Journal of Cosmology, 2010, Vol 6, 1373-1380
Cosmology, March 25, 2010

Galactic Rotation Described by a Thin-Disk Gravitational Model without Dark Matter

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Abstract

A gravitational model is presented consisting of an axisymmetric thin disk of finite radius, capable of describing the measured rotational velocity profiles (i.e., the rotation curves) of mature spiral galaxies. Without loss of generality, the disk is assumed to have a uniform thickness but with a mass density variable in the radial coordinate. The governing integral equation, based on mechanical balance between Newtonian gravitational force and centrifugal force due to galaxy rotation at each and every point on the disk, is solved numerically to determine the radial mass density distribution for a given rotation curve. The nondimensionalized mathematical system contains a dimensionless parameter which we call the "galactic rotation number" that represents the ratio of centrifugal force and gravitational force. Together with a constraint equation for mass conservation, the value of the galactic rotation number can be determine as part of the numerical solution. With a known value of the galactic rotation number, the total galactic mass can be calculated from measured galactic radii and maximum rotation velocities. The predicted total galactic masses are in good agreement with star-count data. Our computed mass density shows a rapid decrease within the central core followed by a slower nearly exponential decay outward from the galactic center and then takes a sharp drop at the galatic edge. However our predicted mass densities show generally slower decaying rates toward the galactic periphery than that of measured brightness, consistent with more ordinary baryonic mass in the outer disk regions having relatively lower light emissivity (and thus appearing darker).

Key words: Galactic rotation; disk gravitational model; rotation curve; Newton's law of gravity;

1 Introduction

The *dark matter* came about for the need to explain "the first signs of trouble" in the galaxy rotation problem (cf. Freeman & McNamara 2006). It is a problem of the apparent discrepancy between the observed rotation speeds of matter in the disk of spiral galaxies and the predictions of Newtonian dynamics considering the *visible* mass. Here, the *predictions of Newtonian dynamics* have often been regarded as the Keplerian rotation curve with orbital velocity decreasing inversely with square root of the the distance from center, as being firmly verified for planetary rotations in the solar system (where the gravitational potential is spherically symmetric due to highly concentrated central mass). But many measurements have shown that the typical galactic rotation curve–the orbital velocity of matter versus radial distance–for mature spiral galaxies differs substantially from the Keplerian rotation curve; therefore, the correctness of the predictions of Newtonian dynamics considering the visible mass becomes questioned. If Newton's laws of motion and gravity are valid, the mass distribution must differ from that of the *visible* mass. Thus, the concept of *dark matter*–a hypothetical matter that is undetectable by its radiation emission, i.e., *invisible*, but its presence can be inferred from gravitational effects on visible matter–was introduced in astronomy and cosmology. Nowadays, the *dark matter* is speculated as something that is not only *dark* but also *non-baryonic* with all kinds of mysterious behavior that has then invoked tremendous efforts of scientific research without much well-founded evidence.

But, what is the *visible* mass? By visible it must emit or reflect 'light'. But, how can the amount of the mass be determined from the measurement of the emitted or reflected light? The answer may not be very convincing. A *mass-luminosity relationship* (as also called *mass-to-light ratio*) is used by astronomers to determine luminous (or visible) mass of a galaxy from the measured brightness (Freeman & McNamara 2006). However, there is no concrete physical basis for the assumption of constant mass-to-light ratio in galaxies. Observations of disk galaxies reveal a dark edge against a brighter background bulge from edge-on views, indicating a substantial radial gradient of the mass-to-light ratio. Thus, any conclusions drawn from the calculations based on a mass distribution inferred from the brightness distribution in a galaxy should be regarded as derived from *irrigorous* logic on a shaky ground.

Yet, the *dark matter* required from the dark halo to produce the observed rotation curves for mature spiral galaxies seems to be just established on such a shaky ground.

Instead of following the Keplerian inverse square root decay, the measurements of galactic rotation curves for mature spiral galaxies (e.g., Volders 1959; Rubin & Ford 1970; Burstein & Rubin 1985; Persic & Salucci 1995) have shown that galactic rotation velocity typically rises linearly from the galactic center (as if the local mass was in rigid body rotation) in a relatively small core, and then reaches a nearly constant (flat) value extending to the galactic periphery. Thus, the essential features of galactic rotation curves may be mathematically represented as

$$V(r) = 1 - e^{-r/R_c}\,, \tag{1}$$

where $V(r)$ denotes the (dimensionless) orbital velocity. The nondimensionalization here is done with velocity measured in units of the maximum velocity in the flat part that may be regarded as the characteristic velocity of galactic rotation V_0, and r the radial coordinate from the galactic center in units of the outermost galactic radius R_g. The parameter R_c corresponds to the observed 'core' radius within which the velocity rises rapidly. Figure 1 shows typical galactic rotation curves described by (1).

With an understanding that Keplerian *orbital velocity law* comes from a simple spherically symmetric gravitational field (e.g., that due to a dominant central point mass as in the solar system), it should not be surprising to find that the rotation curves of mature disk shaped galaxies do not obey the same orbital velocity law, because the gravitational field in disk galaxies having a highly anisotropic mass distribution can be far from spherically symmetric (cf. Binney & Tremaine 1987). But still, many authors kept applying the Keplerian orbital velocity law directly to the a disk galaxy system, to calculate the galactic mass distribution from the measured galactic rotation curves (cf. Rubin 2006; Bennett et al. 2007). Such a questionable practice led to the conclusion that the galactic mass *must increase* with radial position, whereas the measured galactic (surface) brightness suggests otherwise if a constant mass-to-light ratio is assumed. Such a sharp discrepancy between two opposing conclusions seemed to motivate the speculation of dark matter in cosmology.

The gravitational field for disk mass distribution is certainly not as simple as that for spherically symmetric mass distribution, but can still be rigorously computed with the available mathematical tools (cf. Binney & Tremaine 1987). Hence attempts were made to employ gravitational disk models

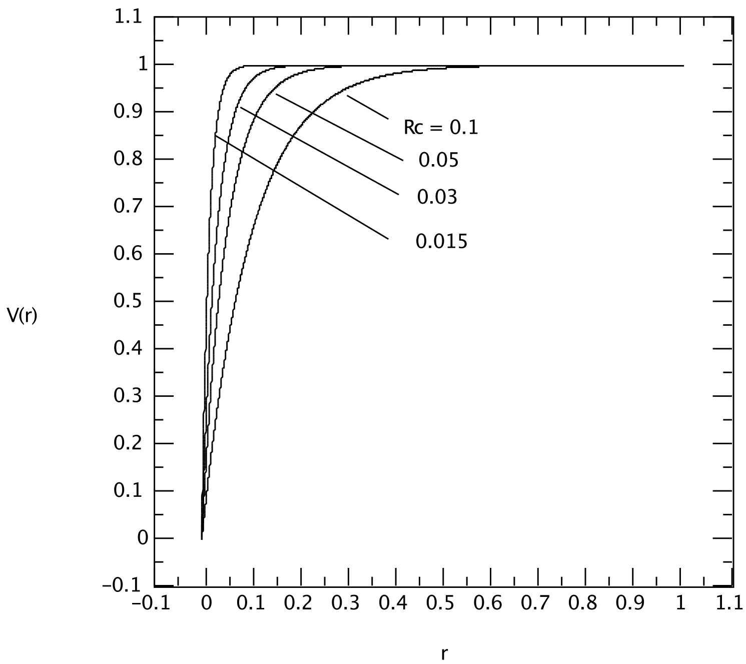

Figure 1: The measured galactic rotation curves represented as nondimensionalized orbital velocity $V(r)$ as a function of the radial distance from the galactic center r according to (1) for galactic "core radius" $R_c = 0.015$, 0.03, 0.05 and 0.1.

to predict galatic rotation curves from given mass distributions. However, the mass distribution considered by those authors is not obtained from direct measurement, but rather from the measured brightness distribution (often exhibiting an exponential decay with radial distance) assuming a *constant* mass-to-light ratio. As the observed (flat) rotation curves were not obtained with those gravitational disk models (e.g., Freeman 1970; Freeman & McNamara 2006), recourse is often made to either mysterious dark matter or modification of Newtonian dynamics (cf. Milgrom 1983) to compensate the discrepancies. Because the assumption of constant mass-to-light ratio does not seem to be established on concrete physical ground, we do not believe models based on such an assumption can lead to convincing conclusions with reasonable logical rigor.

In view of the fact that either the mass distribution or rotation curve for a disk galaxy can be uniquely determined when the other is given, in the present work we choose to solve for the mass distribution with a given rotation curve. Therefore, we do not need to rely on any questionable assumption about the mass-to-light ratio. This approach is also in the same vein as those adopted by several authors (e.g., Baillon & Mizony 1995; Mera et al. 1998; del Rio et al. 1998; Nicholson 2000, 2001, 2003; Mizony 2003, 2007; Marmet 2005; Feng & Gallo 2007, 2008; Gallo & Feng 2009). Because of the difficulties in finding analytical solutions to exactly match a rotation curve of the form shown in Fig. 1 (e.g., Mestel 1963), we compute numerical solutions by discretizing the governing equations with finite-element basis functions and then solving the resultant linear-algebra problem (Feng & Gallo 2007, 2008; Gallo & Feng 2009). When computed based on the well-established physical laws (i.e., classical Newtonian gravity-dynamics or general relativity), the mass distribution exhibits the common feature of generally descreasing density outward from galactic center corresponding to a typically observed rotation curve. No need was suggested for modification

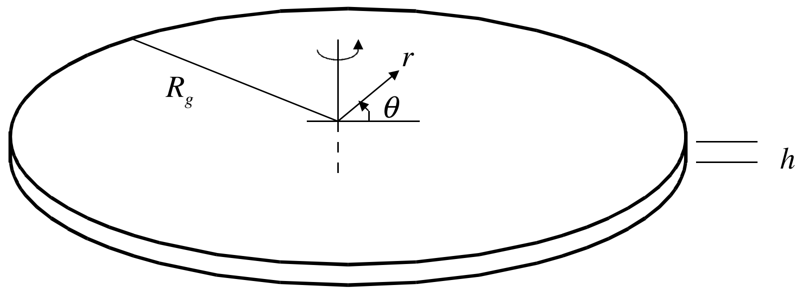

Figure 2: Definition sketch of the thin-disk model considered in the present work. The mass is assumed to distribute axisymmetrically in the circular disk of uniform thickness h with a variable density as a function of radial coordinate r (but independent of the polar angle θ). Here the outmost radius of the disk galaxy is denoted as R_g.

of Newtonian dynamics or introduction of massive peripheral spherical halos of mysterious dark matter.

2 Governing Equations

As shown in Fig. 2, the present model consists of an axisymmetric thin disk of finite size (with an outermost radius R_g), uniform thickness h, but variable mass density ρ as a function of radial coordinate r. The goal is to determine a radial mass density distribution within the disk to satisfy a given rotation curve.

At a steady state, the gravitational forces must balance the centrifugal forces at each and every point in a thin disk as follows.

$$\int_0^1 \left[\int_0^{2\pi} \frac{(\hat{r}\cos\theta - r)d\theta}{(\hat{r}^2 + r^2 - 2\hat{r}r\cos\theta)^{3/2}} \right] \rho(\hat{r}) h \hat{r} d\hat{r} + A \frac{V(r)^2}{r} = 0 \,, \tag{2}$$

where all the variables are made dimensionless by measuring lengths (e.g., r, $\hat{r}$, h) in units of the outermost galactic radius R_g, disk mass density ρ in units of M_g/R_g^3 with M_g denoting the total galactic mass, and velocities $V(r)$ in units of the characteristic galactic rotational velocity V_0 (e.g., the maximum target velocity of galactic rotation as in (1)). The disk thickness h is assumed to be constant and small in comparison with the galactic radius R_g. The results (such as the surface mass density $\rho(r)\,h$) are expected to be insensitive to the exact value of h as long as it is small. The gravitational forces of a series of concentric rings is described by the first term (double integral) while the centrifugal forces are described by the second term.

Our process of nondimensionalization of the force-balance equation yields a dimensionless parameter, which we call the "galactic rotation number" A, as given by

$$A \equiv \frac{V_0^2\, R_g}{M_g\, G} \,, \tag{3}$$

where G ($= 6.67\times10^{-11}$ [$\mathrm{m}^3/(\mathrm{kg\ s}^2)$]) denotes the gravitational constant, R_g is the outermost galactic radius, and V_0 is the characteristic velocity (which is equated here to the maximum asymptotic rotational velocity). This galactic rotation number A simply describes the ratio of centrifugal forces to gravitational forces.

The total mass of the galaxy M_g is assumed to be constant satisfying the constraint

$$2\pi \int_0^1 \rho(\hat{r}) h \hat{r} d\hat{r} = 1. \tag{4}$$

Equations (2) and (4) are used to determine the mass density distribution $\rho(r)$ in the disk, the galactic rotation number A, and the total galactic mass M_g, all from measured values of $V(r)$, R_c, R_g and V_0. This is a well-defined mathematical problem deducible from the input data. With appropriate mathematical treatments, it can be transformed into a set of linear algebra equations with solution computed using a matrix solver (e.g., Feng & Gallo 2007, 2008; Gallo & Feng 2009).

3 Computational Results

To compute numerical solutions, the value of disk thickness h must be provided. Without loss of generality, we assume $h = 0.01$ (based on the estimate for the Milky Way galaxy).

3.1 Mass Distributions

The computed galactic mass distributions corresponding to the galactic rotation curves (given by Eq. (1) and illustrated in Fig. 1) are shown in Fig. 3. From the galactic center, the mass density tends to decrease rapidly (with a slope becoming steeper for a tigher galactic core described by smaller R_c). However, beyond R_c, the mass density decrease more slowly towards the galactic periphery until reaching the galactic edge where it takes a sharp drop. Noteworthy here is that the computed values of galactic rotation number A are within a small range around 1.6 despite an order-of-magnitude variation of the galactic core radius R_c.

As apparent in Fig. 3, the computed $\log\rho$ decreases almost linearly with r except for $r < 0.1$ (within the central galactic core) and $r > 0.9$ (near the galactic edge). This general feature is quite similar to the measured brightness distributions (in typical spiral galaxies) that are commonly fitted in an exponential form with regions of central core and outer edge truncated. For the case of $R_c = 0.015$ and $A = 1.5703$, a least-square fit of our computed $\log\rho$ versus r for $0.1 \le r \le 0.9$ yields

$$\log\rho = 5.4179 - 3.6802\, r\,. \tag{5}$$

This actually corresponds to an exponential function $\rho = \rho_0\, e^{-r/R_d}$ with

$$\rho_0 = 225.4 \text{ and } R_d = 0.2717\,, \tag{6}$$

which is known not to be able to generate the observed flat rotation curves (Binney & Tremaine 1987). Thus the much more rapid decrease of $\log\rho$ in the small intervals [0, 0.1) and (0.9, 1] must play important roles in compensating the deficiencies of the simple exponential mass density distribution for matching the commonly observed rotation curves.

3.2 Predicted Total Galactic Mass

From the knowledge of V_0 and R_g from measured rotation curves, we can determine the value of M_g based on computed value of the galactic rotation number A (cf. (3)) as

$$M_g = \frac{V_0^2\, R_g}{A\, G}\,. \tag{7}$$

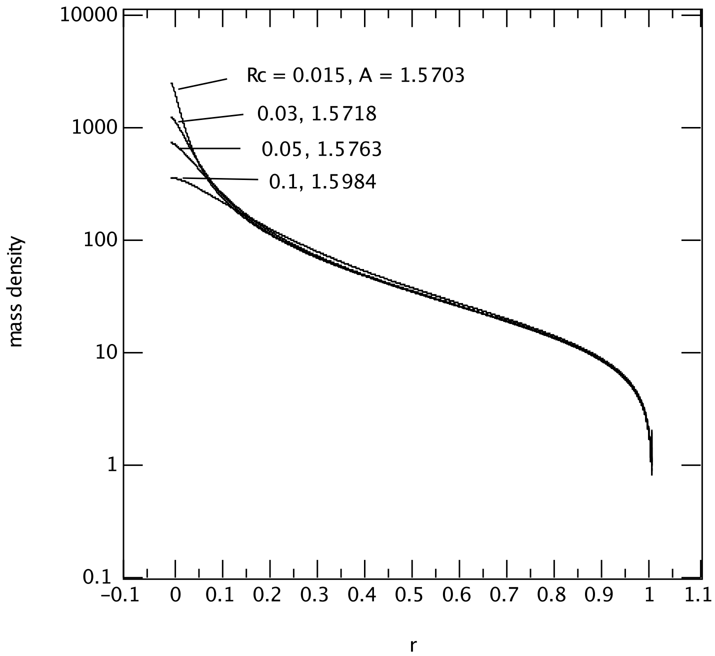

Figure 3: The computed distributions of mass density $\rho(r)$ as function of radial distance from the galactic center r for galactic "core radius" $R_c = 0.015$, 0.03, 0.05, and 0.1 with $A = 1.5703$, 1.5718, 1.5762, and 1.5984 determined as part of the numerical solutions.

If we take the rotation curve with $R_c = 0.015$ in Fig. 1 as that of the Milky Way, we have the galactic rotation number $A = 1.57$. Then, from measured Milky Way values $V_0 = 2.5 \times 10^5$ (m/s) and $R_g = 5 \times 10^4$ (light-years) $= 4.73 \times 10^{20}$ (m), (7) yields

$$M_g = 2.83 \times 10^{41}(\text{kg}) = 1.4 \times 10^{11}(\text{solar-mass})\,.$$

This value is in very good agreement with the Milky Way star counts of 100 billion (Sparke & Gallagher 2007), further including additional dust, grains, lumps, gases and plasma in all galaxies.

With the given values of M_g and R_g, we can estimate the computed 'radial scale' $R_d\, R_g = 4.0$ (kpc) and the exponential disk central (surface) mass density $\rho_0 h M_g / R_g^2 = 1340$ (solar-mass / pc^2) based on (5) for the Milky Way galaxy. Compared with the results from fitting the brightness measurement data, e.g., the radial scale of 2.5 (kpc) and exponential disk central brightness of 867 (solar-luminosity / pc^2) (Freudenreich 1998), our computational results indicate that throughout the Milky Way the mass density decreases at a slower rate than that of the brightness.

Another example is the galaxy NGC3198, which has a (nearly idealized, often cited) rotation curve with $V_0 = 1.5 \times 10^5$ (m/s), $R_g = 30$ (kpc) $= 9.24 \times 10^{20}$ (m), and $R_c \sim 0.015$. Again with $A = 1.57$, we obtain $M_g = 1.98 \times 10^{41}$ (kg) $= 9.9 \times 10^{10}$ (solar-mass). As with the Milky Way, we can also predict the radial scale and exponential central mass density for NGC3198 as 8.16 (kpc) and 250 (solar-mass / pc^2), respectively (based on $\rho_0 = 225.4$ and $R_d = 0.2717$ from (5). Compared with the radial luminosity profile (which suggested an exponential disk with a radial scale of 2.63 (kpc) and central brightness 212 (solar-luminosity / pc^2) based on a total luminosity of 9.0×10^9 (solar-luminosity) (cf. Begeman 1989), our predicted mass density appears to decrease much more slowly with stars generally dimmer than the Sun.

4 Conclusions

The measured rotation behavior of mature spiral galaxies can be consistently described with an axisymmetric thin-disk model and Newton's laws of motion and gravity. Our computed (surface) mass density distribution has similar charateristics as that of measured brightness distribution, with most of the region being represented reasonably with a simple exponential function except in the central core and near the outer edge. However, our computed mass density decreases at a slower rate than that of measured brightness, suggesting the matter in the outer regions has relatively lower light emissivity (and thus appears darker).

If the galactic mass density is assumed to follow the measured brightness distribution (implying a constant mass-to-light ratio), the observed galactic rotation curve cannot be obtained as shown by many previous authors (Freeman & McNamara 2006). But the assumption of constant mass-to-light ratio is not physically reasonable since both the temperature and emissivity can vary. In principle, there cannot be a simple relationship between mass and luminosity. The obvious dark edge line against a bright background bulge commonly seen in edge-on views of spiral galaxies suggests substantial radial mass-to-light gradient. Yet, such an inconvincible assumption that led to failure in describing the observed rotation curves has motivated speculations of mysterious dark matter or modification of Newtonian gravitational theory. If exists, the dark matter must have mysterious (non-baryonic) properties because there is no direct evidence of its existence and it does not respond to gravitational, centrifugal, and electromagnetic forces in any known manner.

By contrast, our computational results indicate that to consistently describe the observed rotational behavior of typical spiral galaxies (with appropriate mathematical treatments and application of well-established physical laws), there is no need to introduce massive peripheral spherical halos of mysterious dark matter and no need to modify Newton's law of gravity.

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