

Farewell to black hole horizons and singularities?

January 14, 2012

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Abstract

We consider the fundamental issues which dominate the question about the existence or non-existence of black hole horizons and singularities from both of the theoretical and observational points of view, and discuss some of the ways that black hole singularities can be prevented from forming at a classical level, i.e. without arguments of quantum gravity. In this way, we argue that black holes could have a different nature with respect

the common belief. In fact, even remaining very compact astrophysics objects, they could be devoid of horizons and singularities.

Our analysis represents a key point within the debate on the path to unification of theories. As recently some scientists partially retrieved the old Einstein's opinion that quantum mechanics has to be subjected to a more general deterministic theory, a way to find solutions to the problem of black hole horizons and singularities at a semi-classical level, i.e. without discussions of quantum gravity, becomes a fundamental framework.

This paper is the latest work co-authored by Darryl Jay Leiter, February 25, 1937 - March 4, 2011

The concept of *black hole* (BH) has fascinated scientists long before the introduction of Einstein's general relativity (see [1] for a historical review). However, an unsolved problem concerning BHs is the presence of a spacetime singularity in their core. Such a problem, originally stated in the historical papers concerning BHs [2, 3, 4], was later generalized in the famous paper by Penrose [5]. It is a common opinion that this problem will be solved when a correct quantum gravity theory is finally constructed (see [6] for recent developments).

In this review, we discuss fundamental issues which dominate the question about the existence or non-existence of BH horizons and singularities from both of the theoretical and observational points of view, and we analyse some ways to avoid the development of BH singularities at a semi-classical level which does not require the need for a quantum gravity theory. On the other hand, at the present time, an absolute quantum gravity theory, which implies a total unification of various interactions has not been obtained and, from a historical point of view, Einstein believed that, in the path to unification of theories, quantum mechanics had to be subjected to a more general deterministic theory, which he called generalized theory of gravitation, but he did not obtain the final equations of such a theory (see for example the biography of Einstein which has been written by Pais [7]). At present, this point of view is partially retrieved by some theorists, starting from the Nobel Laureate G. 't Hooft [8]. However in this context it is a fundamental requirement that solutions to the problem of BH horizons and singularities be obtained at a semi-classical level, i.e. without discussions of quantum gravity. An important point is that it has been recently shown that the true BHs should have $M = 0$, hence the so called BHs could be something else [28].

We begin by noting that in general relativity the Einstein equation relates the curvature tensor of spacetime on the left hand side to the energy-momentum tensor in spacetime on the right hand side [1]. Within the context of the Einstein equation the *strong principle of equivalence* (SPOE) requires that special relativity must hold locally for all of the laws of physics in all of spacetime as seen by time-like observers (see Section 2.1 of [9]). Hence, in the context of the SPOE this implies that the frame of reference of co-moving observers within a gravitationally collapsing object are required to always be able to be connected to the frame of reference of stationary observers by special relativistic transformations with physical velocities which are less than the speed of light. Recently plausible

arguments have been made which support the idea that physically acceptable solutions to the Einstein equation will only be those which preserve the SPOE as a *law of nature* in the universe [10, 11, 12]. The observable consequence of preserving the SPOE as a *law of nature* would be that compact objects which emerge from the process of gravitation collapse could not have event horizons (EHs) because their existence would prevent co-moving observers within a gravitationally collapsing object from being able to be connected to the frame of reference of stationary observers by special relativistic transformations with physical velocities which are less than the speed of light. Hence, as a result of the SPOE, objects having EHs with non-zero mass would be physically prohibited [10, 11, 12]. In particular, the preservation of the SPOE in the Einstein equation would put an overall constraint on the nature of the non-gravitational physical elements which go into the energy-momentum tensor on the right hand side of the Einstein equation. However this constraint would not uniquely determine the specific form of the non-gravitational dynamics of the energy-momentum tensor [10, 11, 12]. For this reason many different theories can be constructed (e.g. *eternally collapsing objects* (ECO), *magnetospheric eternally collapsing objects* (MECO), *nonlinear electrodynamics* (NLED) *objects*, which preserve the SPOE and hence can generate highly redshifted compact objects without EHs [10, 11, 12, 13]. Since each of these different SPOE preserving theories have unique observational predictions associated with the interaction of their non-gravitational components with the environment of their highly redshifted compact objects without EHs, the specific one chosen by Nature can only be determined by astrophysical observations which test these predictions.

Thus, it is quite important to clearly identify some of the observational constraints [10, 11, 12, 14]. There are two major classes of BH candidates, both of which seem to be quite consistent with the possibility that they might be BHs. There are galactic BH candidates (GBHC) found primarily among the x-ray binary systems and supermassive active galactic nuclei (AGN). In quiescent states both are exceedingly faint, consistent with the possibility that they might not have radiating surfaces [10, 11, 12, 14]. For both classes of quiescent objects, it is plausible, though not proven, that matter might be accreting to the central object and disappearing without radiating, as through an EH. In the case of the GBHC, it has been established that many are more massive than the approximately 3 solar mass allowed for objects supported by internal kinetic or degeneracy pressure [10, 11, 12, 14]. The 3 solar mass limit in the compact object is determined by relativistic causality, which requires that the speed of sound must remain less than the speed of light while kinetic pressures become relativistic. To exceed the 3 solar mass limit, as many GBHC do, they must either be BHs or something other than cold catalyzed matter. Since gravity can be locally transformed away it cannot locally determine either the state of matter or its equation of state. Because of this fact exotic matter objects seem to be unlikely candidates for GBHC and AGN. On the other hand, a more likely possibility for GBHC and AGB involves ECO or MECO, which are hot compact objects supported by internal radiation pressure in a pair dominated baryon plasma suspended in an Eddington balance which prevents trapped surfaces

leading to EHs from forming. In this context, in order to satisfy the quiescent luminosity constraints, the surface radiations from the ECO or MECO objects must be extremely redshifted [10, 11, 12, 14].

Among galactic nuclei Sagittarius A* (Sgr A*) has been observed with sufficient resolution to establish that it is very small, already less than a few gravitational radii [15]. Clusters of stars packed tightly enough to account for its mass cannot remain stable for billions of years. Thus, Sgr A* seems likely to be a single object of some kind. The accretion environment of Sgr A* should be capable of supplying about $3 * 10^{-6}$ solar mass per year in a Bondi flow into Sgr A* [10, 11, 12, 14], but only about 10^{-10} solar mass per year is needed to account for its bolometric luminosity. Either the Bondi flow is very radiatively inefficient with most of the mass disappearing through an EH, or there must be some mechanism that ejects most of the mass. In this regard, the polarization of radiation from Sgr A* constrains the flow that can get with a few gravitational radii of Sgr A* to be less than about 10^{-9} solar mass per year. This very strongly suggests that an ejection mechanism operates. Finally, if Sgr A* would have a surface that radiated in equilibrium with an accretion rate, the accretion rate must be less than about 10^{-13} solar mass per year [16].

The MECO model in [10, 11, 12, 14] would have a strong enough magnetic field to eject most of the Bondi flow into Sgr A*. An ECO model with a substantially weaker magnetic field would presumably have to rely on the same kinds of ejection mechanisms that have been proposed for BH models. A similar situation applies to the GBHC. The MECO model provides a strong enough magnetic field to drive the spectral state switches observed for both neutron stars and GBHC in x-ray binaries (and also in AGN). ECO objects with substantially weaker magnetic fields would have to rely on the same accretion disk mechanisms that have been proposed for black hole models.

Thorne showed back in 1965 that pure magnetic energy would not collapse into a BH state [17]. It may be that there are stable gravitationally compact objects that are composed of relatively cool matter and magnetic fields without being either ECO or MECO as presently understood. But such objects would also be subject to the quiescent accretion rate-luminosity constraints. So it seems that the nature of the surface and the strength of intrinsic magnetism are crucial issues. [10, 11, 12, 14].

An ECO is a gravitationally compact mass supported against gravity by internal radiation pressure [18]. In its outer layers of mass, a plasma with some baryonic content is supported by a net outward flux of momentum via radiation at the local Eddington limit L_{Edd} given by (note: we work with in natural units in this paper, i.e. $G = 1$, $c = 1$ and $\hbar = 1$, while ordinary units will be used only for numerical results)

$$L_{Edd,s} = \frac{4\pi M(1 + z_s)}{\kappa}. \quad (1)$$

Here κ is the opacity of the plasma, subscript s refers to the baryon surface layer and z_s is the gravitational redshift at the surface. In general relativity z is given by [1]

$$1 + z = \frac{1}{\sqrt{1 - \frac{2M}{R}}}. \quad (2)$$

For a hydrogen plasma, in ordinary units it is $\kappa = 0.4 \text{ cm}^2 \text{ g}^{-1}$ and

$$L_{Edd,s} = 1.26 \times 10^{38} m(1 + z_s) \text{ erg s}^{-1}, \quad (3)$$

where $m = \frac{M}{M_\odot}$ is the mass in solar units.

Since the temperature at the baryon surface is beyond that of the pair production threshold and the compactness is large enough to guarantee a high rate of photon-photon collisions [19], there is a pair atmosphere further out that remains opaque. The net outward momentum flux continues onward, but diminished by two effects, time dilation of the rate of photon flow and gravitational redshift of the photons. The escaping luminosity at a location, where the redshift is z , is thus reduced by the ratio $\left(\frac{1+z}{1+z_s}\right)^2$, and the net outflow of luminosity as radiation transits the pair atmosphere and beyond is

$$L_{net \ out} = \frac{4\pi M(1+z)^2}{\kappa(1+z_s)}. \quad (4)$$

Finally, as distantly observed where $z \rightarrow 0$, the luminosity is

$$L_\infty = \frac{4\pi M}{\kappa(1+z_s)}. \quad (5)$$

For hydrogen plasma opacity of $0.4 \text{ cm}^2 \text{ g}^{-1}$ and a typical stellar mass GBHC of $7M_\odot$ this equation yields $L_\infty = 8.8 \times 10^{38}/(1+z_s) \text{ erg s}^{-1}$ in standard units. But since the quiescent luminosity of a GBHC must be less than about $10^{31} \text{ erg s}^{-1}$, we see that it is necessary to have $z_s > 8.8 \times 10^7$. Even larger redshifts are needed to satisfy the quiescent luminosity constraints for AGN. This is extraordinary, to say the least, but perhaps no more incredible than the $z = \infty$ of a BH. At the low luminosity of Eq. (3), the gravitational collapse is characterized by an extremely long radiative lifetime, τ [10, 18] given by:

$$\tau = \frac{\kappa(1+z_s)}{4\pi}, \quad (6)$$

i.e. $\approx 4.5 \times 10^8(1+z_s) \text{ years}$ in ordinary units.

With the large redshifts that would be necessary for consistency with quiescent luminosity levels of BHC, it is clear why such a slowly collapsing object would be called an “*eternally collapsing object*”.

For $(1+z) > \sqrt{3}$, radiation is impeded by passage through a small escape cone such that the fraction of radiation that could escape if isotropically emitted at radius R would only be

$$f = 27 \left[\frac{2M}{R(1+z)} \right]^2. \quad (7)$$

For very large z , $R \approx 4M$ and $f = \frac{27}{4(1+z)^2}$.

At the outskirts of the pair atmosphere of an ECO the photosphere is reached. Here the temperature and density of pairs has dropped to a level from which photons can depart without further scattering from positrons or electrons. Nevertheless, the redshift is still large enough that their escape cone is small and most photons will not travel far before falling back through the photosphere. If we let the photosphere temperature, redshift and star's surface be T_p , z_p and σ respectively, the net escaping luminosity is [10]

$$L = \frac{27(2M)^2 4\pi R^2 \sigma T_p^4}{R^2(1+z_p)^2} = \frac{4\pi M(1+z_p)^2}{\kappa(1+z_s)}. \quad (8)$$

But in the radiation dominated region beyond the photosphere, the temperature and redshift are related by

$$T_\infty = \frac{T}{1+z}, \quad (9)$$

where T_∞ is the distantly observed radiation temperature. Substituting into the previous equation, we obtain

$$L_\infty = (27)4\pi(2M)^2 \sigma T_\infty^4 = \frac{4\pi M}{\kappa(1+z_s)}, \quad (10)$$

and, for hydrogen plasma opacity, by restoring ordinary units it is

$$T_\infty = \frac{2.3 \times 10^7}{[m(1+z_s)]^{\frac{1}{4}}} \text{ Kelvins}. \quad (11)$$

The left equality of Eq. (9) can be written in terms of the distantly observed spectral distribution, for which the radiant flux density at distance $R(> 6M)$ and frequency ν_∞ would be

$$F_{\nu_\infty} = 4\pi^2 \nu_\infty \frac{27(2M)^2}{R \left[\exp\left(\frac{2\pi\nu_\infty}{kT_\infty}\right) - 1 \right]}. \quad (12)$$

k in Eq. (12) is the Boltzmann constant.

As previously discussed [10, 12], if one naively assumes that the photon support for an ECO originates from purely thermal processes, one quickly finds that the temperature in the baryon surface layer would be orders of magnitude higher than the pair production threshold. The compactness guarantees that photon-photon collisions would produce numerous electron-positron pairs [19]. This makes the baryon surface a phase transition zone at the base of an electron-positron pair atmosphere. Drift currents proportional to $\frac{\vec{q} \times \vec{B}}{B^2}$ reactively generate extreme magnetic fields there. The stability of the Eddington limited MECO requires that the surface magnetic field, on the Eddington limited baryon surface of the MECO, will be quantum electrodynamically limited to have the values about $10^{20} - 10^{22}$ Gauss required to create a surface density of

bound electron-positron pairs in the baryon plasma [20, 21]. On the other hand the interior magnetic field strength inside of a stellar mass MECO-GBHC will have the much smaller equipartition values that which would be expected from flux compression during stellar collapse. In this context the Einstein-Maxwell equations at the MECO surface radius ($R \approx 4M$) imply that the ratio of tangential field on the exterior surface to the tangential field just under the MECO surface is given by [10, 12]

$$\frac{B_{\theta,S_+}}{B_{\theta,S_-}} = \frac{(1 + z_s)}{2 \ln(1 + z_s)}. \quad (13)$$

In ordinary units the result of Eq. (13) is $\approx \frac{10^{20} \text{ Gauss}}{B_{in} \sqrt{7M_{\odot}/M}}$. We have previously taken $B_{in} = 2.5 \times 10^{13} \text{ Gauss}$ as typical of the interior field that can be produced by flux compression during stellar gravitational collapse. Using this value the solution of Eq. (13) is $1 + z_s = 5.67 \times 10^7 m^{1/2}$. The magnetic moment of a MECO would be $\mu = 1.7 \times 10^{28} m^{5/2} \text{ Gauss cm}^3$ in ordinary units.

This magnetic moment and Eq. (12) give the MECO model a good correspondence with observations of spectral state switches and the radio luminosities of jets for both GBHC and AGN [10, 11, 12].

The authors of [27] describe the detection of a magnetic field of 10^8 Gauss that cannot result from frame dragging. It should be a further support of MECO alternative to BH model.

On the other hand, it has been recently shown that NLED objects can remove BHs singularities [13].

NLED Lagrangian has been used in various analysis in astrophysics, like surface of neutron stars [22] and pulsars [23], and also on cosmological context to remove the big-bang singularity [24, 25].

The effects arising from a NLED become quite important in super-strongly magnetized compact objects, such as pulsars and particular neutron stars [22, 23]. Some examples include the so-called magnetars and strange quark magnetars. In particular, NLED modifies in a fundamental basis the concept of gravitational redshift as compared to the well established method introduced by standard treatments [22]. The analyses proved that, unlike using standard linear electrodynamics, where the gravitational redshift is independent of any background magnetic field, when a NLED is incorporated into the photon dynamics, an effective gravitational redshift appears, which happens to depend decidedly on the magnetic field pervading the pulsar. An analogous result has also been obtained for magnetars and strange quark magnetars [23]. The resulting gravitational redshift tends to infinity as the magnetic field grows larger [22, 23], as opposed to the predictions of standard analyses which involve linear electrodynamics. What it is important is that the gravitational redshift of neutron stars is connected to the mass-radius relation of the object [22, 23]. Thus, NLED effects turn out to be important as regard to the mass-radius relation, which is maximum for a BH.

Following this approach, in [13] a particular non singular exact solution of Einstein field equation has been found adapting to the BH case the cosmological

analysis in [25]. In fact, the conditions concerning the early era of the Universe, when very high values of curvature, temperature and density were present [1, 13, 26], and where matter should be identified with a primordial plasma [1, 13, 26], are similar to the conditions concerning BH physics. This is exactly the motivation because various analyses on BHs can be applied to the Universe and vice versa [1, 13, 26].

The model works on a homogeneous and isotropic star (a collapsing “*ball of dust*”) supported against self-gravity entirely by radiation pressure. Let us consider the Heisenberg-Euler NLED Lagrangian [13, 24, 25]

$$\mathcal{L}_m \equiv -\frac{1}{4}F + c_1 F^2 + c_2 G^2, \quad (14)$$

where $G = \frac{1}{2}\eta_{\alpha\beta\mu\nu}F^{\alpha\beta}F^{\mu\nu}$, $F \equiv F_{\mu\nu}F^{\mu\nu}$ is the electromagnetic scalar and c_1 and c_2 are constants. Through an averaging on electric and magnetic fields [13, 24, 25], the Lagrangian (14) enables a modified radiation-dominated equation of state (p and ρ are the pressure and the density of the collapsing star)

$$p = \frac{1}{3}\rho - \rho_\gamma, \quad (15)$$

where a *quintessential density term* $\rho_\gamma = \frac{16}{3}c_1 B^4$ is present together with the standard term $\frac{1}{3}\rho$ [13, 24, 25]. B is the strength of the magnetic field associated to F . The interior of the star is represented by the well-known Robertson–Walker line-element [1, 13]

$$ds^2 = -dt^2 + a(t)[d\chi^2 + \sin^2 \chi(d\theta^2 + \sin^2 \theta d\varphi^2)]. \quad (16)$$

Using $\sin \chi$ we choose the case of positive curvature, which is the only one of interest because it corresponds to a gas sphere whose dynamics begins at rest with a finite radius [1, 13]. Considering Eq. (14) together with the stress-energy tensor of a relativistic perfect fluid [1, 13, 25]

$$T = \rho u \otimes u - pg, \quad (17)$$

where u is the four-vector velocity of the matter and g is the metric, the Einstein field equation gives the relation [13, 25]

$$t = \int_{a(0)}^{a(t)} dz \left(\frac{B_0^2}{6z^2} - \frac{8c_1 B_0^4}{6z^6} - 1 \right)^{-\frac{1}{2}}, \quad (18)$$

being $B_0 = a^2 B$. The expression (18) is not singular for values of $c_1 > 0$ in Eq. (14) [13, 25]. In fact, the presence of the quintessential density term ρ_γ permits to violate the reasonable energy condition [1, 26] of the singularity theorems. By using elliptic functions of the first and second kind, one gets a parabolic trend for the scale factor near a minimum value a_f in the final stages of gravitational collapse [13].

In concrete terms, by calling l, m, n the solutions of the equation $8c_1 B_0^4 - B_0^2 x + 3x^3 = 0$, it is [13, 25]

$$\begin{aligned}
t &= [-(m-l)^{\frac{1}{2}}\beta_1(\arcsin\sqrt{\frac{z-l}{m-l}},\sqrt{\frac{l-m}{l-n}}) \\
&+n(m-l)^{-\frac{1}{2}}\beta_2(\arcsin\sqrt{\frac{z-l}{m-l}},\sqrt{\frac{l-m}{l-n}})]|_{z=a^2(0)}^{z=a^2(t)},
\end{aligned} \tag{19}$$

where

$$\beta_1(x,y) \equiv \int_0^{\sin x} dz[(1-z^2)^{-1}(1-y^2z^2)^{-1}] \tag{20}$$

is the elliptic function of the first kind and

$$\beta_2(x,y) \equiv \int_0^{\sin x} dz[((1-z^2)^{-1})^{-\frac{1}{2}}((1-y^2z^2)^{-1})^{\frac{1}{2}}] \tag{21}$$

is the elliptic function of the second kind.

Then, recalling that the Schwarzschild radial coordinate, in the case of the BH geometry (16), is $r = a \sin \chi_0$ [1, 13], where χ_0 is the radius of the surface in the coordinates (16), one gets a final radius of the star [13]

$$r_f = a_f \sin \chi_0 > 2M \tag{22}$$

if B_0 has an high strength, where M is the mass of the collapsed star and $2M$ the gravitational radius in natural units [1, 13]. Thus, we find that the mass of the star generates a curved space-time without EHs.

Conclusion remarks

In this work we considered the fundamental issues which dominate the question about the existence or non-existence of BH horizons and singularities from both of the theoretical and observational points of view, and discussed some of the ways that BH singularities can be prevented from forming at a semi-classical level, i.e. without arguments of quantum gravity. In this way, we argued that BHs could have a different nature with respect the common belief. Even remaining very compact astrophysics objects, they could be devoid of horizons and singularities.

In fact, all of the various models discussed fall into the context of preserving the SPOE. Which one will be the correct one will ultimately be determined by their application to observational data on GBHC, AGN, and the compact object in the center of the galaxy. On this fundamental issue, we hope in a further improvement of astrophysical observations in next years.

The discussed framework is a key point within the debate on the path to unification of theories. As recently some scientists, like the Nobel Laureate G. 't Hooft [8], partially retrieved the old Einstein's opinion [7] that quantum mechanics has to be subjected to a more general deterministic theory, it is clear that finding solutions to the problem of black hole horizons and singularities at a semi-classical level, i.e. without discussions of quantum gravity, becomes a fundamental framework.

Acknowledgements

The authors thank Abhas Mitra for various useful discussions on the issues of this paper.

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