conclusion, in light of the necessarily uncertain interpretation of the observations and also the difficulty of fully understanding the theoretical situation in the proposed model. There are some points of interpretation that we are not competent to assess. First, NGC 2158 is quite an open cluster; can we be sure that this star cluster truly is related to the globular clusters, and is not simply a Population I system that chances to masquerade as a globular cluster? The same question might apply to NGC 1866. Also, can we be sure that, in the case of the Small Magellanic Cloud clusters, the lack of a horizontal branch is not an indication of a peculiarity in the composition of the stars rather than in the age of the stars?

There are also three important uncertainties in our general theoretical picture. First, it is at least conceivable that some globular clusters survived as gas clouds (in the second phase) until 2×10^9 years ago. This comment would not apply to globular clusters bound in moderately close orbits to our own Galaxy, because the first pass through the disk would suffice to destroy the cloud or else seed stellar formation. The situation could have been different around the Small Magellanic Cloud, as it would be for truly intergalactic globular clusters.

Second, we place the formation of the protoglobular gas clouds earlier than the formation of the protogalaxies because, for random density perturbations, the variance is larger the smaller the averaging volume. This seems to be a perfectly reasonable general assumption, but one could imagine local exceptions. If, for example, there were a primeval localized magnetic field, it could well have influenced the spectral distribution of the density irregularity. It is conceivable that, in the neighborhood of the Small Magellanic Cloud, events conspired to reverse the normal order of formation, very greatly increasing the time required for the formation and slow contraction of the protoglobular gas clouds, so that the clouds could have fragmented to stars only at a comparatively recent epoch.

Still a third possibility exists. As noted above, under the scalar-tensor theory of gravitation a significantly larger fraction of stars are blue enough to support the gas cloud, permitting a longer life for the cloud. Also, under this theory the strong dependence of the luminosity of stars of 1 solar mass upon the gravitational constant results in the apparent or "evolutionary" ages of these clusters differing by much more than their true ages. Proper evolutionary calculations of Population II stars with varying G have not yet been carried out, but simple homology relations may be adequate to determine these ages. Within 10 per cent, similar homology ages agree with the evolutionary ages of Population I stars for the three examples evaluated by Ezer and Cameron (1966) by detailed numerical integrations. An outlying globular cluster of our own Galaxy, or one loosely bound to the Small Magellanic Cloud, could have developed from a cloud moving freely for a long time. But this change in the life of the gas cloud before development of the globular cluster would be mirrored in a change in the age of the cluster and greatly magnified in the corresponding change in the evolutionary age of the cluster. Using homology relations, we find that the luminosity varies as G^7, and the connection between the evolutionary age t^* of a globular cluster and its true age, t, for a flat-space cosmology with $\omega = 4.9$ (Dicke 1962) is

$$t^* = 4T\{1 - [(T - t)/T]^{1/4}\} . \quad (48)$$

Here $T = 6.6 \times 10^9$ years is the present age of the Universe. The time required for a cluster to fall back to a distance of 5 kpc from the center of the Galaxy is given in Table 4 for several assumptions about the maximum distance of the cloud from the Galaxy. The corresponding possible ranges of evolutionary ages of the globular clusters are also given, assuming that the cluster could have been formed as early as 10^7 years after the fireball and as late as the end of the free-fall period of the gas cloud.

TABLE 4

POSSIBLE "STELLAR-EVOLUTION" AGES IN A GLOBULAR CLUSTER FOR THE SCALAR-TENSOR THEORY

Maximum Distance from Center (kpc)	Maximum Free Time*†	Stellar-Evolution Age‡	
		Min *§	Max *‖
10 .	0 082	17 6	21 2
20	0 25	14 8	21 2
30	0 47	12 8	21 2
40	0 74	11 1	21 2
50 . .	1 04	9 8	21 2

* Time unit is 10^9 years

† Time for the cluster to fall back to 5 kpc from center of Galaxy, if maximum distance from Galaxy is given in the first column

‡ Apparent or evolutionary ages assuming $\omega = 4\ 9$.

§ Based on the assumption that the cluster stars formed at the time given in the second column.

‖ Based on the assumption that the cluster stars formed at $t = 10^7$ years.

With the wider range of possible evolutionary ages of outlying clusters, it would not be surprising if such clusters exhibited color-magnitude diagrams requiring a second parameter for their classification, or if some clusters appeared to be decidedly younger than the average. It should be emphasized that this explanation for apparently younger globular clusters should not be considered to be more than a possibility until the status of the gravitational theory is clarified.

VI. CONCLUSIONS

We have considered the expected properties of the first bound systems to have formed out of the expanding Universe. We believe that the coincidence between the properties of the globular clusters and the computed mass, estimated radius at fragmentation into stars, and globular shape for the clouds argues strongly for the general validity of the view that these first systems are in fact protoglobular clusters.

If truly extragalactic globular clusters exist, it is a very important result, for it at least shows that globular clusters need not be formed in the collapse of a protogalaxy. On the present theory, we would in fact expect to find numerous extragalactic globular clusters. It is perhaps too much to expect that gas clouds in the second phase of the evolution still exist; nevertheless, a search for such extragalactic objects would be of considerable interest.

Peebles and Dicke (1968) are quite right that the size, globular shape, uniform brightness, Jeans mass, suggest they are bound systems, but they are not the first bound systems: these are the plasma protogalaxies PGs that fragmented at density minima provided by fossil big bang turbulent combustion vortex lines. The Jeans clumps of dark matter planets are the second bound system. PGs are the first.

The "intermediate" clusters at the moment appear to present the most severe challenge to the picture. We have reasoned, however, that the evidence here is not compelling at the moment, and there are possible explanations for these objects within the framework of the proposed theory.

We greatly benefited by numerous discussions of this subject with, among others, A. G. W. Cameron, J. Ostriker, R. B. Partridge, M. Schwarzschild, P. Solomon, L. Spitzer, and D. T. Wilkinson. We should like to take this opportunity to thank our colleagues for these discussions, without, of course, implying that any of them necessarily subscribe to the views we have presented.

REFERENCES

Allen, C. W. 1963, *Astrophysical Quantities* (2d ed.; London: Athlone Press), p. 254.
Alpher, R. A. 1948, *Phys. Rev.*, **74**, 1577.
Arp, H. C. 1965, in *Stars and Stellar Systems*, Vol. 5: *Galactic Structure*, ed. A. Blaauw and M. Schmidt (Chicago: University of Chicago Press), p. 401.
Arp, H. C , and Cuffey, J. 1962, *Ap. J.*, **136**, 51.
Arp, H. C., and Thackeray, A. D. 1967, *Ap. J.*, **149**, 73.
Bates, D. R. 1951, *M.N.R.A.S.*, **111**, 303.
Bergh, S. van den. 1967, *Pub. A.S.P.*, **79**, 460.
Boardman, W. J. 1964, *Ap. J. Suppl.*, No. 9, **90**, 185.
Bok, B. J. 1966, *Ann. Rev. Astr. and Ap.*, **4**, 95.
Brans, C., and Dicke, R. H. 1961, *Phys. Rev.*, **124**, 925.
Callan, C., Dicke, R. H., and Peebles, P. J. E. 1965, *Am. J. Phys.*, **33**, 105.
Chandrasekhar, S. 1958, *Ap. J.*, **128**, 114.
Dicke, R. H. 1962, *Rev. Mod. Phys.*, **34**, 110.
———. 1968, *Ap. J.*, **152**, 1.
Dicke, R. H., Peebles, P. J. E., Roll, P. G., and Wilkinson, D. T. 1965, *Ap. J.*, **142**, 414.
Doroshkevich, A. G., Zel'dovich, Ya. B., and Novikov, I. D. 1967, *Astr. Zh.*, **44**, 295; English trans. in *Soviet Astr.—AJ*, **11**, 233, 1967.
Emden, R. 1907, *Gaskugeln* (Berlin: Teubner).
Ezer, D., and Cameron, A. G. W. 1966, *Canadian J. Phys.*, **44**, 593.
Gamow, G. 1948, *Phys. Rev.*, **74**, 505.
Gascoigne, S. C. B. 1966, *M N.R.A.S* , **134**, 59.
Hodge, P. W. 1965, *Ap. J.*, **141**, 308.
Layzer, D. 1964, *Ann. Rev. Astr. and Ap.*, **2**, 341.
McCrea, W. H., and Milne, E. A. 1934, *Quart. J. Math., Oxford Series*, **5**, 73.
McDowell, M. R C. 1961, *Observatory*, **81**, 240.
Partridge, R. B., and Peebles, P. J. E. 1967, *Ap. J.*, **147**, 868.
Peebles, P. J. E. 1965, *Ap. J.*, **142**, 1317.
———. 1967*a*, Proc. 4th Texas Conf. Relativistic Ap., New York, January 1967 (to be published).
———. 1967*b*, *Ap. J.*, **147**, 859.
———. 1968, *ibid.*, **153**, 1.
Sandage, A. R., and Wildey, R. 1967, *Ap. J.*, **150**, 469.
Schmeltekopf, A. L., Fehsenfeld, F. C., and Ferguson, E. E. 1967, *Ap. J. (Letters)*, **148**, L155.
Seaton, M. J. 1955, *Ann. d'ap* , **18**, 188.
Spitzer, L., Jr. 1949, *Ap. J.*, **109**, 337.
Stecher, T P., and Williams, D. A. 1967, *Ap. J. (Letters)*, **149**, L29.
Stokes, R. A , Partridge, R. B , and Wilkinson, D. T. 1967, *Phys. Rev. Letters*, **19**, 1199.
Takayanagi, K., and Nishimura, S. 1960, *Publ. Astr. Soc. Japan*, **12**, 77.
Weizsäcker, C. F. von. 1955, *Zs. f. Ap.*, **35**, 252.
Woolf, N. J. 1965, Proc. 3d Texas Conf. Relativistic Ap., Miami, December 1965.

The present paper does not show how plasma epoch kinematic viscosity and plasma epoch turbulence affect the formation of plasma epoch protogalaxies at 10^12 seconds. These PGs form the nucleus of all galaxies at the plasma to gas transition at 10^13 seconds (0.3 Myr) when protoglobularstarcluster PGCs and dark matter planets simultaneously fragmented with the density and rate of strain fossilized from the earlier time when structure formation began. PGCs fragmented at fossil plasma turbulence vortex line density minima. PGs fragmented at fossil big bang turbulent combustion vortex line density minima. Both are at Kolmogorov scales determined by the kinematic viscosity of the plasma for PGs and the gas for PGCs. This HGD conclusion fits observations.