

# The Journal of Cosmology

Journal of Cosmology, 2011, Vol 13.

JournalofCosmology.com, 2011

## Identifying a Kaluza Klein Treatment of a Graviton Permitting a Deceleration Parameter $Q(Z)$ As An Alternative to Standard DE

A. Beckwith

Chongqing University, Department of Physics; Chonqing, PRC, 400044  
Emeritus, Department of Physics, the University of Trieste,  
the Abdus Salam International Centre for Theoretical Physics, Trieste, and  
Istituto Nazionale di Fisica Nucleare, Sezione di Trieste, Italy.

### ABSTRACT

The case for a four-dimensional graviton mass (non-zero) influencing reacceleration of the universe in five dimensions is stated, with particular emphasis on whether five-dimensional geometries as given below give us new physical insights as to cosmological evolution. A comparison with the quantum gas hypothesis of Glinka shows how stochastic GW/gravitons may emerge in vacuum-nucleated space, with emphasis on comparing their number in phase space with different strain values. The final question is, can DM/DE be explained by a Kaluza Klein particle construction? I.e., the author presents a Kaluza Klein particle representation of a graviton mass with the first term to the right equal to a DM contribution and with the 2<sup>nd</sup> term to the right being effective DE. We propose obtaining the rate of production of relic universe Kaluza Klein gravitons, based on an analogy to the production of axions from the Sun over a wide range of frequencies. This last statement is a work in progress being developed which is being developed at Chongqing University.

**Keywords:** “massive” gravitons, entropy, Kaluza Klein, dark matter, dark energy

## 1. INTRODUCTION

We presently investigate how a speed up of cosmological expansion could occur by invoking the deceleration parameter as defined below (Beckwith 2010 a, 2011) [1] as a way to look for a mechanism to explain how such a speed up of expansion may be possible. We investigate the utility of the graviton in this investigation, since if the graviton has a very slight rest mass in four dimensions, we have a way to unify both dark matter and dark energy expressions (Beckwith 2010 a, 2011).

We start by setting the mass of the graviton equal to a dark matter expression plus a contributing factor for dark energy. A very light, but non zero rest mass of the graviton in four dimensions would correspond to dark energy. As evidenced by considerable cosmological expansion history with a slow decrease in the rate of deceleration of the expansion of the universe, an ultra light rest mass is selected for the graviton to match a red shift value of .423 one billion years ago, eventually contributing to a speed up of cosmological expansion.

We next consider what sort of signal contribution could be identified that would allow GW generation in connection with massive gravitons. Actual hardware requirements which may enable such a the search for such GW are described in (Woods et. al., 2011).

The analogy with axions is picked in part, with a nod to solar axion research, to help researchers identify signal detection regimes of massive gravitons which could be sensed by appropriate GW/graviton detector technology (Woods et. al., 2011). The expectation is that the peak distribution of graviton generation and therefore detectable GW would be in a comparatively narrow band of frequencies, and that it is unlikely that there would be a smooth curve of graviton emission rates, similar to the expected results for axions. If graviton detection is possible confirmation of their effective mass will lead to partial experimental confirmation that the graviton contributed to the reacceleration of the universe one billion years ago. The effective “current” of “massive” gravitons as a contribution to reacceleration of the universe, if confirmed, would have many exciting cosmological implications including experimentally falsifiable predictions for GW astronomy. The ramifications of “massive” graviton current is described more fully in the ‘discussions’ section of this document.

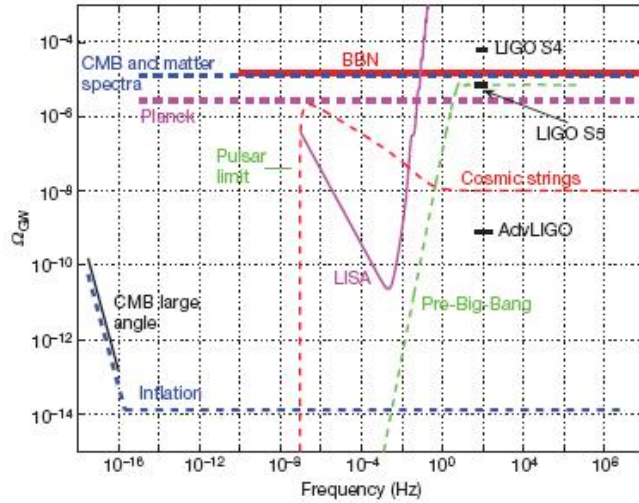
## 2. BASIC THEME, HOW TO IDENTIFY THE ‘FOOT PRINT’ OF “MASSIVE” GRAVITONS

This article begins by a question of how to identify a Kaluza Klein treatment of a graviton in five dimensions, with a DM component and a very small rest mass in four dimensions, similar to the behavior of DE a billion years ago. After grounding in gravitational wave density, the issue of a rate equation for production of relic particles from initial cosmology will be proposed as a research goal. The analogy of DM from axions from the Sun having a rate equation graph plotted against frequency is suggested as a worthy goal of researchers if Kaluza Klein gravitons can be detected and measured. However, production of axions from the Sun is broadband whereas in most likelihood, the relic particle production of gravitons would be in narrow frequency ranges. as predicted by (Grishchuk 2006) . Gravitational wave density will be presented as necessary background.

We will start with a first-principle introduction to determination of gravitational wave energy density  $\Omega_{gw}$  using the definition given by (Maggiore 2008)  $\Omega_{gw}$  is defined as a way to measure gravitational wave strength as a function of general random energy fluctuation backgrounds. Eq. (1) measures the strength of the gravitational wave signal.

$$\Omega_{gw} \equiv \frac{\rho_{gw}}{\rho_c} \equiv \int_{f=0}^{f=\infty} d(\log f) \cdot \Omega_{gw}(v) \Rightarrow h_0^2 \Omega_{gw}(v) \cong 3.6 \cdot \left[ \frac{n_f}{10^{37}} \right] \cdot \left( \frac{v}{1kHz} \right)^4 \quad (1)$$

where  $n_f$  is the frequency-based numerical count of gravitons per unit phase space. Since  $\Omega_{gw}$  is usually an extremely small fraction of very noisy relic cosmological background conditions, the representation of  $\Omega_{gw}$  is based on scaling its relative strength. I.e., Figure 1 shows the magnitude of  $\Omega_{gw}$  for different cosmological models. Here,  $\Omega_{gw}$  is the same as  $\Omega_{GW} \equiv \rho(t)/\rho_{critical}$  as of figure 1 below. Combining experimental confirmation of Eq. (1) with observations using different values of  $H = \frac{\dot{a}}{a}$  and  $\Omega_{GW} \equiv \rho(t)/\rho_{critical}$  will be tied in with analysis of the plots in Figure 1. Note, these models are not consistent with each other.



**Figure 1.** (Abbott et al., 2009) shows the relation between GW frequency and GW energy density for different cosmological models

(Grishchuk, 2001) states the relation between  $\Omega_{gw}$  (which he refers to as  $\Omega_g$ ) and the frequency spectrum  $h(v_g, \tau)$  as given in equation (2). The models brought up in figure 1 need to be compared with each other, as part of experimental inquiry. Eq. (2) and Eq. (3) are part of that comparison.

$$\Omega_{gw} \approx \frac{\pi^2}{3} \left( \frac{v}{v_H} \right)^2 h^2(v, \tau) \quad (2)$$

We will address the divergences between models presented by Maggiore, Abott, and Grishchuk

and will develop candidate discriminating criteria for a number count  $n_f$  based on (Glinka, 2007) graviton gas work:

$$n_f = [1/4] \cdot \left[ \sqrt{\frac{v(a_{initial})}{v(a)}} - \sqrt{\frac{v(a)}{v(a_{final})}} \right] \quad (3)$$

If  $h_0 \sim .75$

$$\Omega_{gw}(v) \cong \frac{3.6}{h_0^2} \cdot \left[ \frac{n_f}{10^{37}} \right] \cdot \left( \frac{v}{1kHz} \right)^4 \quad (4)$$

If we assume  $a \sim a_{final}$ , and substituting (Glinka's, 2007) approximation of a graviton "gas" by treating gravitons as boson "particles" in the early universe -- permitting a rapid accumulation of "gravitons" in the initial phase of the big bang -- then for very high  $v(a_{initial})$  values, Eq. (3) will in most cases be approximately,

$$n_f = [1/4] \cdot \left[ \sqrt{\frac{v(a_{initial})}{v(a)}} - 1 \right] \sim [1/4] \cdot \left[ \sqrt{\frac{v(a_{initial})}{v(a)}} \right] \quad (5)$$

Frequently, a researcher will be looking at  $\Omega_{gw} \approx 10^{-5} - 10^{-14}$ , and comparing that value with what one gets for equation (5) with  $\Omega_{gw} \approx 10^{-5}$  in pre-big-bang scenarios, with initial values of frequency set for  $v(a_{initial}) \approx 10^8 - 10^{10}$  Hz, as specified by (Grishchuk, 2006). Notice that  $v(a_{final}) \approx 10^0 - 10^2$  Hz near the present era. Table 1, Table 2, and Table 3 below are based on Eq. 4 and Eq. 5, as to forming a suitable counting algorithm. The idea is to use a semi-classical approximation via the Wheeler De Witt equation as described by (Glinka, 2007) for early-universe gravitons. This semi classical approximation, by treating gravitons as bosons, will permit up to  $10^6$  particles as entropy units being manufactured within Planck length values of space-time volume, i.e., roughly  $10^{120}$  times smaller

**Table 1.** If one assumes  $\Omega_{gw} \approx 10^{-5}$

$v(a) \approx v(a_{final}) \approx 10 - 10^2$	$v(a_{initial})$	$n_f$ ( Eq. 5)
***	$10^3$	$10^{32}$
***	$10^8$	$10^{12}$
***	$10^{10}$	$10^3$

**Table 2.** If one assumes  $\Omega_{gw} \approx 10^{-10}$

$v(a) \approx v(a_{final}) \approx 10 - 10^2$	$v(a_{initial})$	$n_f$ ( Eq. 5)
***	$10^3$	$\sim 10^{27}$
***	$10^8$	$\sim 10^7$
***	$10^{10}$	$\sim 10^{-2}$ (not really measurable)

**Table 3.** If one assumes  $\Omega_{gw} \approx 10^{-14}$

$v(a) \approx v(a_{final}) \approx 10 - 10^2$	$v(a_{initial})$	$n_f$ ( Eq. 5)
***	$10^3$	$\sim 10^{23}$
***	$10^8$	$\sim 10^3$
***	$10^{10}$	$\sim 10^{-6}$ ( not really measurable)

As will be explained in the Appendix, there is a way to relate graviton count and entropy, so then the numbers associated with  $n_f$  are a de facto counting algorithm for entropy per unit phase space. Note that the highest counting numbers for entropy are associated with  $\Omega_g \approx 10^{-5}$ , which, according to Figure 1, is associated with pre-big-bang GW/graviton production.  $\Omega_g \approx 10^{-14}$  is associated with usual inflation, as given in Figure 1. I.e., if one is looking for standard creation of entropy paradigms associated with the early universe, a typical phase transition argument for early entropy production is given by (Tawfik,2008), which for QCD regimes is shown in Eq.(6).

$$S \equiv V_3 \cdot T^3 \sim 2.05 \cdot 10^{58} \quad (6)$$

Where S = entropy. We assume here that  $S_{total} \sim 10^{58}$  may be associated with gravitons/GW at the very end of inflation, and not the beginning (where one would have a far lower initial count, i.e.,  $S \sim 10^7$ ) and with frequencies initially on the order of  $10^8$  to  $10^{10}$  in the beginning of cosmological evolution. Such a huge burst of graviton production for temperatures on the order of  $T \sim 174 \cdot MeV$  would lead to measurable consequences.

### 3. GRAVITONS WITH A NON ZERO REST MASS. THE KK TREATMENT

Consider if there is then also a small graviton mass, i.e., as stated by (Beckwith, 2010a, 2011):

$$m_n(Graviton) = \sqrt{\frac{n^2}{L^2} + (m_{graviton-rest-mass} = 10^{-65} \text{ grams})^2} = \frac{n}{L} + 10^{-65} \text{ Grams} \quad (7)$$

Note that (Rubakov, 2002) works with KK gravitons, without the tiny mass term for a 4

dimensional rest mass included in Eq.(7). To obtain the KK graviton/DM candidate representation along RS dS brane world, Rubakov obtains his values for graviton mass and graviton physical states in space-time after using the following normalization:  $\int \frac{dz}{a(z)} [h_m(z) \cdot h_{\tilde{m}}(z)] \equiv \delta(m - \tilde{m})$ . (Rubakov, 2002) uses  $J_1, J_2, N_1, N_2$  which are different forms of Bessel functions. His representation of a graviton state is given by Eq. (8), which is almost completely acceptable for our problem, since the rest mass of a graviton in four dimensions is so small. If so, then the wave function for a graviton with a tiny 4 dimensional space time rest mass can be written as (Rubakov, 2002).

$$h_m(z) = \sqrt{m/k} \cdot \frac{J_1(m/k) \cdot N_2([m/k] \cdot \exp(k \cdot z)) - N_1(m/k) \cdot J_2([m/k] \cdot \exp(k \cdot z))}{\sqrt{[J_1(m/k)]^2 + [N_1(m/k)]^2}} \quad (8)$$

Eq. (8) is for KK gravitons having a TeV magnitude mass  $M_z \sim k$  (i.e., for mass values at .5 TeV to above 1 TeV) on a negative tension RS brane. It would be useful to relate this KK graviton, which is moving with a speed proportional to  $H^{-1}$  with regards to the negative tension brane with  $h \equiv h_m(z \rightarrow 0) = \text{const} \cdot \sqrt{\frac{m}{k}}$  as an initial starting value for the KK graviton mass. If Eq. (8) is for a “massive” graviton with a small 4 dimensional graviton rest mass and if  $h \equiv h_m(z \rightarrow 0) = \text{const} \cdot \sqrt{\frac{m}{k}}$  represents an initial state, then one may relate the mass of the KK graviton moving at high speed with the initial rest mass of the graviton. This rest mass of a graviton is  $m_{\text{graviton}} (4\text{-Dim GR}) \sim 10^{-48} eV$ , opposed to  $M_x \sim M_{\text{KK-Graviton}} \sim .5 \times 10^9 eV$ . Whatever the range of the graviton mass, it may be a way to make sense of what was presented by (Dubovsky et al., 2009) [11], who argue for a graviton mass, using CMBR measurements, of  $M_{\text{KK-Graviton}} \sim 10^{-20} eV$ . Also, Eq. (9) will be the starting point used for a KK tower version of Eq. (9). So from (Maartens, 2004 and 2005) ,

$$\dot{a}^2 = \left[ \left( \frac{\tilde{\kappa}^2}{3} \left[ \rho + \frac{\rho^2}{2\lambda} \right] \right) a^2 + \frac{\Lambda \cdot a^2}{3} + \frac{m}{a^2} - K \right] \quad (9)$$

(Maartens, 2004 and 2005) also gives a 2<sup>nd</sup> Friedman equation:

$$\dot{H}^2 = \left[ - \left( \frac{\tilde{\kappa}^2}{2} \cdot [p + \rho] \cdot \left[ 1 + \frac{\rho^2}{\lambda} \right] \right) + \frac{\Lambda \cdot a^2}{3} - 2 \frac{m}{a^4} + \frac{K}{a^2} \right] \quad (10)$$

Also, an observer is in the low red shift regime for cosmology, for which  $\rho \cong -P$ , for red-shift values  $z$  from zero to 1.0-1.5. One obtains exact equality,  $\rho = -P$ , for  $z$  between zero to .5. The net effect will be to obtain, a phenomenologically based description of re acceleration of the universe based on Eq. (10), assuming  $\Lambda = 0 = K$  and using  $a \equiv [a_0 = 1] / (1 + z)$  to get a deceleration parameter  $q$  as given in Eq. (11).

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -1 + \frac{2}{1 + \tilde{\kappa}^2 [\rho/m] \cdot (1+z)^4 \cdot (1 + \rho/2\lambda)} \approx -1 + \frac{2}{2 + \delta(z)} \quad (11)$$

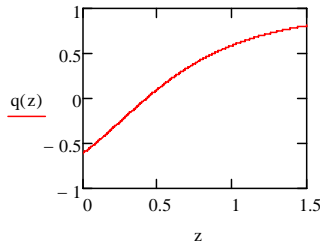
These set values, along with a revised Eq. (10) allow a graviton-based substitute for DE.  $\Lambda = 0 = K$  plus a small rest mass for a graviton in four dimensions allows for "massive gravitons" that behave the same as DE. Setting  $\Lambda = 0 = K$ , while having a modified behavior for the density expression, for a Friedman equation with small 4 dimensional graviton mass, means that dark energy is being replaced by a small 4 dimensional rest mass for a graviton.

#### 4. CONSEQUENCES OF SMALL GRAVITON MASS FOR REACCELERATION OF THE UNIVERSE

In a revision of (Alves *et. al.*, 2009) , (Beckwith, 2010a and 2011) used a higher-dimensional model of the brane world combined with KK graviton towers per (Maartens, 2004) . The energy density  $\rho$  of the brane world in the Friedman equation is used in a form similar to (Alves *et al.*, 2009) by (Beckwith, 2010a, 2011) for a non-zero graviton:

$$\rho \equiv \rho_0 \cdot (1+z)^3 - \left[ \frac{m_g \cdot (c=1)^6}{8\pi G(\hbar=1)^2} \right] \cdot \left( \frac{1}{14 \cdot (1+z)^3} + \frac{2}{5 \cdot (1+z)^2} - \frac{1}{2} \right) \quad (12)$$

(Beckwith, 2010a and 2011) suggests that at  $z \sim .4$ , a billion years ago, acceleration of the universe increased, as shown in Figure 2. Figure 2 is, if confirmed is a good verification of the (Ng, 2008) hypothesis, and would be a starting point to investigate the role of gravitons in cosmology. The author notes that (Buonanno,2006) assumes a much lower range of initial frequencies for relic GW than the author. (Beckwith, 2010a and 2011) obtained a re-acceleration of the universe result as given in figure 2. The contribution of a low rest mass for 4 dimensional gravitons, as given in equation 7 leads to a speed-up of acceleration of the expansion of the universe a billion years ago, i.e. for a red shift slightly smaller than .5. Figure 2 below is predicated upon a small 4 dimensional rest mass (stated in Eq. (7) for a graviton behaving the same as dark energy)... We will state in our discussions section as to what is needed to give experimental confirmation as to what is a current for a "massive" graviton which is appropriate for explaining in part, figure 2 below.



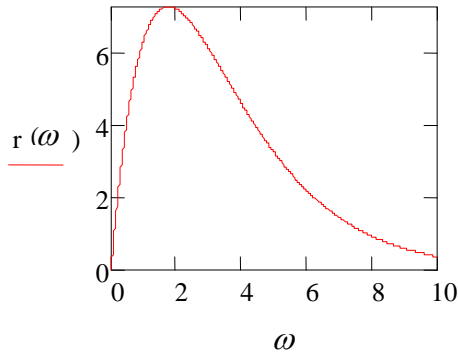
**Figure 2.** Re-acceleration of the universe based on (Beckwith ,2010a); (note that  $q(z) < 0$  if  $z < .423$ ).

## 5. COMPARISON WITH AXION FLUX RESULTS FROM THE SUN: WHAT WOULD BE NEEDED TO MEASURE DM FLUX FOR A NEW MODEL OF DM/ DE?

This section is intended to explain a rate of particle production as given for solar axions, and to determine what may be necessary to adapt such an approach for Kaluza-Klein gravitons in the onset of inflation. Figure 3 is a redo, using the (Dimpoulou et al, 1986). value of axion flux, while noting that it is similar to the axion flux from the sun, per (Lazaruth et al., 1992) . It would be appropriate to do the same with the DM implied by Eq. (7). Figure 3 is based on Eq. 13, given by (Buoanno, 2006), for applied frequency  $\omega$  vs.  $r(\omega)$ , axion flow in KeV values of solar axions:

$$r(\omega) \sim \omega \cdot \exp[-\omega + \varpi] \quad (13)$$

Eq. (13) models what happens for axions in the sun, and we hope to eventually obtain a similar rate expression of graviton production for Kaluza Klein gravitons as given in Eq. (7) versus energy, similar to what is shown in figure 3, in the case of solar axions.



**Figure 3.** : Beckwith's revision of axion flux from the Sun, in terms of (frequency=energy, if  $\hbar = 1$ ), with the plot of the number of axions produced by the Sun in terms of KeV values of solar axions.

The rate of production of solar axions  $r(\omega)$  is plotted against frequency,  $\omega$  . Beckwith's goal is to eventually duplicate figure 3 for relic gravitons, using data collection for Kaluza-Klein gravitons. This rate equation plotted in figure 3, as given in equation (13) as used by Beckwith, is dominated by  $\omega \cdot \exp[-\omega + \varpi]$  ( $\varpi$  a plasma interaction effect that is to be determined, and  $\omega$  is part of the expression for permitted solar axion energy values.  $\omega \gg \varpi$  in most cases. We suggests finding grounds for a similar energy plot of DM values from a suitably modified version of Eq. (7) for KK dark-matter candidates. Doing so would mean understanding how a rate equation based upon Eq. (13) for DM production could commence using a model of KK DM production/ evolution. We suggest that it would be appropriate to use an early universe counterpart to the known model of axion production in stars to illustrate the correspondence of axion production in the sun with relic particle production in the early universe. For Eq. (14),  $n_z$  is the number density of Z atoms with an ionized K shell,  $n_e$  the number density of free electrons,  $\rho$  a general density of states for the axion-producing background. Furthermore  $\sigma$  is the axio-recombination (free-bound) cross-section given by (Dimopoulos,1986). However, this



leaves open the question of whether the cross section  $\sigma$  for the LHC values of massive gravitons is similar to what is done in stars. The usual assumption is that the definitions correspond. Eq. 14 is a rate of production of axions.

$$\Gamma_{\text{axion-production-in-stars}} = \sum_z \frac{n_z n_e \langle \sigma \cdot v_e \rangle}{\tilde{\rho}} \quad (14)$$

Eq. (14) has corresponds partially with Eq. (13), but differs due to the density value  $\tilde{\rho}$ , which may be derived experimentally. In the early universe, for a KK dark matter counterpart, we would still have a density value  $\tilde{\rho}$  to consider and a possible  $\sigma$  for cross section for some interaction of KK DM production. However, due to early universe conditions, there would be no counterpart to  $n_z$  or  $n_e$ . (Durer, Marissa, and Renaldo, 2009) have used very early universe plasmas, going back to the electroweak transition and turbulence, as a model for early-universe GW production. One would need to specify how to obtain  $\sigma$  for some interaction of KK DM production, which is why observation of the mass and width (or cross section) of one or more KK gravitons, as part of a DM candidate, at the LHC, as remarked by (Grzadkowski et al., 2006) may be the only way to obtain experimental inputs into a graviton production / KK DM version of Eq. (14). However, this leaves open the question of whether the cross section  $\sigma$  for the LHC values of massive gravitons, etc, would be the same as what would occur for early universe conditions. So far, the only known theoretical calculations of the above are along the lines of  $\sigma$  arising from photon and neutrino annihilation rates, as given by (Hewett, 2006) [20]. The author is attempting to obtain suitable values of  $\sigma$  to put in the given calculation at the start of the inflationary era.

## 6. FINDINGS, DISCUSSION OF RESULTS.

We can use Figure 3 as an idea of how to identify rate of graviton collection opportunities against frequency for a detector along the lines of reference given by (Woods, et. al, 2011). What is expected though is that instead of the smoothly varying curve that if gravitons were matched against rates, in a similar manner, that there would be spikes, instead of smooth variations. Once GW astronomy becomes a fact, figure 3 for gravitons will emerge. The sharpness of the spikes, if analyzed properly will say much about the supposition given in Eq. (7) about a KK decomposition of the mass of a graviton into dark matter and dark energy contributions.

What is most intriguing, is the possibility of Eq. (11) having a non uniform frontier of re acceleration of the universe, a billion years ago. Not a “perfect sphere” of re acceleration, but one with a jagged edge of moving space time regions. I.e. a complicated structure. In part, adding more details to supposed ‘cosmic voids’ and regions of space time distributions of galaxies in a fractal geometric manner, as given in (Ribeiro and Miguelote, 1998) . If and when GW astronomy becomes a fact, suppositions as to how galaxies are distributed through space time may obtain a phenomenological descriptive rationale, which we hope leads to falsifiable experimental measurements.. The main point we wish to emphasize is that to do all of this, the following current behavior in a GW / graviton detector would have to be verified for massive gravitons. This is the geometry of space time which may be confirmed by appropriate analysis of Eq. (11). So, next, how to confirm the reality in falsifiable experimental conditions for obtaining “massive gravitons” making all of this possible?

What (Li et al., 2003) showed which (Beckwith, 2010b) commented upon and made an extension of is to obtain a way to present first order perturbative electromagnetic power flux, i.e. what was called  $\mathbf{T}^{uv(1)}$  in terms of a non zero four dimensional graviton rest mass, in a detector, in the presence of uniform magnetic field, when examining the following situation, i.e what if we have curved space time with say an energy momentum tensor of the electro magnetic fields in GW fields as given by

$$\mathbf{T}^{uv} = \frac{1}{\mu_0} \cdot \left[ -F_\alpha^\mu F^{\nu\alpha} + \frac{1}{4} \cdot g^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right] \quad (15)$$

(Li et al, 2003) state that  $F_{\mu\nu} = F_{\mu\nu}^{(0)} + \tilde{F}_{\mu\nu}^{(1)}$ , with  $|\tilde{F}_{\mu\nu}^{(1)}| \ll |F_{\mu\nu}^{(0)}|$  will lead to

$$\mathbf{T}^{uv} = \mathbf{T}^{uv(0)} + \mathbf{T}^{uv(1)} + \mathbf{T}^{uv(2)} \quad (16)$$

The 1<sup>st</sup> term to the right hand side of Eq. (16) is the energy – momentum tensor of the back ground electro magnetic field, and the 2<sup>nd</sup> term to the right hand side of Eq. (16) is the first order perturbation of an electro magnetic field due to the presence of gravitational waves. The above Eq. (15) and Eq. (16) will eventually lead to a curved space version of the Maxwell equation. As was given in (Li et. al, 2003)

$$\frac{1}{\sqrt{-g}} \cdot \frac{\partial}{\partial x^\nu} \cdot (\sqrt{-g} \cdot g^{\mu\alpha} g^{\nu\beta} F_{\alpha\beta}) = \mu_0 J^\mu \quad (17)$$

As well as

$$F_{[\mu\nu,\alpha]} = 0 \quad (18)$$

Eventually, with GW affecting the above two equations, we have a way to isolate  $\mathbf{T}^{uv(1)}$ . If one looks at if a four dimensional graviton with a very small rest mass included (Beckwith, 2010b) we can write

$$\frac{1}{\sqrt{-g}} \cdot \frac{\partial}{\partial x^\nu} \cdot (\sqrt{-g} \cdot g^{\mu\alpha} g^{\nu\beta} F_{\alpha\beta}) = \mu_0 J^\mu + J_{effective} \quad (19)$$

Where for  $\varepsilon^+ \neq 0$  but very small

$$F_{[\mu\nu,\alpha]} \sim \varepsilon^+ \quad (20)$$

The claim (Beckwith, 2010b) made is that

$$J_{effective} \cong n_{count} \cdot m_{4-D-Graviton} \quad (21)$$

As stated by (Beckwith,,2010b)  $m_{4-D-Graviton} \sim 10^{-65}$  grams , while  $n_{count}$  is the number of gravitons which may be in the detector sample. What researchers intend to do is to try to isolate out an appropriate  $T^{(i)}$  assuming a non zero graviton rest mass. Finding an appropriate  $T^{(i)}$  if successfully done would be enough to help obtain some experimental confirmation as to figure 2 above, as well as give more understanding as to the physics inherent as to figure 3, in its GW / graviton physics configuration. The hope is that proving Eq. (21) above will lead to falsifiable experimental results as to many topics in cosmology which so far are a province of many disparate cosmology models. A situation we hope to rectify in the 21 century.

## 7. CONCLUSIONS

Can GW/Gravitons do double duty as DM/DE candidates in cosmic evolution? (Beckwith, 2010a and 2011) investigated whether gravitons could be a graviton gas as a substitute for a vacuum energy. He also considered a suggestion by (Yurov, 2002) of double inflation, which if verified would support the models in Figure 1. What is to be done is to obtain a rate of production of KK relic gravitons over a frequency range different from that of axions from the Sun. The rate of production of axions is given in Figure 3. Assuming gravitons as a bosonic gas permits up to a million gravitons in a volume of space  $10^{120}$  smaller than what is normally predicted. Eventually, deriving a new rate of graviton production for early universe conditions will be necessary, and the author expects that it will be over a far narrower range of frequencies than given in figure 3. This is what will be investigated (Woods et al.,2011). We are also intrigued by the possibility that as given in.( Ribeiro and Miguelote,2006) that proper analysis of Eq. (7) via instrumentation (Ribeiro and Miguelote , 2006 ) will allow us to understand the space time geometry of galaxy clusters, and of the density of space time just at the point of re acceleration of the universe a billion years ago. To do this, we should attempt to , if a detector can be built with a uniform magnetic field which may allow us to identify and confirm a current from “ massive “ gravitons equivalent to Eq. (21) above. We also state that confirmation of Eq. (21) above would give credence to a unified DM/DE model as an alternative to the Claperyon Gas models (Shilo and Ghez,2008) have been discussed repeatedly. Finally, it also would give an insight as to additional dimensions which may be revealed by gravitational wave astronomy (Clarkson and Seahra, 2007) which so far has been a theoretical enterprise with no experimental input so far.

## 8. NOMENCLATURE

$Z_N$  = partition function, a concept usually from statistical physics

$\varpi$  = frequency of a space plasma

$n_z$  = number density, i.e., the numerical count for Z atoms in a phase-space regime.

$\lambda$  = wavelength of a "particle." (Wave-particle duality of quantum mechanics)

$\sigma$  = cross section, in QM, usually in units of  $\pi$  times spatial distances, squared.

$z$  = red shift =  $\sqrt{1 + (v/c)/1 - v/c} - 1$  , where v is the speed of a "particle" and c is speed of light

$\rho$  = density of “physical states,” i.e., usually associated with physical states in a unit of phase space "volume."

KK. = Kaluza-Klein. A model that seeks to unify the two fundamental forces of gravitation and electromagnetism. In the case of this paper, it is for particles obeying a unification of gravitation and electromagnetism.

$q = -\frac{\ddot{a}a}{\dot{a}^2}$  is a "deceleration" parameter; when positive, it means cosmic acceleration is slowing down, and when negative, it means cosmic acceleration is speeding up.

$a$  = scale factor, and the dots refer to time derivatives.

$m_g$  = rest mass of a graviton. Usually in four dimensions:  $\sim 10^{-65}$  grams

$\Omega_{gw} \equiv \rho_{gw}/\rho_c$  = Gravitational wave density, rescaled, with  $\rho_{gw}$  and  $\rho_c$ , critical density. Here,  $\Omega_{gw}$  is a ration of the relative strength of a gravitational wave over a general energy background observed in cosmology.

P = pressure. I.e.,  $\rho = -P$ , with regards to density and pressure.

K = curvature, i.e. K = 0 means zero space-time curvature (flat space)

DM = Dark Matter.

DE = Dark energy.

## 9. ACKNOWLEDGEMENTS

The author wishes to thank Dr. Fangyu Li as well as Stuart Allen, of international media associates whom freed the author to think about physics, and get back to his work. This work is supported in part by National Nature Science Foundation of China grant No. 11075224

## 10. APPENDIX

The appendix section presents how a modification of infinite quantum statistics by (Ng,2008) permits a one-to-one identification of entropy with relic "particle" production in the early universe. This step by (Ng,2008) uses a small graviton creation volume, V; for high frequency (short wavelength) relic gravitons right after the big bang. What follows is consistent with Ng if the graviton volume V for nucleation is tiny, well inside inflation values. So the log factor drops out of entropy S if V as an initial space-time volume is chosen properly for both Eq (1) and Eq. (2)

$$Z_N \sim \left(\frac{1}{N!}\right) \cdot \left(\frac{V}{\lambda^3}\right)^N \quad (1)$$

This result begins with modification of the entropy/partition function that Ng used in an approximation of temperature, starting with early temperature  $T \approx R_H^{-1}$  ( $R_H$  represents the region of space of the particles in question). Eq. (1), according to Ng, leads to entropy of the limiting value of a counting algorithm,  $S = (\log[Z_N])$  will be modified by what (Ng,2008) refers as using infinite quantum statistics, as given in Eq, (2).

$$S \approx N \cdot (\log[V/N\lambda^3] + 5/2) \xrightarrow{\text{Ng-inf inite-Quantum-Statistics}} N \cdot (\log[V/\lambda^3] + 5/2) \approx N \quad (2)$$

Furthermore, assume that the volume of space is of the form  $V \approx R_H^3$  and look at a numerical factor  $N \sim (R_H/l_p)^2$ , where the denominator is Planck's length (on the order of  $10^{-35}$  centimeters). We also specify a "wavelength"  $\lambda \approx T^{-1}$ . So the value of  $\lambda \approx T^{-1}$  and of  $R_H$  are the same order of magnitude. Note (Ng,2008) changed conventional statistics: he outlined how to get  $S \approx N$  or  $S \approx \langle n \rangle$  (where  $\langle n \rangle$  is graviton density), beginning with a partition function as in Eq. (1).

## REFERENCES

Abbott B. P. *et al.* (2009) [The LIGO Scientific Collaboration & The Virgo Collaboration], "[An Upper limit on the stochastic gravitational – wave background of cosmological origin The LIGO Scientific Collaboration & The Virgo Collaboration](#)," *Nature*, **460**, (2009), pp. 991 – 993.

Alves, E. , Miranda O., and de Araujo, J. (2009) "Can Massive Gravitons be an Alternative to Dark Energy?" *ArXiv*, <http://arxiv.org/pdf/0907.5190>.

Beckwith, A. W. (2010) "Applications of Euclidian Snyder Geometry to the Foundations of Space Time Physics", *EJTP* **7**, **No. 24** (2010) 241 – 266, *Vixra* <http://vixra.org/abs/0912.0012>,v 6 (2010).

Beckwith, A.W. (2010) in a talk at Chonqing University, November 4 , 2010

Beckwith A. W. (2011) "Energy Content of Gravitation as a Way to Quantify Both Entropy and Information Generation in the Early Universe", PP.58-61 DOI: 10.4236/jmp.2011.22010

Buonanno, A. (2007) "Gravitational waves" pp 10-52, LXXXVI, *Particle physics and Cosmology, the fabric of space-time*, Elsevier, Oxford, UK, 2007.

Clarkson, C. and Seahra, S. S. (2007) " A gravitational wave window on extra dimensions" *Class. Quant. Grav.* 24, F33 (2007).

Dimopoulos, S. *et al.*, (1986) "Axion recombination, a new mechanism for axion stellar production," *SLAC PUB 3992*, June 1986 <http://www.slac.stanford.edu/cgi-wrap/getdoc/slac-pub-3992.pdf> (1986).

Dubovsky, S. , *et al.* (2009) "Signatures of a Graviton Mass in the Cosmic Microwave Background," *report UTTG-06-09, TCC-23-09, PhysRevD.81.023523*, <http://arxiv.org/abs/0907.1658> ( 2009).

[Durrer](#), R., [G. Marozzi](#) , G. ., and [Rinaldi](#). M. (2009) "On Adiabatic Renormalization of Inflationary Perturbations," *Phys.Rev.D* 80:065024,2009; <http://arxiv.org/abs/0906.4772>

Glinka, L.A. (2007), "Preliminaries in Many-Particle Quantum Gravity. Einstein--Friedmann Spacetime," *ArXiv* [http://arxiv.org/PS\\_cache/arxiv/pdf/0711/0711.1380v4.pdf](http://arxiv.org/PS_cache/arxiv/pdf/0711/0711.1380v4.pdf)

Grishchuk, L. P.,(2001) “Relic gravitational waves and their detection,” *Lect. Notes Phys.* 562, 167 (2001), <http://arxiv.org/abs/gr-qc/0002035>

Grishchuk, L. P. (2006) “Discovering Relic Gravitational Waves in Cosmic Microwave Background Radiation,” Invited lecture at the first J.A.Wheeler School on Astrophysical Relativity, *ArXiv*, June 2006. <http://arxiv.org/abs/0707.3319> (2010).

Grzadkowski, B., and Gunion, J. (2006) , “KK Gravitons and Unitarity Violation in the Randall-Sundrum Model,” Report number: *UCD-2006-15* (2006). *Arxiv*, <http://arxiv.org/abs/hep-ph/0610105>.

Hewett, J. (2006) “Phenomenology of Extra Dimensions,” pp. 231-262, in *Particle Physics Beyond the Standard Model, Les Houches, LXXXIV*, Ecole d'été de physique , with Kazakov, D. Lavigna, S., and Dalibard, J., editors (2006).

Lazarus, D. et al. (1992), “A search for solar axions,” *PRL* **69**, 16, 2333, 1992.

Li, F., Tang, M., and Shi, D. (2003) “ Electromagnetic response of a Gaussian beam to high frequency relic gravitational waves in quintessential inflationary models”, *PRD* **67**,104008 (2003), pp1-17

Maggiore. , M. (2008) , *Gravitational Waves, Volume 1: Theory and Experiment*, Oxford Univ. Press (2008).

Maartens. R. , (2004) “Brane-World Gravity,” *Living Rev. Relativity* 7, (2004) ,<http://www.livingreviews.org/lrr-2004-7> (2004).

Maartens , R., (2005) “Brane world cosmology,” pp 213-247 from the conference *The physics of the Early Universe*, editor Papantronopoulos, (Lect. notes in phys., Vol **653**, Springer Verlag, 2005).

Ng. Y., J. ,(2008) “Spacetime Foam:From Entropy and Holography to Infinite Statistics and Nonlocality,” *Entropy* **10**( 4), pp. 441-461 (2008).

Ribeiro M. B. and Miguelote, A. Y.(1998) “Fractals and the Distribution of Galaxies”, *Braz. J. Phys.* vol. 28 n. 2 São Paulo June 1998

Rubakov, V.A.,(2002) *Classical theory of gauge fields* Princeton University Press NJ., USA. May 6, 2002

Shilo, D. and Ghez, R. (2008) “New Wine in Old Flasks, a New Solution to the Clapeyron Equation”, *Eur. J. Phys.* 29 25 (2008) doi: 10.1088/0143-0807/29/1/003

Tawfik , A., (2009) “Cosmological Consequences of QCD Phase Transition(s) in Early Universe,” *AIP Conf. Proc.* 1115:239-247,2009, <http://arxiv.org/abs/0809.3825>

Woods, R.C., et. al. (2011) “A new theoretical technique for the measurement of high-frequency

relic gravitational waves”, Accepted by J.M.P for May 2011, [vixra.org/pdf/1010.0062v1.pdf](http://vixra.org/pdf/1010.0062v1.pdf)

Yurov, A.V. , (2002) “Complex field as inflaton and quintessence,” *arXIV*,  
<http://arxiv.org/abs/hep-th/0208129> (2002).