Quasar-microlensing versus star-microlensing evidence of small-planetary-mass objects as the dominant inner-halo galactic dark matter

Carl H. Gibson

Mechanical and Aerospace Engineering and Scripps Institution

of Oceanography Departments, University of California, San Diego, CA 92093-0411;

cgibson@ucsd.edu

and

Rudolph E. Schild

Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138;

rschild@cfa.harvard.edu

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ABSTRACT

We examine recent results of two kinds of microlensing experiments intended to detect galactic dark matter objects, and we suggest that the lack of short period star-microlensing events observed for stars near the Galaxy does not preclude either the "rogue planets" identified from quasar-microlensing by Schild 1996 as the missing-mass of a lens galaxy, or the "Primordial Fog Particles" (PFPs) in Proto-Globular-star-Cluster (PGC) clumps predicted by Gibson 1996 – 2000 as the dominant inner-halo galactic dark matter component from a new hydrodynamic gravitational structure formation theory. We point out that hydro-gravitational processes acting on a massive population of such micro-brown-dwarfs in their nonlinear accretional cascades to form stars gives intermittent lognormal number density n_p distributions for the PFPs within the PGC gas-stabilized-clumps. Hence, star-microlensing searches that focus on a small fraction of the sky assuming a uniform distribution for n_p are subject to vast underestimates of the mean $\langle n_p \rangle_{mean}$. Sparse independent samples give modes $10^{-4} - 10^{-6}$ smaller than means of the highly skewed lognormal distributions expected. Quasar-microlensing searches with higher optical depths are less affected by n_p intermittency. We attempt to reconcile the results of the star-microlensing and quasar-microlensing studies, with particular reference to the necessarily hydrogenous and primordial small-planetary-mass range. We conclude that star microlensing searches cannot exclude and are unlikely even to detect these low-mass candidate-galactic-dark-matter-objects so easily observed by quasar-microlensing and so robustly predicted by the new theory.

Subject headings: cosmology: theory, observations — dark matter — Galaxy: halo — gravitational lensing — turbulence

1. Introduction

Clearly the missing-mass problem is central to astronomy today since all attempts to understand the growth of structure in the universe, as well as the dynamics of galaxies and their clusters, require an understanding of the nature and location of this dominant mass component (Carr 1994). Because most of the inner-halo galactic missing-mass is probably in compact objects, attempts to detect it have been based upon quasar-microlensing and star-microlensing which can detect objects of any mass but cannot detect a smooth distribution of matter.

The first published quasar-microlensing reports were from lensed quasars by Vanderriest et al. 1989 and Irwin et al. 1989, although certainly the high optical depth Q0957+561A,B quasar-microlensing reported by Vanderriest et al. was known by Schild and Cholfin 1986 who did not presume to discuss microlensing until their measured time delay difference of 1.1 years between the A and B quasar images was confirmed. A secure image time delay difference is required to allow correction for intrinsic fluctuations of the source brightness at $0.1 - 0.3$ year planetary-mass periods by subtraction of the image light curves matched at the time their light was lensed. The 1.1 year Schild and Cholfin 1986 delay value has been confirmed and is now generally accepted, Kundic et al. 1997. Star-microlensing searches at low optical depth in directions of large star densities toward the Large Magellanic Cloud (LMC) and the Galactic Bulge resulted in detections (Alcock et al. 1993, Auborg et al. 1993, Udalski et al. 1993). However the interpretation of the microlensing statistics from the two kinds of programs has produced disagreement.

The quasar-microlensing in Q2237 was first to be analyzed, and earliest reports suggesting solar mass stars (Wambsganss et al. 1990, Corrigan et al. 1991, Yee and De Robertis 1992) were later amended by Refsdal and Stabell 1993 who demonstrated that the apparently rapid fluctuations detected in the observations could indicate a dominant population of terrestrial mass microlenses, but called for more accurate lightcurves over longer time spans to make the indication conclusive. Interpretation of the Q0957 microlensing was delayed by a long controversy about the time delay, but the critical fact that rapid microlensing is observed was already noted by Schild and Smith 1991. Exhaustive statistical analyses of the Q0957 data caused Schild and Thomson 1995 to again conclude that rapid microlensing is observed, and a power spectrum by Thomson and Schild 1996, also shown in Schild 1996, led to the Schild conclusion that the microlensing is caused by "rogue planets ... likely to be the missing mass." A primordial origin for such micro-brown-dwarf (MBD) objects as the baryonic galactic dark matter had already been proposed by Gibson 1996, who predicted a condensation mass of about 10^{-7} $M_{\odot}$ for the relatively inviscid gas formed at photon decoupling from the cooling viscous plasma. Recent best estimates from fossil turbulence theory by Gibson 2000abc of 10^{-6} $M_{\odot}$ for the "Primordial Fog Particle" (PFP) mass are closer to the "rogue planet" value found by Schild 1996 for Q0957, and are compatible with results for Q2237 as tentatively interpreted by Refsdal and Stabell 1993. The Gibson 1996 theory predicts simultaneous fragmentation of the primordial gas into 10^{6} $M_{\odot}$ proto-globular-cluster (PGC) clumps as expected from the Jeans 1902 theory (but for different reasons), adding yet another source of unaccounted for undersampling error to the star-microlensing searches.

Microlensing of stars has to date produced many detections, but their interpretation is clouded by uncertainties in the models of the Halo of the Galaxy. The original theoretical work that justified these searches (Paczynski 1986, Griest 1991) did not take into account that microlensing of LMC stars might be produced by dark matter or foreground stars on the near side of the LMC itself, as pointed out by Sahu 1994. This produces uncertainties in the interpretation of star-microlensing so large that parallax observations from one or more satellites may be needed to accurately compute the locations and parameters of the lenses (Gaudi and Gould 1997). Moreover, it was anticipated in the MACHO project experimental design (Griest et al. 1991) that any Halo dark matter objects would have mass $(10^{-3} - 10^{-1})\, M_{\odot}$ much larger than $10^{-7} M_{\odot}$, so search frequencies and strategies prevented detection of lower mass objects for the first several years. In all MACHO/EROS calculations of exclusion estimates a spatially uniform distribution is assumed, rather than the highly intermittent distributions expected (§5) if the Halo dark matter mass is dominated by hydrogenous planetoids (PFPs, MBDs) in dark PGC clumps. A small fraction of the LMC stars sampled may intersect PGCs, but even these have little chance of any PFP microlensing within practical observational periods from the extremely intermittent PFP number density n_p we show will occur within most PGCs due to their accretional cascades.

Whereas the available statistics of star-microlensing do not appear to have convincingly produced a dark matter detection (Kerins and Evans 1998), they have sometimes, but not consistently, been presented as precluding the population discovered in quasar-microlensing. The purpose of the present manuscript is to compare these two microlensing approaches to the missing-mass detection problem, with particular reference to the PGC clumps of PFP planetoids proposed by Gibson 1996 as the dominant inner-halo component of the galaxy missing-mass based on nonlinear (non-Jeans) fluid mechanical theory (§4). We show in our Conclusions (§6) that results from quasar-microlensing observations (§2) and star-microlensing observations (§3) are actually compatible, considering the intermittent lognormal number density expected for these objects as they cluster in a nonlinear cascade to form larger clusters, larger objects and finally, for a small percentage, stars (§5).

2. Quasar-microlensing observations

The first dark matter conclusion based upon microlensing was by Keel 1982, who confirmed from 80 years of archived images the remarkable concentration of bright quasars near bright galaxies noted by Burbidge 1979 and Arp 1980. Keel found QSO surface densities σ increased by more than two orders of magnitude at separations $R \approx 10$ kpc on the galaxy plane above background values with separations $R \geq 100$ kpc, but showed low amplitudes of quasar brightness fluctuations (less than 1 magnitude or 17%) at stellar-mass frequencies. Keel therefore concluded that "if gravitational amplification is involved in producing an excess of bright QSOs near galaxies, ordinary halo stars are not responsible." The effective radius of the bright QSO density distribution was 17 kpc, "slightly larger than globular cluster systems of the Galaxy and M31." From this evidence, the mass of the microlensing objects must either be much larger than $M_{\odot}$ or much smaller, and concentrated within the 10^{21} m (32 kpc) inner-galaxy-halo range observed for globular clusters around galaxies.

The directly measured microlensing (by Vanderriest et al. 1989) for Q0957 for a 415 day time delay was not analyzed for missing-mass implications for many years, possibly in part because the time delay would remain controversial. The detected microlensing in Q2237, first reported in Irwin et al. 1989 and initially interpreted as due to masses in the range $0.001 M_{\odot} < M < 0.1 M_{\odot}$, was reinterpreted in terms of a statistical microlensing theory by Refsdal and Stabell 1993, applicable to the case of resolved accretion disks, giving possible microlensing masses as small as $10^{-7} M_{\odot}$.

With time delay issues for the Q0957 system resolved in favor of the original (Schild and Cholfin 1986) time delay value, the microlensing was discussed by Schild and Thomson 1994, 1995, who removed the intrinsic source fluctuations of the A image by subtracting the B image shifted 404 days forward, revealing a continuously fluctuating pattern in the difference signal, due only to microlensing, that would be evidence of the missing-mass. A Fourier power spectrum of these microlensing fluctuations in Schild and Thomson 1995, and updated in Schild 1996, shows that the microlensing power spectral area is dominated by a broad peak whose center frequency corresponds to a microlensing mass of $10^{-5.5} M_{\odot}$ (4.4×10^{24} kg). The dominant microlens mass might be smaller than this due to clumping, but not much larger.

To understand why quasar-microlensing and star-microlensing experiments seem to give different results, it is necessary to understand how the higher optical depth in quasar-microlensing affects the outcome differently for microlensing masses with the intermittent spatial distribution expected for any primordial population of small self-gravitating masses. In quasar-microlensing, two or more images of the same quasar are observed, and a normal galaxy, acting as a lens, will produce unit optical depth to microlensing. The definition of optical depth is given precisely in Schneider et al. 1992. It is formulated for an unresolved source, and is effectively the probability that a microlensing event is underway. The surface optical depths for the (A,B) images of the Q0957 system are (0.3, 1.3), which means that for image B it is virtually certain that at least one microlensing event is underway at any time. For image A, the probability is only about 30% that at any statistical moment a microlensing event is underway. These probabilities are independent of the microlensing mass, which can be estimated from the event duration.

A more complex situation, more favorable for the detection of microlensing, exists for the Q0957 system, because the Einstein rings of all microlensing masses are substantially smaller than the size of the luminous quasar accretion disk. Schild 1996 used the historic record of Q0957 brightness fluctuations, in combination with the appropriate Refsdal and Stabell 1993 statistical theory, to estimate the size of the quasar source as 6 times larger than the stellar-microlens Einstein ring. For a half-solar mass typical microlens, presumed to be present from the lens galaxy's spectrum, this gives an accretion disk diameter of 3×10^{15} m. More importantly, this result means that at any time there are 6^2 independent lines of sight to the quasar B image that each has a probability of 1.3 of hosting a statistically independent stellar-microlensing test, or a net probability of 1.3×6^2 stellar mass microlenses at any moment, and the observed brightness fluctuations on a 30-year time scale should simply reflect the Poisson statistics of this number of occupied stellar microlensing sites.

The luminous matter described above is known to exist from the lens galaxy's light, whose spectrum is dominated by stars of approximately half-solar mass. From the amplitudes and durations of its stellar-microlensing events we have made inferences about the area of the luminous quasar source. But Schild 1996 emphasizes the microlensing signature of a dominant second population of objects, whose 10 to 100 day fluctuation pattern corresponds to a microlens mass of 10^{-6} $M_{\odot}$. For these objects to be the missing-mass, there would need to be at least 10^6 of them for each half-solar mass star with the usual assumption that the lens galaxy is dominated by the missing-mass at the 90% level or more. Consistent with the estimates for the quasar source luminous area given above, the line of sight to image B should have $1.3 \times 36 \times 10^7$ micro-brown-dwarfs, and image A should have $0.3 \times 36 \times 10^7$, out of $\approx 10^{17}$ in a galaxy. The brightness fluctuations observed should reflect the statistical distribution in the numbers of these microlenses on time scales appropriate for such small particles (10 to 100 days). The pattern of brightness fluctuations in each quasar image should be continuous and should be symmetrical (there should be equal positive and negative fluctuations), as observed in both the A and B images, and in their phased difference. We discuss in §5 the implications of this large number of statistical tests on estimates of the statistical parameters of a primordial population of missing-mass objects.

A reanalysis of the time delay and microlensing in this lens system by Pelt et al. 1998 leads to a smaller source size, but this does not affect the conclusion that many MBDs would be projected against the luminous quasar source at all times. It appears likely that the microlensing interpretation is beginning to confront the real quasar source structure, and that the simple, uniformly bright, luminous disk source models are becoming inadequate. It is beyond the scope of the present paper to model the hydro-gravitational processes of quasar accretion, but it is not unreasonable to speculate that such regions of large turbulence dissipation rate ε, high gas density ρ, and enormous luminosity could rapidly produce a large number of powerful point sources of light. Stars form in turbulent regions with mass $M_T = (\varepsilon^6 \rho^{-5} G^{-9})^{1/4}$ in times $\tau_G = (\rho G)^{-1/2}$, Gibson 1996, with M_T limited only by radiation pressure differences that decrease as the ambient luminosity increases (the Eddington limit with small ambient luminosity is $M_{E0} \approx 100 M_{\odot}$). The luminosity of such quasar superstars should increase rapidly with $M_T \gg M_{E0}$. The Tully-Fisher relationship, where spiral galaxy luminosity increases with rotation rate, is another candidate for enhanced turbulence causing an excess of large stars with large luminosities.

3. Star-microlensing observations

The star-microlensing, or MACHO/EROS/OGLE experiments, are operating in a very different optical depth realm, and are therefore affected by the detection statistics in profoundly different ways. These experiments were justified by the statistical inference of Paczynski 1986, later confirmed by Griest 1991, that if the missing-mass consists of stellar objects the optical depth for microlensing by a star in the Galaxy's Halo of a star in the disk of the Large Magellanic cloud would be $\approx (1/2) \times 10^{-6}$. Thus, by observing up to 8×10^6 LMC stars nightly, the probability of detecting a microlensing event would hopefully be above unity. Early reports of the detection of such MACHO/EROS/OGLE events and claims of the detection of the Halo missing-mass had to be corrected by Sahu 1994, who demonstrated that microlenses in the LMC halo were about as important as microlenses in the Galaxy's Halo, and the events would be indistinguishable to available experiments.

To date these experiments have reported several LMC events (hundreds of events toward the Galaxy bulge) but no detection of the missing-mass, partly because of the ambiguity about whether the LMC microlenses are in the Halo of the Galaxy or in the LMC itself, and partly, we suggest, because the likelihood of MACHO clumping and intermittency have been neglected for the $10^{-6}M_{\odot}$ mass range.

The statistics of star-microlensing are profoundly different than those for quasar-microlensing because of the very different optical depth regimes. For star-microlensing, on any night, of order 2×10^6 stars are sampled, and so there are 2 000 000 independent statistical tests asking "is a solar-mass microlensing event underway." Since an event for solar-mass microlenses lasts about 30 days, a new sample is available monthly. However, it is not an independent sample since it involves the same objects only slightly displaced by their relative velocities. The time required to assure complete temporal independence of the sample is a few million years and spatial independence of the sample would require a different set of stars elsewhere on the sky. If the missing-mass is not approximately solar but rather terrestrial (i.e., $10^{-6}M_{\odot}$), then events should be of duration only hours, and most would have escaped detection in the main star-microlensing searches. However, some higher frequency searches for such events have been undertaken (Alcock et al. 1998, Renault et al. 1998) and the claim made that non-detection constitutes proof that they do not exist. No explanation is given for the fact that rapid microlensing is observed in quasar monitoring, Schild 1996, and we conclude that star-microlensing is unable to detect and study the missing matter of galaxies. Why?

In §5 we show that the failure of star-microlensing to detect the missing Galactic Halo matter is readily understood from the sparse number of independent samples and because of the expected non-Gaussian distributions of the PFP masses of the microlensing objects within PGC clumps as well as the likely non-Gaussian distributions of the PGC clumps.

Our purpose is to show that any massive, primordial, population of small condensed objects would be expected to have entered a nonlinear gravitational cascade of particle aggregation, and the resulting highly intermittent lognormal number density distribution would leave too much uncertainty in the star-microlensing statistics for any conclusions to be drawn about the mean number density n_p of $10^{-6} M_{\odot}$ objects from limited star-microlensing searches. Because quasar-microlensing involves optical depths and detection probabilities at least a factor of 10^6 higher, it offers greater prospects for robust sampling of mean masses and mean densities of such small dark-matter objects. Kerins and Evans 1998 have also suggested inner-outer Halo population differences might contribute to the failure of star microlensing searches to find the baryonic dark matter.

4. Nonlinear hydro-gravitational condensation of the primordial gas

Star microlensing surveys have not aggressively searched for planetoids as the missing Galactic mass because the existence of such objects is impossible by the generally accepted Jeans 1902 theory of gravitational instability. However, Jeans's linear acoustic theory is subject to vast errors, especially for very strongly turbulent and very weakly turbulent flows and for weakly collisional materials such as neutrinos with enormous diffusivities D, Gibson 2000abc. Cold dark matter theories for galaxy formation from nonbaryonic dark matter condensations fail because the diffusivity D for such material is so large that the diffusive Schwarz scale is larger than the corresponding Jeans scale, $L_{SD} \equiv (D^2/\rho G)^{1/4} \gg L_J \equiv V_S/(\rho G)^{1/2}$, where L_J values in CDM theories are made small enough for galaxy size CDM halos to form by assuming the nonbaryonic dark matter is cold to reduce the sound speed V_S. However, no structures can form by gravity on scales smaller than the largest Schwarz scale according to Gibson 1996, and in this case the largest Schwarz scale will be L_{SD}. For a weakly collisional nonbaryonic fluid, the criterion

$L_{SD} \gg L_J$ is equivalent to ${m_p}^{3/2}/\sigma \gg (kT\rho/G)^{1/2}$, where m_p is the particle mass, σ is the collision cross section, and k is Boltzmann's constant. Substitution of ρ and T values from the plasma epoch, when CDM halos were presumably formed, give values about 10^{-12} $\mathrm{kg}^{3/2}\mathrm{m}^{-2}$. Typically assumed CDM particle masses $m_p \geq 10^{-27}$ kg would require σ values $\approx 10^{-28}\,\mathrm{m}^2$ for the equality to fail, but realistic σ values for CDM must be many orders of magnitude less for its particles to escape detection, $<\approx 10^{-43}\,\mathrm{m}^2$. Observed outer-halo galaxy scales of 10^{22} m (324 kpc) suggest from L_{SD} that the nonbaryonic dark matter has weakly collisional particles with m_p values closer to 10^{-35} kg, and this material makes only a small ($\approx 1\%$) contribution to the inner-halo galaxy density, Gibson 2000b.

The case of strong turbulence was first recognized by Chandrasekhar 1951, who suggested that a "turbulent pressure" $p_T = \rho V_T^2$ should simply be added to the gas pressure p in the Jeans length scale definition $L_J \equiv V_S/(\rho G)^{1/2} \approx (p/\rho^2 G)^{1/2}$, where $V_S \approx (p/\rho)^{1/2}$ is the sound velocity, p is pressure, ρ is density, and G is Newton's constant of gravitation. This argument is flawed for two reasons: firstly because pressure without gradients offers no resistance to gravitational forces (the basic flaw in Jean's theory), and secondly because the turbulent velocity is a function of length scale L. Substituting p_T for p in the expression for L_J rather than adding and using the Kolmogorov expression $V_T \sim (\varepsilon L)^{1/3}$ gives the turbulent Schwarz scale $L_{ST} \equiv \varepsilon^{1/2}/(\rho G)^{3/4}$ of Gibson 1996, where ε is the viscous dissipation rate of the strong turbulence. If the dissipation rate $\varepsilon \leq \rho\nu G$, where ν is the kinematic viscosity of the gas, then the flow is non-turbulent at the scale of gravitational domination and is controlled by viscous forces at the viscous Schwarz scale $L_{SV} \equiv (\gamma\nu/\rho G)^{1/2}$, where γ is the rate-of-strain of the flow, Gibson 1996.

In gas where $L_{ST} \gg L_J$, Jeans mass stars cannot form because the turbulence forces overwhelm gravitational forces at all scales smaller than L_{ST}. Such a situation may occur due to supernova shock turbulence, in starburst regions and in cold molecular clouds, and due to gravitational infall turbulence near galaxy and PGC cores with $\varepsilon \geq RT(\rho G)^{1/2}$ for strongly turbulent gas regions on scales larger than L_{ST}, where $V_S^2 \approx RT$ and R is the gas constant. The case of interest in the present paper has been overlooked by standard cosmological models; that is, the hot gasses of the early universe emerging from the plasma epoch with weak or nonexistent turbulence, where both L_{SV} and $L_{ST} \ll L_J$.

The physical significance of the Jeans scale L_J is found by deriving the maximum length scale of pressure equilibrium in a fluctuating density field with self gravitational forces. Pressure equilibrium occurs if the gravitational free fall time $\tau_G \equiv (\rho G)^{-1/2}$ is greater than or equal to the time required for acoustic propagation of the resulting adjustment in the pressure field on scale L; that is, $\tau_G \geq \tau_P \equiv L/V_S$. Solving gives $L \leq L_J$. Thus L_J is not a minimum scale of gravitational instability as usually supposed, but is a maximum length scale of acoustical pressure equilibrium.

This shows that there should be fragmentation of the primordial gas at both the Jeans scale $L_J \approx 10^4 L_{ST}$ and the weakly turbulent or nonturbulent Schwarz scales $L_{ST} \approx L_{SV}$, Gibson 1996. From the ideal gas law $p = \rho RT$, the pressure can adjust to density increases and decreases due to gravitational condensations and void formations on scales $L_J \geq L \geq L_{ST}$ keeping the temperature nearly constant without radiant heat transfer. Voids forming at scales larger than L_J cannot adjust their pressure fast enough to maintain constant temperature, causing a temperature decrease within the voids. Radiant heating of such voids larger than L_J by the surrounding warmer gas therefore accelerates their formation in the expanding proto-galaxy, causing fragmentation of the primordial gas into proto-globular-cluster (PGC) blobs with mass of order 10^{36} kg, or $10^6 M_{\odot}$. Since this instability has nothing to do with the Jeans theory, the scale $L_{IC} \equiv (RT/\rho G)^{1/2}$ has been termed the initial condensation scale, Gibson and Schild 1999ab.

Because pressure forces propagate by electromagnetic radiation in plasmas the sound speed in the plasma epoch is nearly the light speed c, giving Jeans scales L_J larger than the Hubble scale of causal connection $L_H \equiv ct$ and therefore no possibility of structure formation in the baryonic component of matter by the Jeans theory. However, we have seen that the Jeans scale is generally not the criterion for gravitational structure formation, but instead gives the maximum scale of pressure equilibrium by acoustic propagation. Thus we can conclude that the small inhomogeneities in the CMB observed by COBE and other experiments are not acoustic as often assumed, but are more likely remnants of the first formation of structure due to gravity. The power spectrum of COBE temperature fluctuations peaks at a length scale $L_{HFS} \ll L_H$ much smaller than the horizon, or Hubble, scale L_H at the time of photon decoupling, suggesting the first structure formation must have occurred previously to match the spectral peak scale to a smaller horizon scale L_{HFS} existing at some earlier time of first structure, Gibson 2000b.

Formation of proto-super-cluster to proto-galaxy mass fragments from void formations was possible in the plasma epoch, but with slight increases in density from gravitational condensation because the universe age t was at all times less than the gravitational free fall time $\tau_G \equiv (\rho G)^{-1/2}$ and because of non-baryonic diffusive compensation, Gibson and Schild 1999b. Expansion of the universe enhances formation of voids and retards condensation of fragments, but amplitudes of both are damped by rapid diffusive density compensation as the non-baryonic component fills the minima and exits the maxima formed in the baryonic component. Fragmentation and fractionation of the primordial plasma was first triggered when the Hubble scale of causal connection $L_H \equiv ct$ decreased to scales less than the viscous or weakly turbulent Schwarz length scales $L_{SV} \approx L_{ST}$, Gibson 1996. Remarkably, the baryonic matter density $\rho(t)$ from Einstein's equation at $t = 10^{12}$ s of 10^{-17} kg m^{-3}, Weinberg 1972, matches the density of globular star clusters (GCs). We take this ρ value to be that of the primordial gas at $t = 10^{13}$ s, and further evidence of structure formation beginning at about $t = 10^{12}$ s. Observations of $\rho \approx 10^{-17}$ kg m^{-3} in dim and luminous globular clusters suggests the internal evolution of PGCs has been quite gentle and nondisruptive so that dark PGCs with PFPs in less advanced stages of their accretional cascades may be considered metastable, with average densities $\rho_{PGC} \approx \rho_{IC}$ and masses $M_{PGC} \approx M_{IC}$ that depart slowly from these initial values.

Additional evidence suggesting the time of first structure was about $t_{FS} \approx 10^{12}$ s, or 30 000 years, is that this is the time when the supercluster mass of 10^{46} kg matches the Hubble mass $\rho(t)(ct)^3$, Gibson 1997a. Taking the horizon scale at this time equal to the viscous Schwarz scale $L_{SV} \equiv (\gamma\nu/\rho G)^{1/2}$ with $\gamma \approx t^{-1}$ gives a kinematic viscosity ν of 5×10^{26} m^2 s^{-1}, which matches the photon viscosity $\nu \equiv l_C c$ obtained from the mean free path for Thomson scattering $l_C = 1/n_e\sigma_T$ with free electrons of the plasma times the speed of light c, where n_e is the free electron number density and σ_T is the Thomson scattering cross section, Gibson 1999b. This enormous viscosity (a million times that of the Earth's upper mantle) gives a horizon scale Reynolds number $\mathrm{Re} \equiv \mathrm{c^2 t}/\nu \approx 200$ which is slightly above critical, consistent with evidence from COBE of weak turbulence in the primordial plasma, and precludes any interpretation of the observed CMB temperature anisotropies as "sonic peaks" since any such powerful plasma sound would be promptly damped by viscosity, even if its unknown source could be identified, Gibson 2000b.

Evidence of proto-supercluster to proto-galaxy formation in the plasma epoch can also be inferred from the advanced morphology of superclusters, with 10^{24} m voids bounded by "great walls" as shown by deep galaxy maps, and by galaxy rotation rates of order 10^{-15} rad s^{-1} consistent from angular momentum conservation as the galaxies expand with our estimated time of proto-galaxy fragmentation at $\approx 10^{13}$ s, with angular velocity Ω and strain rate γ both near 10^{-13} rad s^{-1}. Thus, we assume that the neutral primordial gas emerging from the plasma epoch was fragmented into $\approx 10^{41}$ kg proto-galaxy blobs of hydrogen and helium with a very uniform density $\rho \approx 10^{-17}$ kg m^{-3} reflecting the time

$t_{FS} \approx 10^{12}$ s of first fragmentation and proto-superclusters, and $\Omega \approx 10^{-13}$ rad s^{-1}.

Turbulent mixing scrambles density fluctuations from large scales to small to produce nonacoustic density nuclei that move with the fluid, and these trigger gravitational condensation on maxima and void formation at minima. Condensation on acoustic density nuclei; that is, density maxima moving with the sound speed as envisaged by Jeans, is not a realistic description of gravitational structure formation under any circumstances, as shown by Gibson 1996. The smallest scale ρ fluctuation produced by turbulent mixing is at the Batchelor length scale $L_B \equiv (D/\gamma)^{1/2}$, Gibson 1991, corresponding to a local equilibrium between convective enhancement of $\nabla\rho$ by the rate-of-strain γ and smoothing of $\nabla\rho$ by the molecular diffusivity D, near points of maximum and minimum ρ moving with the fluid velocity $\vec{\mathbf{v}}$, Gibson 1968 and Gibson and Schild 1999a.

The Jeans theory fails because it relies on the Euler momentum equations that neglect viscous forces, and because it employs linear perturbation stability analysis, which neglects the nonlinear inertial-vortex forces that dominate turbulent flows, Gibson 1999a. Viscous forces are necessary to stabilize the rapidly expanding flow of the early universe for times t before structures form, with rate-of-strain γ values of order t^{-1}, Gibson 1999b. Expanding inviscid flows are absolutely unstable (Landau and Lifshitz 1959, Gibson 2000b), so a major result of the COsmic Background Experiment (COBE) was a demonstration that temperature fluctuations $\delta T/T$ were $\leq 10^{-5}$ rather than $\geq 10^{-2}$ to be expected if plasma epoch flows before transition to neutral gas were fully turbulent. In the absence of structure to produce buoyancy forces, only viscous forces (small Reynolds numbers) can explain the absence of turbulence that is manifest in the homogeneity of the cosmic microwave background (CMB) temperature fluctuations.

Linearity and the Jeans theory cannot be salvaged by arguing that $\delta\rho/\rho$ is small so that very long times would be required for the gravitational structure formation to "go nonlinear". The time required to form structure by gravity is $\approx \tau_G \equiv (\rho G)^{-1/2}$ where τ_G is the gravitational free fall time, and nearly independent of the magnitude of the dominant density fluctuations on scales larger than the largest Schwarz scale. The mass of such a fluctuation $M'(t) \approx M'(0) exp[2\pi(t/\tau_G)^2]$, Gibson 1999b. Thus, the 10^{-3} difference between the observed CMB density fluctuations and the larger values expected for turbulence only causes a 45% increase in the time required for $M'(t)$ to reach planetoidal values. For $\rho = 10^{-17}$ kg m^{-3}, $\tau_G = 3.9 \times 10^{13}$ s or 1.2×10^6 years. Starting from 10^{-17} kg m^{-3}, the time to reach density $\rho = 10^0$ kg m^{-3} is 3.1×10^6 years (10^{14} s) and $< 10\%$ more to reach 10^3 kg m^{-3} and quasi-hydrostatic equilibrium. This first condensation process of the universe has a profound effect on everything else that happens afterwards. The dynamics shifts from that of a collisional gas to that of weakly collisional warming gas blobs in an increasingly viscous, cooling, gaseous medium, evolving slowly toward a cold dark metastable state of no motion as the baryonic dark matter of galaxies, as observed by Schild 1996. The sites with the highest likelihood of instability to form the first stars and quasars are the cores of PGCs and the cores of PGs, at this same time $t \approx 10^{14}$ s with $z \approx 264$. A large population of quasars at extremely high redshifts z is indicated by dim point sources of hard X-rays detected by the Chandra satellite, Mushotzky et al. 2000. A survey of star-forming galaxies for $3.8 \leq z \leq 4.5$ after improved dust extinction corrections shows no indication of a peak in the (star formation rate) SFR/comoving-volume and therefore a monotonic increase in SFR/proper-volume with increasing z toward a peak at some high $z \geq 4.5$ value, Steidel et al. 1999, presumably also at $z \approx 264$, compared to a peak at $z = 1 - 2$ assumed by many authors following the "collapse" of gas into CDM halos to form galaxies at $z \approx 5$ for the standard CDM model. Galaxies never collapse after their formation at $z \approx 1000$ in the present scenario. Clusters of galaxies observed at z = 3.09, Steidel et al. 2000, also present no difficulties for our scenario of first structure growth.

The kinematic viscosity of the neutral primordial gas formed from the cooling plasma at decoupling is dramatically less than that of the plasma, with $\nu \approx 5 \times 10^{12}$ m^2 s^{-1}, Gibson 1999b. Thus the viscous Schwarz scale $L_{SV} = (5 \times 10^{12} \times 10^{-13}/10^{-17} \times 6.7 \times 10^{-11})^{1/2}$ $= 2.7 \times 10^{13}$ m, and the viscous Schwarz mass $M_{SV} \equiv L_{SV}^3 \rho = (2.7 \times 10^{13})^3 \times 10^{-17}$ is 2×10^{23} kg, or $10^{-7} M_{\odot}$, which we take to be the minimum expected mass of a primordial fog particle. The minimum PFP mass increases to $\approx 5 \times 10^{24}$ kg if fossil turbulence effects are included, taking $\gamma_o = 10^{-12}$ s^{-1} as a fossil vorticity turbulence remnant of the time of first structure 10^{12} s, Gibson 2000bc. The weak turbulence levels possible from the COBE observations increase the condensation mass to maximum $M_{ST} \leq 10^{25}$ kg values that are still in the small-planetary-mass range. The formation of these gaseous planetoids represents the first true condensation by gravity since the Big Bang, where the mass density ρ increases with time. Whatever small turbulence may have existed at decoupling will now be damped by buoyancy forces as the PFPs begin their gradual evolution from hot gas blobs to freezing cold liquid-solid Neptunes at $z \approx 3$, with a gentle, complex, accretional cascade to form stars that results in practical invisibility of these PFPs to star-microlensing searches. Frozen hydrogenous planetoids are stable to evaporation at the temperature of the present universe for masses larger than about $10^{-8} M_{\odot}$, De Rujula et al. 1992. No account has been made for possible increases in PFP atmosphere size or increased PFP number density on their fraction of supernova energy absorbed with increasing z, even though $\Lambda \neq 0$ and "dark energy" inferences depend on only 30% dimming of supernova Type Ia events at the larger z values to justify claims of accelerations in the universe expansion.

5. Intermittent lognormal number density of hydrogenous, primordial, small-planetary-mass objects

Star-microlensing estimates (Alcock et al. 1998) of the contribution to the missing-mass of the Galaxy Halo by compact Halo objects in various mass ranges have assumed that the objects are uniformly distributed in space. This assumption is highly questionable, especially for objects in the small-planetary-mass range $M_p \approx 10^{-6} M_{\odot}$. If such objects are sufficiently compact to cause microlensing, and sufficiently numerous to dominate the Galaxy mass, they must be primordial, and consist of H-He in primordial proportions. They comprise the initial material of construction for the first small stars in PGCs, and an important stage in the process of such star formation. Non-baryonic materials are too diffusive to become compact, and no cosmological model has produced this much baryonic mass that is not H-He.

Starting from a nearly uniform distribution of $10^6 M_{\odot}$ PGC-mass clumps of hot-gas PFPs in the proto-galaxy (PG), all of the PFPs must have participated to some extent in nonlinear, gravitational, accretion cascades culminating for luminous GCs in the formation of the first small stars simultaneously, giving an abrupt end to the "dark ages" of the universe at $z \approx 264$, versus 5 much later as usually assumed, with the first large stars forming simultaneously with reevaporation and the first strong turbulence at PGC-cores near PG-cores as Population III super-stars, with their supernovas, black holes, and proto-quasars, and probably one or less for each proto-galaxy so that the universe never reionized as often assumed. Those PGCs in the outer regions of PGs should cool more rapidly and be less successful in accretion cascades of their PFPs than those nearer PG-cores. Pairs of PFPs should form first, and then pairs of pairs, etc., producing a complex array of nested clumps within each PGC, and large voids in between. Large drag forces from the fairly dense, weakly turbulent, inter-PFP gas should produce small PFP terminal velocities from its drag compared to virial velocities, and the large number of neighboring PFPs and their clumps should tend to randomize PFP velocity directions.

Such an accretion process produces a highly non-uniform PFP number density n_p, with the potential for large undersampling errors in star-microlensing estimates of the mean number density $\langle n_p \rangle$ of such objects in the Halo from the resulting small numbers of independent samples of n_p. A sample consists of several observations of each star for times longer than the expected microlensing time for a number of stars sufficient to give at least one event, where the sampling time required depends on how many objects are expected and whether the Galaxy Halo consists of a uniform distribution of objects in the sampled mass range or a nonuniform distribution. It takes longer to acquire a sample if the probability density function (pdf) of n_p is nonuniform and much longer if it is extremely intermittent. An independent sample is one taken from stars separated from other samples by distances larger than the largest size of clumps of such objects or any clumps of such clumps. If the same stars are used for each sample, one must allow time between samples so that the sampled field of objects can move away and be replaced by an independent field. To avoid undersampling error, one must account for nonuniformity of the objects sampled and take sufficient numbers of independent samples to achieve an acceptable level of statistical uncertainty in the estimated mean $\langle n_p \rangle$.

5.0.1. Clustering models for the accretion of primordial-fog-particles (PFPs)

Detailed modeling of the accretion of PFP objects within the proto-globular-cluster (PGC) initial condensation masses is an important topic, but is rather complex and therefore beyond the scope of the present paper. Rates of PFP merging depend on relative velocities, frictional damping, and degrees of condensation of the PFP gaseous atmospheres. A gas epoch existed before $z = 3$ with no possibility for condensation of the primordial gas to liquid or solid states because universe temperatures were above hydrogen and helium dew point and freezing point temperatures. Thus rates of merging of the large, entirely gaseous PFPs existing then should have been more rapid than the colder, more compact PFPs to appear later with condensation to liquid and solid states in the colder universe. Frictional and tidal forces between gassy PFPs were significant, reducing velocities and merging rates and permitting radiative cooling and further condensation toward more collisionless PFP states. Principles of collisionless dynamics (Binney and Tremaine 1987) describing star interactions often may not apply to PFP interactions, particularly in the early stages of PFP merging when large scale gas atmospheres about the objects existed. Neither must stellar virial theorems apply to PFPs because frictional forces of PFP extended gaseous atmospheres will reduce their relative velocities below virial values if they have been exposed to radiation, tidal, or convective heating flows that can cause re-evaporation.

As an exercise, we can compare star clustering with PFP clustering. It is well known from stellar dynamics that the third star in a triple must lie at least three times the radius of a binary away from the center. An accretion cascade of triples to form triples of triples, etc., within a uniform cluster of stars, would result in an ever decreasing average mass density of the star cluster, rather than a constant or increasing density as expected for frictionally interacting PFPs within a proto-globular-cluster (PGC) as they accrete in a bottom-up cascade to form globular cluster stars. If there were n stages in a tripling cascade and we assumed the third member of the k-th triple cluster must be four times the radius R_k separating the associated binary of $(k-1)$-th clusters, where $1 \leq k \leq n$, then the mass of the n-th cluster is $M_n = 3^n M_{star}$, and the radius $R_n = (4m)^n d$, with star size d and binary star separation $R_{bs} = md, m \gg 1$. This would result in a monotonic decrease in the mass density of star clusters $M_n = 3^n M_{star}/(4m)^{3n} {R_{bs}}^3$ with increasing n. If the initial mass density of unclustered stars is $\rho_0 = M_{star}/(4md)^3 \approx \rho_{star}/(4m)^3$, where ρ_{star} is the star density, then $\rho_n/\rho_{star} = 3^n(\rho_0/\rho_{star})^{3n}$, which also decreases rapidly with increasing values of n since $\rho_0/\rho_{star} \ll 1$.

Such a star tripling cascade is possible, but is slow. Times between collisions $t_{col} = 1/(\sigma_{star} v n)$ are large. For example, in a virialized globular cluster with $\sigma_{star} \approx 10^{18}$

m^2, $v \approx 8 \times 10^3$ m s^{-1}, and $n \approx 10^{-51}$ m^{-3} we find $t_{col} \approx 10^{29}$ s, which is $\approx 3 \times 10^{11}$ larger than the age of the universe. On the other hand, times t_{PFP} for collisions of PFPs $t_{PFP} \approx L_{sep}(\rho_{PFP}/\rho_0)^{2/3}/v$ are much shorter, where $L_{sep} = (M_{PFP}/\rho_0)^{1/3}$ is the PFP separation distance, ρ_0 is the mass density of the PGC, v is the relative velocity, and ρ_{PFP} is the PFP density. For the same relative velocity $v \approx 8 \times 10^3$ m s^{-1}, for $\rho_{PFP}/\rho_0 = 10$, $\rho_0 = 10^{-17}$ kg m^{-3}, and $L_{sep} = 2 \times 10^{13}$ m, we find $t_{PFP} \approx 10^{10}$ s, which is much less than the age of the universe of 10^{13} s when PFPs were first formed. Relative velocity values for PFPs then were certainly much less than virial values $(2GM/R)^{1/2}$, with collision times as much as a few times the universe age as a lower bound based on $v \approx \gamma R$ from the universe expansion value $\gamma = 1/t$.

As another exercise, consider the commonly assumed virial theorem for collisionless objects such as stars. The virial theorem applied to PFPs within a PGC soon after fragmentation would incorrectly equate the kinetic energy per unit mass $v^2/2$ of each PFP to its potential energy per unit mass GM/R within the PGC of mass M and radius R. By Hegge's law (Binney and Tremaine 1987), the outer orbital velocity of stars, from the application of the virial theorem to a star cluster, must exceed the dispersion velocity of the stars in the cluster, or else any internal hierarchical star structures will be destroyed. Hegge's law is appropriate to star clusters because stars are virtually collisionless. However, frictional and tidal forces between interacting PFPs prevent virialization and enhance merging. For the purposes of the present paper, it suffices to postulate that the aggregation cascade at every stage from PFP to star mass within a PGC is nonlinear and approximately self-similar, leading to clumping and increased intermittency of the PFP number density as the number of stages increases.

It is not known what fraction of the gas of a proto-galaxy fragments into proto-globular-clusters simultaneous with PGC fragmentation into PFPs. Since there are only about 200 globular clusters (GCs) in the Milky Way Halo, and this number is typical for nearby galaxies, one might think that PGC formation were a rare event since about a million PGCs are required to equal the mass of a proto-galaxy and only 0.02% are observed in our nearby galaxies close enough to sample. A more likely scenario is that $\approx 10^6$ PGCs formed per galaxy as expected, but most PGCs have remained dark with so few stars that they have escaped detection, either in the bright, dusty, galaxy core where the clustering of stars is more difficult to discern, or in the dark halo where external triggers of star formation are rare and most PGCs are still only partially evolved. Some fraction of the original PGCs of a galaxy will be disrupted to form the interstellar medium of the disk and core where rogue PFPs will generally dominate the ISM mass. Dynamical constraints on dark clusters of objects such as PGCs are examined by Carr and Sakellariadou 1999, and our value of $[M_C/M_\odot, R_C/pc] = [10^6, 8]$ is well within their range of values not excluded.

An authoritative book on globular cluster systems suggests that observations of these systems have "outstripped theoretical work" on how the systems and the globular clusters themselves are formed, especially young globular clusters, Ashman and Zepf 1998. The accumulation of evidence supports early suggestions of these authors that galaxy merger-induced starbursts are favorable environments for globular cluster formation, as dramatically confirmed in the Hubble Space Telescope (HST) images of the merger galaxy NGC 3256 where more than 1000 bright blue objects have been identified as young globular clusters, Zepf et al. 1999. Our suggestion is that the bright system of young GCs observed within 10^{20} m of the central region of the NGC 3256 merger can be readily explained as a small fraction of the many dark, primordial, PGC-PFP systems available in the merging galaxies that have been brought out of cold storage by tidal forces of the merger process accelerating the accretion of PFPs to form stars.

With the recent high resolution and infrared capabilities of the HST, many dense star clusters near the core of the Milky Way have been revealed. These clusters have masses of order $10^6 M_\odot$ expected for PGC objects formed at the initial condensation L_{IC} scale at decoupling. Densities up to 10^{-13} kg m^{-3} are indicated, with massive $100M_\odot$ stars reflecting the large L_{ST} scales caused by the high turbulence levels from the frequent supernovas and powerful radiation and winds. Supernovas, radiation and winds are also mechanisms for increasing PGC density for PGCs close to a galaxy core because they increase friction by producing higher gas densities in the inter-PFP medium within evolving PGCs.

Super-star clusters with mass $10^6 M_\odot$ and $\rho \approx 10^{-15}$ kg m^{-3} are reported by O'Connell et al. 1994 in the core of the dwarf galaxy NGC 1569, observed from the HST even before its repair, confirming the 1985 claim of Arp and Sandage that these objects were not stars but super-star-clusters. De Marchi et al. 1997 conclude that the brightest of the several super-star-clusters NGC 1569A in the NGC 1569 galaxy is actually a superposition of two clusters separated by a distance close to their sizes, so the clusters are apparently in orbit. They are identified as "young globular star clusters". Similar young globular star clusters are reported by Ho and Filippenko 1996 at the center of NGC 1705, an amorphous galaxy, by Holtzman et al. 1996 in the interacting galaxy 1275, and by Watson et al. 1996 in the starburst galaxy NGC 253. In the two colliding galaxies NGC 4038/4039 termed The Antennae, a population of over 700 blue pointlike objects were identified by Whitmore and Schweizer 1995 as young globular clusters formed as a result of the merger, and are therefore possibly dark PGCs brought out of cold storage. The size of the objects is given as 18 pc, giving a density of 10^{-17} kg m^{-3} that is very close to the density at the time of PGC formation from primordial gas, and represents a typical density of Halo GCs. Thus we have evidence that dark proto-globular-clusters (DPGCs) exist in abundance in a variety of galaxy types. If DPGCs consist of PFPs, no significant fraction of their original number of PFPs have apparently evaporated by collisionless dynamics mechanisms. Instead, the

GC mass-density ρ changes observed are to larger rather than smaller values. The original condensation mass $10^6 M_\odot$ has not changed.

For PGCs formed in the outer regions of proto-galaxies (PGs) that were most likely to have maximum turbulence and minimum gas density and thus maximum PFP mass and separation and most rapid cooling to a collisionless state, a more stellar-like dynamics of clustering might seem appropriate with consequent decrease in the average PGC mass density. However, we find no evidence that GC densities outside galaxy core regions ($\approx 10^{20}$ m) deviate appreciably from $\rho \approx 10^{-17}$ kg m^{-3}, suggesting that PGCs are gas stabilized by the possibility of reevaporation of their PFPs. The PGCs themselves presumably dispersed, along with the luminous GCs, from locations within an initial maximum galaxy diameter of 10^{20} m to the present size of 10^{21} m, partly due to the expansion of the universe and perhaps partly from their more collisionless dynamics compared to those near galaxy cores. Initial Hubble flow velocity differences at PG scales after decoupling were $\delta v_r = \gamma d = 10^{-13} \times 10^{20} = 10^7$ m s^{-1}, giving rapid expansion of the galactic scale protovoids. The average mass density of galaxies is certainly substantially less today, with $\rho \approx 10^{-21}$ kg m^{-3}, than the average mass density of $\rho \approx 10^{-17}$ kg m^{-3} for the PGs discussed in §4.

Evidence for the existence of Halo dark matter in the form of cold H_2 gas clouds clumped in dark clusters is claimed from measurements of diffuse γ-ray flux, De Paolis et al. 1999. Similarly, cold self-gravitating hydrogen clouds explain “extreme scattering events” of compact radio quasars, Walker and Wardle 1998. Both of these gas-cloud interpretations are consistent with and support our postulated dark metastable PGC-PFPs as the inner-galaxy-halo dark-matter, with their inter-PFP-gas and PFP-atmospheres, except that we require that most of the PGC mass be in the form of mostly-frozen solid-liquid PFPs as the source of this gas that resists changes in the average PGC mass density by shifts in the equilibrium gas density. For cold H_2 clouds to exist completely as gas in clumps with PGC densities and sizes, huge turbulence dissipation rates of $\varepsilon \geq 3.4 \times 10^{-6}$ m^2 s^{-3} would be required to prevent their condensation to form stars at L_{ST} scales, compared to primordial ε values of only $\approx 10^{-13}$ m^2 s^{-3} estimated at the plasma-gas transition from the measured CMB temperature fluctuations leading to PFPs, Gibson 1999b. No source of kinetic energy is available to produce or sustain such turbulence, and without such a source to prevent gravitational condensation the turbulence would dissipate in a few million years and the PGC mass gas cloud would collapse in a powerful star-burst of super-star formation.

5.0.2. Intermittent lognormal n_p probability density function

The average separation distance L_p between small-planetary-objects (PFPs) for a galaxy at present is about $(M_p/\rho_{halo})^{1/3} \approx 7 \times 10^{14}$ m for a galactic density $\rho \approx 10^{-21}$ kg m^{-3}, with object size $r_p \approx (M_p/\rho_p)^{1/3} \approx 4 \times 10^6$ m for a density $\rho_p \approx 3 \times 10^3$ kg m^{-3} corresponding to condensed H-He at halo temperatures, giving approximately 8 decades for the cascade from separation to accretion scales of M_p mass objects. The number density $n_p(r)$ of such planetary objects per unit volume becomes an increasingly intermittent random variable (rv) as the averaging volume size r decreases. If we assume most of the nonlinear gravitational cascade is self-similar, then the probability density function of $n_p(r)$ will be lognormal with an intermittency factor $I_p(R/r) \equiv \sigma^2_{ln[n_p(r)/n_p(R)]}$, where σ^2 is the variance about the mean, that increases as the range of the cascade R/r increases as

$$\sigma^2_{ln[n_p(r)/n_p(R)]} \approx \mu_p ln(R/r) \tag{1}$$

where R is the largest scale of the cascade, $r_p \leq r$ is the smallest, and μ_p is a universal constant of order one. The expression is derived by expressing $n_p(r)/n_p(R)$ as the product of a large number of breakdown coefficients $b_m = n_p(k^m r)/n_p(k^{m+1} r)$ for the $0 \leq m \leq l$ intermediate stages of the cascade with scale ratio k sufficiently large for the random variables b_m to be independent. Taking the natural logarithm of the product

$$\frac{n_p(r)}{n_p(R)} = \frac{n_p(r)}{n_p(kr)} \frac{n_p(kr)}{n_p(k^2r)} \frac{n_p(k^2r)}{n_p(k^3r)} \cdots \frac{n_p(k^{l-1}r)}{n_p(R)} = b_1 b_2 \ldots b_l \tag{2}$$

gives

$$ln[n_p(r)/n_p(R)] = ln(b_1) + ln(b_2) + \ldots + ln(b_l). \tag{3}$$

By the central limit theorem, the random variable $ln[n_p(r)/n_p(R)]$ is Gaussian, or normal, because it is the sum of many identically distributed, independent random variables $ln(b_m)$, which makes the random variable $[n_p(r)/n_p(R)]$ lognormal. The variance of a normal rv formed from a sum of rvs is the sum of the variances of its components. Since there are l breakdown coefficients b_m, and $R/r = k^l$, then

$$\sigma^2_{ln[n_p(r)/n_p(R)]} \approx l \times \sigma^2_{ln(b_m)} = \frac{ln(R/r)}{ln(k)} \sigma^2_{ln(b_m)} \tag{4}$$

which gives Eq. (1) and its constant

$$\mu_p \approx \frac{\sigma^2_{ln(b_m)}}{ln(k)}. \tag{5}$$

The universal constant μ_p is always positive because $\sigma^2_{ln(b_m)}$ is positive and $k > 1$.

If the intermittency factor I_p is large, then estimation of the mean number density $\langle n_p \rangle = n_p(R)$ from a small number of samples becomes difficult. A single sample gives an estimate of the mode, or most probable value, of a distribution, and the mean to mode ratio G (the Gurvich number) for a lognormal n_p distribution is

$$G \equiv \frac{\langle n_p \rangle_{mean}}{\langle n_p \rangle_{mode}} = exp(3I_p/2). \tag{6}$$

Taking $\mu_p \approx 1/2$, Bershadskii and Gibson 1994, and substituting $R/r_p = 7 \times 10^{14}/4 \times 10^6$ gives $I_p = 9.5$. Thus, $G = 1.5 \times 10^6$ from Eq. (6), which is the probable undersampling error for $\langle n_p \rangle$ from a single sample. Increasing M_p has no effect on the intermittency factor I_p or the undersampling error G as long as the aggregation cascade is complete since the ratio R/r_p is unaffected by M_p.

The intermittent lognormal distribution function for $n_p(r)$ inferred above is generic for nonlinear cascades over a wide range of scales and masses and with a wide range of physical mechanisms, since it is based on the central limit theorem which gives nearly Gaussian distributions even when the summed component random variables are not identically distributed and are not perfectly independent as long as there are many components.

5.0.3. Implications for star-microlensing versus quasar-microlensing studies

The EROS and MACHO collaborations (Alcock et al. 1997, Renault et al. 1998, Alcock et al. 1998) estimate they should have encountered about 20 star-microlensing events in their two years of observing with time periods corresponding to $M_p \approx 10^{-6} M_{\odot}$, assuming such objects are uniformly distributed in the Halo with $\langle n_p \rangle R^3 M_p \approx M_{Halo}$. Because they encountered none, they conclude that the contribution of M_p objects can be no more than 1/20 of M_{Halo}. However, since n_p is expected to be a lognormal random variable (LNrv) with intermittency factor $I_p = 9.5$, the most that can be said from this information is that the modal mass $\langle n_p \rangle_{mode} R^3 M_p$ of such objects is less than $M_{Halo}/20$. Since the mean mass $\langle n_p \rangle R^3 M_p \approx G \langle n_p \rangle_{mode} R^3 M_p \leq M_{Halo} G/20$, this is no constraint at all on $M_p \approx 10^{-6} M_{\odot}$ as the mass of Halo objects dominating M_{Halo} because $G \approx 10^6 \gg 20$. About $2G/20$ y of observing time, with no detections, would be needed to make the claimed exclusion, or 150,000 years. Furthermore, 150,000 years assumes all LMC stars are seen through PGCs even though these cover a small fraction of the sky and the PGC density itself is likely to be an intermittent lognormal.

As mentioned, another serious problem with the EROS/MACHO sampling method is a lack of independence of the samples. Both collaborations are constrained because the solid angles occupied by the stars of the Large Magellanic Cloud are smaller than those expected for dark proto-globular-clusters (PGCs) of PFPs. Assuming a PGC diameter of

4.6×10^{17} m at a distance of 10^{21} m, each PGC occupies a fraction 1.7×10^{-8} of the sky, or 1.7% for a million PGCs per galaxy. Therefore, most MACHO/EROS stars are likely to be in areas between PGCs, so they probably will see nothing, as they have done. In order to give independent samples of the Halo mass, stars chosen for star-microlensing tests must be separated by angular distances larger than the separation between the largest clumps. The time for a dark PGC to drift its diameter in the Halo is about 10^{12} s. Three hundred independent samples are required to achieve 50% accuracy with 95% confidence in estimating the expected value of a LNrv with $I_p = 9.5$ (Baker and Gibson 1987), requiring $\approx 10^7$ y of observations toward the LMC to achieve this accuracy. This is very impractical.

A million PGCs in the Galaxy Halo should occupy about 2% of the sky, so if they were uniformly distributed only 2% of the LMC stars would intersect a PGC. This in itself would justify a claim that PFPs fail as the missing mass based on the assumption (questioned herein) that particle density n_p distribution function is uniform. We have shown that even if all the LMC stars were seen through PGCs, the indicated $\langle n_p \rangle$ would still be underestimated since the Gurvich factor for any PGC is likely to be much larger than 200 ($G \approx 10^6$). Quasar microlensing is much less affected by intermittency effects since as shown in §2 the number of microlensing masses seen projected in front of the quasar's luminous disk is likely to be of order 10^8. Because no model as yet exists for the rapid brightness fluctuations observed, we do not yet know the role of structure in the quasar accretion disk and the distribution of implied masses.

6. Conclusions

Quasar-microlensing signals have continuous fluctuations at short time scales indicating the lensing galaxy mass is dominated by small-planetary-mass objects (Schild 1996). From a new theory of hydrodynamic gravitational condensation (Gibson 1996−2000) these objects are identified as PFP "primordial fog particles" that fragmented within PGC fragments within proto-galaxy blobs of neutral gas emerging from the plasma epoch at 300 000 years. The proto-galaxies emerged embedded in a nested foam of proto-supercluster to proto-galaxy structures formed previously within the superviscous Big Bang plasma starting at about 30 000 years, with the non-baryonic component diffusing to fill the voids and damp gravitational formation of ρ fluctuations with amplitudes larger than the $\delta\rho/\rho \approx 8 \times 10^{-5}$ peak level observed, (§4). Observations of quasars, galaxies, and galaxy clusters at large redshifts support predictions of the theory that proto-superclusters formed at $z \approx 5700$, proto-galaxies at $z \approx 1000$, and the first stars, super-stars, proto-quasars, as well as the Schild 1996 "rogue planets", near $z \approx 260$, (§4). The large population of clumped, frozen, hydrogenous planetoids predicted can masquerade as dark energy, quintessence, and the cosmological constant by causing high z supernova dimming as an unanticipated time dependent radiation energy sink and overestimates of the cluster baryon fraction because of similar unanticipated gas clumping and cooling effects. Because the baryonic matter is mostly sequestered as PGC clumps of PFPs before the appearance of quasars, the Gunn-Peterson paradox of the missing intergalactic gas in quasar spectra is resolved.

Failure of star-microlensing studies to observe equivalent microlensing events from the Bulge and LMC is not in conflict with the quasar-microlensing observations because fragmentation of primordial gas as PGCs and the nonlinear clustering of PFP fragments within PGCs during their accretion to form stars produces extremely intermittent lognormal distributions of the object number density n_p, therefore increasing the minimum number of independent samples required for statistically significant estimates of the mean density $\langle n_p \rangle$ by star-microlensing far beyond practical limits. Sample calculations (§4, Gibson 1996, 1997ab, 1999ab, 2000abc) give an estimated primordial fog particle (PFP, MBD) condensation mass range $M_{PFP} \approx 10^{-7} - 10^{-6} M_{\odot}$, supporting the Schild 1996 quasar-microlensing conclusion that the lens galaxy mass of Q0957+561A,B is dominated by "rogue planets." The EROS/MACHO star-microlensing exclusion of such micro-brown-dwarf objects as the missing Halo mass (Alcock et al. 1998) is attributed to the unwarranted assumption that the spatial distribution of such objects is uniform rather than extremely intermittent.

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