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DOES THE PHOTON CARRY THE ARROW OF TIME? AN EXPERIMENTAL TEST

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Abstract

If the photon has a negative parity under Wigner time reversal this generates a spontaneous CPT symmetry breaking effect that causes the photon to carry the quantum electrodynamic arrow of time (Leiter, D., 2009, 2010). In order to demonstrate the validity of this idea we show that a classic nonlinear optics experiment in the scientific literature, which involves a Michelson interferometer using combinations of ordinary mirrors and phase conjugate mirrors, contains experimental results which support the idea that the photon has a negative parity under Wigner time reversal.

SECTION 1: INTRODUCTION

It has been demonstrated (Leiter, D., 2009, 2010) that the quantum electrodynamic measurement process can be completed by inserting an operator symmetry of microscopic observer-participation called "Measurement Color" (MC) into Quantum Electrodynamics (QED).

The resultant Measurement Color Quantum Electrodynamics (MC-QED) formalism was shown to be a nonlocal quantum field theory which contains a time reversal violating description of the quantum electrodynamic measurement process which is independent of thermodynamic or cosmological assumptions. This occurred because the Measurement Color operator symmetry within MC-QED caused the photon operator to have a negative parity under Wigner time reversal. Then the requirement of a stable vacuum state generated a spontaneous CPT symmetry breaking effect which dynamically generated a quantum electrodynamic arrow of time in the Heisenberg operator equations of motion. This result differs from the case of QED which does not contain an intrinsic arrow of time since its photon operator has a positive parity under Wigner time reversal.

In this context we present analytical arguments which show that a well-known classic nonlinear optics experiment in the scientific literature, which uses combinations of ordinary mirrors and phase conjugate mirrors in a Michelson interferometer, has produced experimental results which represents strong evidence in favor of the MC-QED prediction that the photon has a negative parity under Wigner time reversal and that "the photon carries the quantum electrodynamic arrow of time".

SECTION II: EXPERIMENT TO TEST FOR THE TIME PARITY OF THE PHOTON

In order to verify the correctness of the microscopic operator observer-participant paradigm underlying the structure of the MC-QED formalism it is necessary to demonstrate that experiments can be performed which can provide a test for its underlying prediction that the photon has a negative parity under Wigner time reversal.

The purpose of this paper is to demonstrate that the results of such an experimental test involving nonlinear optics in Michelson interferometers already exists in the literature and supports the predictions of the MC-QED formalism.

In order to begin our analysis we will consider an experimental arrangement involving a Michelson interferometer in which a coherent optical laser beam sent thru a beam splitter to creates interference fringes due to multiple reflections in the vertical and horizontal arms of the apparatus.

Two different experimental scenarios are considered and their results are compared. The first scenario (M-M) is a Michelson interferometer involving the combination of two conventional mirrors while the second scenario (PCM-M) is a Michelson interferometer involving the combination of a Phase Conjugate mirror PCM and a conventional mirror M.

The schematic drawing of this experimental setup (Wolf, Mandel et, al 1987; Jacobs, et. al. 1987; Boyd, et. al. 1987) shown in figure 1 below is one in which one has the option of replacing one of the two conventional mirrors with a phase-conjugate mirror PCM.

A laser sends a coherent optical beam thru the interferometer and creates multiple interference fringes. Selective changes in the phase α of the internal beams can be generated by the use of a gas cell located in positions A, B, or C. The interference fringes can be recorded by the photo-detector, for both the M-M and the M-PCM configurations, and the results compared to the predictions of the QED and the MC-QED formalisms.

If the location of interference fringes recorded by the photo-detector are observed for the case A, where the phase shift α introduced by a gas cell located at position A in the figure induces a change the phase α of the incident waves on the PC mirror, the results which are obtained can experimentally distinguish between the predictions of the QED and the MC-QED formalisms.

In order to understand how this experiment has the potential to be able to distinguish between the QED and the MC-QED formalisms we will now discuss the underlying theoretical and experimental structure of it in more detail. In a series of three seminal papers (Wolf, Mandel et, al 1987; Jacobs, et. al. 1987; Boyd, et. al. 1987) published a detailed theoretical analysis, later supported by experimental observations, which demonstrated how experiments of this type could distinguish between the classical nature of the interference patterns produced by Michelson interferometers the M-M and the M-PCM configurations. More modern reviews of this type of experiment (Garuccio, 2007) have discussed the additional possibility of using it to test for quantum non-locality and anti-coherence effects in light.

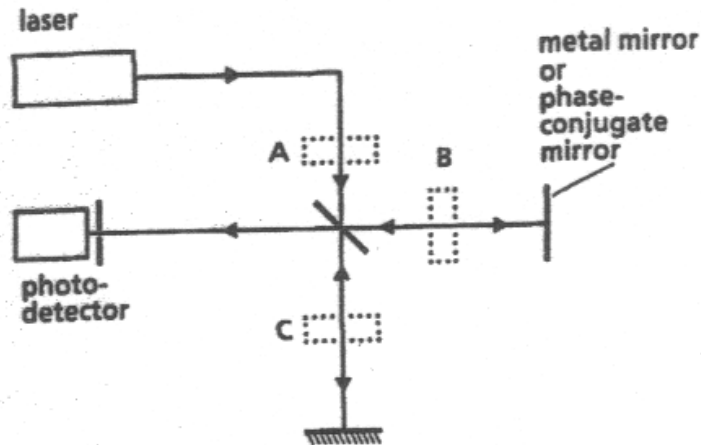


Figure 1. A modified version of a Michelson interferometer (Wolf, Mandel et, al 1987; Jacobs, et. al. 1987; Boyd, et. al. 1987) in which one has the option of replacing one of the two conventional mirrors with a phase-conjugate mirror PCM. A laser sends a coherent optical beam thru the interferometer and creates multiple interference fringes. Selective changes in the phase α of the internal beams can be generated by the use of a gas cell located in positions A, B, or C. The interference fringes are recorded by the photo-detector, for both the M-M and the M-PCM configurations, and the results compared to the predictions of the QED and the MC-QED formalisms.

In particular for the Michelson interferometer in the M-M configuration, which involved normally incident linearly polarized light with complex amplitude $|A|e^{i\alpha}$, it was shown that the locations of the bright maxima and dark minima of the time-averaged interference fringes were given respectively by:

$$\begin{aligned} \text{Bright maxima of interference fringes} \\ z = n(\lambda / 2) \qquad \qquad \qquad (n = 0,1,2,\dots) \end{aligned}$$

$$\begin{aligned} \text{Dark minimum of interference fringes} \\ z = (n + 1/2)(\lambda / 2) \qquad \qquad \qquad (n = 0,1,2,\dots) \end{aligned}$$

On the other hand for the Michelson interferometer in the M-PCM configuration, which involved normally incident linearly polarized light with complex amplitude $|A|e^{i\alpha}$ and where the complex amplitude reflectivity of the phase-conjugate mirror was assumed to be given by $\mu = |\mu|e^{i\phi}$ (here $|\mu| = 1$ and the phase shift ϕ was given in radians) it was shown that the locations of the bright maxima and dark minima of the time-averaged interference fringes were given respectively by:

$$\begin{aligned} \text{Bright maxima of interference fringes} \\ z = n(\lambda / 2) + (\lambda / 2) [(\phi / 2 - \alpha) / \pi] \qquad \qquad \qquad (n = 0,1,2,\dots) \end{aligned}$$

$$\begin{aligned} \text{Dark minimum of interference fringes} \\ z = (n + 1/2)(\lambda / 2) + (\lambda / 2) [(\phi / 2 - \alpha) / \pi] \qquad \qquad \qquad (n = 0,1,2,\dots) \end{aligned}$$

Hence in the context of (Wolf, Mandel et, al 1987; Jacobs, et. al. 1987; Boyd, et. al. 1987) it was demonstrated and later shown experimentally that the difference between the location of the maximum and minimum of the interference fringes for Michelson interferometers in the M-PCM and the M-M configurations was associated with a displacement of the interference pattern in radians given by

$$\Delta(\phi, \alpha) = [(\phi / 2) - \alpha] \text{ radians}$$

where $e^{i\phi}$ was the internal phase shift generated by the phase-conjugate mirror and $e^{i\alpha}$ was the phase of incident linearly polarized light

Next we point out that it can be shown (see Appendix I and II) that in MC-QED the connection between the coherent photon state vectors $|a(\alpha), \lambda\rangle$, and the corresponding coherent classical electromagnetic photon fields which they represent, is given by the expectation value of its negative Wigner time parity photon operator over the coherent states $|a(\alpha), \lambda\rangle$ in the formalism. Hence in the context of MC-QED a coherent photon state associated with propagation vector \mathbf{k} and phase α is predicted to transform with a negative time parity into a coherent photon state associated with propagation vector $-\mathbf{k}$ and phase $-(\alpha + \pi)$ under Wigner time reversal. This is different from the case of QED, where a coherent photon state associated with propagation vector \mathbf{k} and phase α is predicted to transform with a positive time parity into a coherent photon state associated with propagation vector $-\mathbf{k}$ and phase $-\alpha$ under Wigner time reversal.

In the context of this difference in the Wigner time reversal properties of coherent photon states in QED and MC-QED it is important to note that it has been shown (Chew, Habashi 1985) that the “healing effect” associated with removal of intermediate distortions of optical images generated by a phase conjugate mirror is physically equivalent, within a constant phase factor $\exp(i\phi)$ of modulus one, to the effects of time reversal. On this basis we conclude that both QED and MC-QED will predict the same phase conjugate mirror “healing effect” on the distortion of optical images since for QED the constant phase factor is $\phi = 0$ while for MC-QED the constant factor is $\phi = -\pi$.

Hence from the above experimental and theoretical discussion we find that the difference between the location of the maximum and minimum of the interference fringes for Michelson interferometers in the M-PCM and the M-M configurations will associated with a displacement of the interference pattern in radians is given respectively for QED and MC-QED by

$$\Delta(0, \alpha) = -[\alpha] \text{ radians} \quad \text{QED}$$

$$\Delta(-\pi, \alpha) = -[(\pi / 2) + \alpha] \text{ radians} \quad \text{MC-QED}$$

In the limiting case where $\alpha = 0$ this corresponds to a displacement of the interference pattern in radians given by

$$\Delta(0, 0) = 0 \text{ radians} \quad \text{QED}$$

$$\Delta(-\pi, 0) = -(\pi / 2) \text{ radians} \quad \text{MC-QED}$$

The difference between the QED and the MC-QED predictions is a reflection of the fact that the nonlocal photon operator acting within the quantum field theoretic structure of the MC-QED formalism has a negative parity under Wigner time reversal and hence carries the quantum electrodynamic arrow of time in the formalism. For this reason the apparatus discussed in figure 1 allows an experimental test to be performed to determine if the photon carries the arrow of time as predicted by the MC-QED formalism.

SECTION III: DISCUSSION INTERPRETING THE RESULTS OF THE EXPERIMENT

Results taken from the classic experiment performed by (Jacobs, et al. 1987) are shown in figure 2 below. While straight lines have been drawn through the data points for the M-PCM and M-M configurations in figure 2, it appears that nonlinear internal processes in the gas cell introduce oscillation errors into the data which break the predicted linearity for non-zero values of the phase shift. However these nonlinear gas cell oscillation errors will not affect the individual data points at phase shift equal to zero, since for these data points the gas cell is not active in the interferometer. For this reason only the data points at phase shift equal zero, for which the gas cell does not act in the interferometer, will be relevant in the analysis which follows.

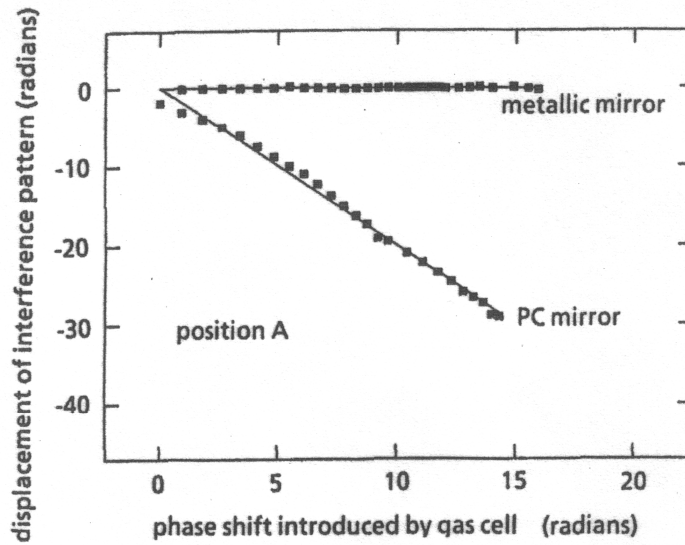


Figure 2. Measured displacement of the interference fringe pattern, for a Michelson interferometer for the M-M (metallic mirror) configuration and the M-PCM (PC mirror) configuration (taken from figure 3 of Jacobs, et al. 1987). The displacement of the interference pattern in radians is produced by interference between the signal and the phase-conjugate waves. It is plotted as a function of the phase shift α introduced by a gas cell whose location (in position A in figure 1 above) induces a change the phase α of the incident waves acting on the PC mirror. Note that the difference between the displacement of the interference pattern for the M-M (metallic mirror) and the M-PCM (PC mirror) configuration for the case of zero phase shift $\alpha = 0$ appears to be consistent with the MC-QED value $\Delta(-\pi, 0) = -(\pi/2) = -1.57$. Hence the results of this experiment appears to offer strong support in favor of MC-QED and its prediction that the photon carries the arrow of time.

In the graph the displacement of the interference pattern in radians, produced by interference between the signal and the phase-conjugate waves, is plotted as a function of the phase shift α introduced by a gas cell whose location, at position A as shown in figure 1, allows it to induce a change the phase α of the incident waves on the PC mirror.

Note that the difference between the displacement of the interference pattern for the M-M (metallic mirror) and the M-PCM (PC mirror) configuration for the case of zero phase shift $\alpha = 0$ appears to be consistent with the predicted MC-QED value of $\Delta(-\pi, 0) = -(\pi/2) = -1.57$ radians and not with the predicted QED value of $\Delta(0, 0) = 0$ radians.

On this basis of these results, this experiment appears to offer strong support in favor of MC-QED and its prediction that the photon carries the arrow of time.

SECTION III: CONCLUSIONS

By incorporating an operator symmetry of microscopic observer-participation called “Measurement Color” into Quantum Electrodynamics (QED) the resultant Measurement Color Quantum Electrodynamics (MC-QED) contains a time reversal violating description of the quantum electrodynamic measurement process which is independent of thermodynamic or cosmological assumptions. This occurred because Measurement Color symmetry within MC-QED caused the photon operator in the formalism to have a negative parity under Wigner time reversal. This created a spontaneous CPT symmetry breaking effect which dynamically determined a causal quantum electrodynamic arrow of time in the formalism (Leiter, 2009, 2010).

On this basis it was shown (see Appendix I and II) that in MC-QED a coherent photon state, with propagation vector \mathbf{k} and phase (α), transformed under Wigner time reversal with negative time parity into a coherent photon state with propagation vector $-\mathbf{k}$ and phase $-(\alpha + \pi)$. This was to be compared to the case of a coherent photon state in QED, with a propagation vector \mathbf{k} and phase (α), which was shown to transform under Wigner time reversal with a positive time parity into a coherent photon state with propagation vector $-\mathbf{k}$ and phase $(-\alpha)$ under Wigner time reversal.

Because of this difference between the Wigner time reversal symmetry of the photon in MC-QED and QED, we demonstrated that a Michelson interferometer experiment involving a combination of ordinary mirrors and phase conjugate mirrors could experimentally determine if the photon operator has a negative parity under Wigner time reversal and in this way test the MC-QED prediction that “the photon carries the arrow of time”.

The experiment shown schematically in figure 1 involved measuring and comparing the displacement of the interference fringe pattern, for a Michelson interferometer for the M-M (metallic mirror) configuration and the M-PCM (PC mirror) configuration. In the context of this experiment it was shown that the displacement in radians $\Delta(\phi, \alpha)$ of the interference pattern in produced by interference between the signal and the phase-conjugate waves was

given by $\Delta(\phi, \alpha) = [(\phi / 2) - \alpha]$, where ϕ was the internal phase shift generated by the phase-conjugate mirror and α was the phase of incident linearly polarized light.

In the limiting case of zero phase shift ($\alpha = 0$) the difference between the displacement of the interference pattern for the M-M (metallic mirror) configuration and the M-PCM (PC mirror) configuration predicted by MC-QED has a value of $\Delta(-\pi, 0) = -\pi / 2$ radians which is distinctly different from the value of $\Delta(0, 0) = 0$ radians predicted by QED.

Because of the difference between the Wigner time reversal symmetry properties of the photon operator in QED and MC-QED we have shown that well-know classic Michelson interferometer experiment discussed in the literature, (Wolf, Mandel et, al 1987; Jacobs, et. al. 1987; Boyd, et. al. 1987) which involves combinations of ordinary mirrors and phase conjugate mirrors, appears to have experimentally demonstrated that the photon operator has a negative parity under Wigner time reversal and hence the MC-QED prediction that “the photon carries the quantum electrodynamic arrow of time”. It is hoped that this paper will encourage experimental physicists to perform more modern versions of the Michelson interferometer experiment described in this paper, in order to further verify this experimental result within the context of the high accuracy of twenty-first century technology.

APPENDIX I: PHOTON BARE STATE STRUCTURE IN THE MC-QED FORMALISM

It has been shown (Leiter, D. 2009, 2010, <http://journalofcosmology.com/Contents.html>) that the Measurement Color symmetric charge field photon Hamiltonian operator in the MC-QED formalism is given by

$$H_{ph} = \sum_{(j)} \{ : \int dx^3 [-1/2 (\partial_t \mathbf{A}^{\mu(j)}_{(rad)}) \partial_t \mathbf{A}_{\mu}^{(j)}_{(rad)}{}^{(obs)} + \nabla \mathbf{A}^{\mu(j)}_{(rad)} \cdot \nabla \mathbf{A}_{\mu}^{(j)}_{(rad)}{}^{(obs)}] : \}$$

where the symbols $: \cdot :$ denote operator normal ordering, $(j=1,2,\dots,N \rightarrow \infty)$, and

$$\begin{aligned} \mathbf{A}_{\mu}^{(j)}_{(rad)}(x) &= (\alpha_{\mu}(x) - \mathbf{A}_{\mu}^{(j)}_{(-)}(x)) \\ \mathbf{A}_{\mu}^{(j)}_{(rad)}{}^{(obs)}(x) &= \sum_{(k) \neq (j)} \mathbf{A}_{\mu}^{(k)}_{(rad)}(x) = \mathbf{A}_{\mu}^{(j)}_{(-)}(x) \\ \alpha_{\mu}(x) &= \sum_{(j)} \mathbf{A}_{\mu}^{(j)}_{(-)}(x) / (N-1) \end{aligned}$$

are linear functions of the negative time parity operator $\mathbf{A}_{\mu}^{(j)}_{(-)}(x) = \int dx'^4 D_{(-)}(x-x') J_{\mu}^{(j)}(x')$ where $\mathbf{A}_{\mu}^{(j)}_{(-)}(x) = (\mathbf{A}_{\mu}^{(j)}_{(-)}(x))^{\dagger}$ which implies that $\square^2 \mathbf{A}_{\mu}^{(j)}_{(-)}(x) = 0$ which from the above also implies that $\square^2 \mathbf{A}_{\mu}^{(j)}_{(rad)}{}^{(obs)} = \square^2 \mathbf{A}_{\mu}^{(j)}_{(rad)} = \square^2 \alpha_{\mu}(x) = 0$.

The negative time parity operators $\mathbf{A}_{\mu}^{(j)}_{(rad)}$, $\mathbf{A}_{\mu}^{(j)}_{(rad)}{}^{(obs)}$ and α_{μ} ($j=1,2, \dots, N \rightarrow \infty$) can be respectively expanded as

$$\begin{aligned} \mathbf{A}_{\mu}^{(j)}(x) &= (\alpha_{\mu}(x) - \mathbf{A}_{\mu}^{(j)}_{(-)}(x)) = \int dk^3 / \sqrt{[2(2\pi)^3 k^2]} \{ \mathbf{a}_{\mu}^{(j)}(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{x}} + \mathbf{a}_{\mu}^{(j)}(\mathbf{k})^{\dagger} e^{i\mathbf{k}\cdot\mathbf{x}} \} \\ \mathbf{A}_{\mu}^{(j)}_{(rad)}{}^{(obs)}(x) &= \mathbf{A}_{\mu}^{(j)}_{(-)}(x) = \int dk^3 / \sqrt{[2(2\pi)^3 k^2]} \{ \mathbf{a}_{\mu}^{(j)}_{(-)}(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{x}} + \mathbf{a}_{\mu}^{(j)}_{(-)}(\mathbf{k})^{\dagger} e^{i\mathbf{k}\cdot\mathbf{x}} \} \\ \alpha_{\mu}(x) &= \sum_{(j)} \mathbf{A}_{\mu}^{(j)}_{(-)}(x) / (N-1) = \int dk^3 / \sqrt{[2(2\pi)^3 k^2]} \{ \alpha_{\mu}(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{x}} + \alpha_{\mu}(\mathbf{k})^{\dagger} e^{i\mathbf{k}\cdot\mathbf{x}} \} \end{aligned}$$

where $\mathbf{k}\cdot\mathbf{x} = k_{\nu} x^{\nu} = \mathbf{k}\cdot\mathbf{x} + k_0 x^0$ and $\mathbf{k} = \mathbf{n} / \lambda$, $k_0 = \nu / c$

From the above we see that

$$\begin{aligned} \mathbf{a}_{\mu}^{(j)}_{(-)}(\mathbf{k}) &= \mathbf{a}_{\mu}^{(j)}_{(rad)}{}^{(obs)}(\mathbf{k}), \quad \alpha_{\mu}(\mathbf{k}) = \sum_{(j)} \mathbf{a}_{\mu}^{(j)}_{(-)}(\mathbf{k}) / (N-1) = \sum_{(j)} \mathbf{a}_{\mu}^{(j)}(\mathbf{k}) \\ \mathbf{a}_{\mu}^{(j)}(\mathbf{k}) &= \alpha_{\mu}(\mathbf{k}) - \mathbf{a}_{\mu}^{(j)}_{(-)}(\mathbf{k}) = \sum_{(j)} \mathbf{a}_{\mu}^{(j)}_{(-)}(\mathbf{k}) / (N-1) - \mathbf{a}_{\mu}^{(j)}_{(-)}(\mathbf{k}) \end{aligned}$$

We now show that the time reversal violating Measurement Color symmetric operators $\alpha_\mu(\mathbf{k})$ and $\alpha_\mu(\mathbf{k})^\dagger$ act respectively as destruction and creation operators for Measurement Color symmetric charge field photon states in MC-QED.

We begin by substituting the above representations of $A_\mu^{(j)}(x)$, $A_\mu^{(j)}(\text{obs})(x)$, and $\alpha_\mu(x)$ into the above MC-QED commutation relations to find ($j, m = 1, 2, \dots, N \rightarrow \infty$)

$$\begin{aligned} [a_{\mu}^{(j)}(-)(\mathbf{k}), a_v^{(m)}(-)^\dagger(\mathbf{k}')] &= (1 - \delta^{jm}) (-\eta_{\mu\nu} \lambda_0 \delta^3(\mathbf{k} - \mathbf{k}')) \\ [\alpha_\mu(\mathbf{k}), a_v^{(j)}(-)^\dagger(\mathbf{k}')] &= -\eta_{\mu\nu} \lambda_0 \delta^3(\mathbf{k} - \mathbf{k}') \\ [\alpha_\mu(\mathbf{k}), \alpha_\nu(\mathbf{k}')] &= (-\eta_{\mu\nu} \lambda_0 \delta^3(\mathbf{k} - \mathbf{k}'))(N / (N-1)) \\ [\alpha_\mu(\mathbf{k}), a_v^{(j)}(\mathbf{k}')^\dagger] &= 0 \\ [a_{\mu}^{(j)}(-)^\dagger(\mathbf{k}), a_v^{(m)}(-)^\dagger(\mathbf{k}')] &= 0 \\ [a_{\mu}^{(j)}(-)(\mathbf{k}), a_v^{(m)}(-)(\mathbf{k}')] &= 0 \end{aligned}$$

where $k_0 = \sqrt{(\mathbf{k}^2)} = \omega(\mathbf{k})$ and all other commutators vanish.

Next we substitute above representations of $A_\mu^{(j)}(x)$ and $A_\mu^{(j)}(\text{obs})(x)$ into the charge field photon hamiltonian H_{ph} which gives the charge field photon Hamiltonian as\

$$H_{ph} = \sum_{(j)} \{ : [- \int dk^3 / k_0 (\omega(\mathbf{k}) a_{\mu}^{(j)}(-)^\dagger(\mathbf{k}) a^{\mu(j)}(-)(\mathbf{k}))] : \}$$

and normal ordering of operators inside of the symbols $\{ : : \}$ has been taken.

In addition by inserting

$$a_{\mu}^{(j)}(\mathbf{k}) = \alpha_\mu(\mathbf{k}) - a_{\mu}^{(j)}(-)(\mathbf{k}) \quad \text{and} \quad \alpha_\mu(\mathbf{k}) = \sum_{(j)} a_{\mu}^{(j)}(-)(\mathbf{k}) / (N-1) = \sum_{(j)} a_{\mu}^{(j)}(\mathbf{k})$$

into H_{ph} the hermetian property of the photon hamiltonian $H_{ph} = H_{ph}^\dagger$ follows directly.

In this context if the bare MC-QED charge field photon vacuum state $|0_{ph}\rangle$ is defined by

$$a_{\mu}^{(j)}(-)(\mathbf{k}) |0_{ph}\rangle = 0 \quad (j = 1, 2, \dots, N \rightarrow \infty)$$

this implies that $H_{ph} |0_{ph}\rangle = 0$ as required.

Now since $\alpha_\mu(\mathbf{k}) = \sum_{(j)} \mathbf{a}_\mu^{(j)}(-)(\mathbf{k}) / (N-1)$ the above definition of $|0_{ph}\rangle$ also implies that the bare charge field photon vacuum state also obeys

$$\alpha_\mu(\mathbf{k}) |0_{ph}\rangle = 0$$

In this context the bare single charge-field photon in MC-QED can be defined as

$$|\lambda \mathbf{k}_1\rangle = \alpha_\mu(\mathbf{k}_1)^\dagger |0\rangle = (1 / (N-1)) \sum_{(j)} \mathbf{a}_\mu^{(j)}(-)^\dagger(\mathbf{k}_1)^{(j)} |0\rangle$$

This can be seen by calculating

$$H_{ph} |\mathbf{k}_1\rangle = - \sum_{(j)} \int d\mathbf{k}^3 / k_0 (\omega(\mathbf{k}) \mathbf{a}_v^{(j)}(\mathbf{k})^\dagger \mathbf{a}^{v(j)}(-)(\mathbf{k}) \alpha_\mu(\mathbf{k}_1)^\dagger) |0\rangle$$

Then using the fact that

$$[\alpha_\mu(\mathbf{k}), \mathbf{a}^{v(j)}(-)^\dagger(\mathbf{k}')] = -\delta_{\mu v} k_0 \delta^3(\mathbf{k} - \mathbf{k}') \quad \text{and} \quad \alpha_\mu(\mathbf{k}_1) = \sum_{(j)} \mathbf{a}_\mu^{(j)}(\mathbf{k}_1)$$

we have

$$\begin{aligned} H_{ph} |\mathbf{k}_1\rangle &= \sum_{(j)} \int d\mathbf{k}^3 (\omega(\lambda) \mathbf{a}_v^{(j)}(\lambda)^\dagger (-\delta_\mu^v) \delta^3(\lambda - \lambda_1) |0\rangle \\ &= \omega(\mathbf{k}_1) \sum_{(j)} \mathbf{a}_\mu^{(j)\dagger}(\mathbf{k}_1) |0\rangle \\ &= \omega(\mathbf{k}_1) (\alpha_\mu(\mathbf{k}_1)^\dagger |0\rangle = \omega(\mathbf{k}_1) |\mathbf{k}_1\rangle \quad \text{as required} \end{aligned}$$

Hence the N-bare charge-field photon states in MC-QED are defined as

$$|\mathbf{k}_{\alpha 1}, \mathbf{k}_{2\beta}, \mathbf{k}_{3\gamma}, \dots\rangle = (1/N!)^{1/2} \alpha_\alpha(\mathbf{k}_1)^\dagger \alpha_\beta(\mathbf{k}_2)^\dagger \alpha_\gamma(\mathbf{k}_3)^\dagger \dots |0\rangle$$

In a similar manner as that of the covariant form of QED, consistency with the expectation value of the operator form of Maxwell equations in the covariant form of MC-QED requires that an Indefinite Metric Hilbert space must be used.

In the context of an Indefinite Metric Hilbert space, the subset of physical bare charge field photon states in MC-QED contained within the above set of multiple charge field photon eigenstates of H_{ph} are required to obey the Weak Subsidiary Condition $\lambda^\mu \mathbf{a}_\mu^{(j)}(\mathbf{k}) |\Psi\rangle = 0$ where

$$\mathbf{a}_\mu^{(j)}(\mathbf{k}) = \alpha_\mu(\mathbf{k}) - \mathbf{a}_\mu^{(j)}(-)(\mathbf{k}) = \sum_{(j)} \mathbf{a}_\mu^{(j)}(-)(\mathbf{k}) / (N-1) - \mathbf{a}_\mu^{(j)}(-)(\mathbf{k})$$

which requires them to contain equal numbers of timelike and longitudinal charge field photons. Since the Indefinite Metric Hilbert space implies that charge field photon states with an odd number of time-like charge field photons have an additional negative sign associate with their inner product, the combination of the Weak Subsidiary Condition and the Indefinite Metric Hilbert space together imply that the physical bare charge field photon states have a positive semi-definite norm and energy momentum expectation values similar to that of the QED formalism.

APPENDIX II: COHERENT PHOTON STATES IN THE MC-QED FORMALISM

While the fermion current operators $J^{(k)\mu}(\mathbf{x},t)$ in MC-QED transform under Wigner time reversal as $T_w J^{(k)\mu}(\mathbf{x},t)T_w^{-1} = J^{(k)}_{\mu}(\mathbf{x},-t)$ similar to that of QED, the charge-field photon operators

$$A_{\mu}^{(j)}{}_{(rad)}{}^{(obs)}(\mathbf{x}) = \sum_{(m) \neq (j)} A_{\mu}^{(m)}{}_{(rad)}(\mathbf{x}) = A_{\mu}^{(j)}{}_{(-)}(\mathbf{x})$$

$$A_{\mu}^{(j)}{}_{(-)}(\mathbf{x}) = \int dk^3 / \sqrt{[2(2\pi)^3 k^2]} \{ a_{\mu}^{(j)}{}_{(-)}(\mathbf{k})e^{-i\mathbf{k} \cdot \mathbf{x}} + a_{\mu}^{(j)}{}_{(-)}(\mathbf{k})^{\dagger} e^{i\mathbf{k} \cdot \mathbf{x}} \}$$

$$\alpha_{\mu}(\mathbf{x}) = \sum_{(j)} A_{\mu}^{(j)}{}_{(-)}(\mathbf{x}) / (N-1) = \int dk^3 / \sqrt{[2(2\pi)^3 k^2]} \{ \alpha_{\mu}(\mathbf{k})e^{-i\mathbf{k} \cdot \mathbf{x}} + \alpha_{\mu}(\mathbf{k})^{\dagger} e^{i\mathbf{k} \cdot \mathbf{x}} \}$$

where $\mathbf{k} \cdot \mathbf{x} = k_{\nu} x^{\nu} = \mathbf{k} \cdot \mathbf{x} + k_0 x^0$ and $\mathbf{k} = \mathbf{n} / \lambda$, $k_0 = \nu / c$, have a negative parity under Wigner Reversal operator T_w as

$$T_w A_{\mu}^{(j)}{}_{(-)}(\mathbf{x},t)T_w^{-1} = -A^{\mu(j)}{}_{(-)}(\mathbf{x},-t)$$

$$T_w \alpha_{\mu}(\mathbf{x},t)T_w^{-1} = -\alpha^{\mu}(\mathbf{x},-t)$$

Hence this implies that under Wigner Time reversal T_w the charge-field photon creation and annihilation operators in MC-QED also transform with a negative time parity respectively as

$$\begin{aligned} T_w a_{\mu}^{(j)}{}_{(-)}(\mathbf{k})^{\dagger} T_w^{-1} &= -a^{\mu(j)}{}_{(-)}(-\mathbf{k})^{\dagger} & T_w a_{\mu}^{(j)}{}_{(-)}(\mathbf{k}) T_w^{-1} &= -a^{\mu(j)}{}_{(-)}(-\mathbf{k}) \\ T_w \alpha^{\mu}(\mathbf{k})^{\dagger} T_w^{-1} &= -\alpha^{\mu}(-\mathbf{k})^{\dagger} & T_w \alpha^{\mu}(\mathbf{k}) T_w^{-1} &= -\alpha^{\mu}(-\mathbf{k}) \end{aligned}$$

Now in MC-QED a coherent photon state $|a(\alpha), \mathbf{k}\rangle$ of frequency $\omega(\mathbf{k}) = \sqrt{(\mathbf{k}^2)} = k_0$ and complex phase $a(\alpha)$, is an eigenstate of the charge-field photon destruction operator $\alpha^{\mu}(\mathbf{k}) = \varepsilon^{\mu} \alpha(\mathbf{k})$ as $\alpha^{\mu}(\mathbf{k}) |a(\alpha), \mathbf{k}\rangle = \varepsilon^{\mu} a(\alpha) |a(\alpha), \mathbf{k}\rangle$ which is satisfied if

$$\alpha(\mathbf{k}) |a(\alpha), \mathbf{k}\rangle = a(\alpha) |a(\alpha), \mathbf{k}\rangle$$

Solving for a coherent photon state $|a(\alpha), \mathbf{k}\rangle$ with a mean photon number $\langle N \rangle$ yields

$$|a(\alpha), \mathbf{k}\rangle = \exp(-\langle N \rangle / 2) \exp[a(\alpha) \alpha_{\mu}(\mathbf{k})^{\dagger}] |0\rangle$$

where $a(\alpha) = (\langle N \rangle)^{1/2} \exp(i\alpha)$. In terms of the n-photon state of frequency $\omega(\mathbf{k})$ is given by

$$|n, \lambda\rangle = (1/(n!))^{1/2} \alpha^{\mu}(\mathbf{k})^{\dagger} \alpha^{\nu}(\mathbf{k})^{\dagger} \alpha^{\eta}(\mathbf{k})^{\dagger} \dots |0\rangle$$

the above expression for $|a(\alpha), \mathbf{k}\rangle$ can be written in the more explicit form

$$|a(\alpha), \mathbf{k}\rangle = \sum_{\mathbf{n}=0,1,\dots,\infty} [\exp(-\langle N \rangle / 2) (\langle N \rangle^{\mathbf{n}} / \mathbf{n}!)^{1/2} \exp(i\mathbf{n}\alpha) |\mathbf{n}, \mathbf{k}\rangle]$$

By calculating $\langle a(\alpha), \mathbf{k} | a(\alpha), \mathbf{k} \rangle$ we see that the distribution of photons in the coherent state obeys a Poisson statistical distribution $\prod_{\mathbf{n}} (\langle N \rangle) = [\exp(-\langle N \rangle) (\langle N \rangle^{\mathbf{n}} / \mathbf{n}!)]$ since

$$\langle a(\alpha), \mathbf{k} | a(\alpha), \mathbf{k} \rangle = \sum_{\mathbf{n}=0,1,\dots,\infty} [\exp(-\langle N \rangle) (\langle N \rangle^{\mathbf{n}} / \mathbf{n}!)] = \sum_{\mathbf{n}=0,1,\dots,\infty} \prod_{\mathbf{n}} (\langle N \rangle)$$

Since $\alpha^{\mu}(\mathbf{k}) = \varepsilon^{\mu} \alpha(\mathbf{k})$ and the bare charge field photon creation operators $\alpha(\mathbf{k})^{\dagger}$ have a negative parity under Wigner Time reversal T_w given by $T_w \varepsilon^{\mu} \alpha(\mathbf{k})^{\dagger} T_w^{-1} = -\varepsilon_{\mu} \alpha(-\mathbf{k})^{\dagger}$, we find that Wigner time reversal T_w acting on the coherent photon state $|a(\alpha), \mathbf{k}\rangle$ gives

$$\begin{aligned} |a(\alpha), \mathbf{k}\rangle_{T_w} &= T_w |a(\alpha), \mathbf{k}\rangle = \sum_{\mathbf{n}=0,1,\dots,\infty} [\exp(-\langle N \rangle / 2) (\langle N \rangle^{\mathbf{n}} / \mathbf{n}!)^{1/2} \exp(-i\mathbf{n}\alpha) T_w |\mathbf{n}, \mathbf{k}\rangle] \\ &= \sum_{\mathbf{n}=0,1,\dots,\infty} [\exp(-\langle N \rangle / 2) (\langle N \rangle^{\mathbf{n}} / \mathbf{n}!)^{1/2} \exp(-i\mathbf{n}\alpha) (-1)^{\mathbf{n}} |\mathbf{n}, -\mathbf{k}\rangle] \\ &= \sum_{\mathbf{n}=0,1,\dots,\infty} [\exp(-\langle N \rangle / 2) (\langle N \rangle^{\mathbf{n}} / \mathbf{n}!)^{1/2} \exp(-i\mathbf{n}(\alpha + \pi)) |\mathbf{n}, -\mathbf{k}\rangle] = |a(-[\alpha + \pi]), -\mathbf{k}\rangle \end{aligned}$$

Hence the negative time parity of the Wigner time reversed coherent photon state in MC-QED implies an observable difference between time reversed phase-conjugate coherent states in QED and MC-QED.

This is because for MC-QED

$$|a(\alpha), \mathbf{k}\rangle_{T_w} = T_w |a(\alpha), \mathbf{k}\rangle = |a(-[\alpha + \pi]), -\mathbf{k}\rangle$$

while for QED

$$|a(\alpha), \mathbf{k}\rangle_{T_w} = T_w |a(\alpha), \mathbf{k}\rangle = |a(-\alpha), -\mathbf{k}\rangle$$

Hence in QED a coherent photon state associated with wave vector \mathbf{k} and phase α is predicted to transform into a coherent photon state associated with wave vector $-\mathbf{k}$ and phase $-(\alpha)$ under Wigner time reversal, while in MC-QED a coherent photon states associated with wave vector \mathbf{k} and phase α is predicted to transform into a coherent photon state associated with wave vector $-\mathbf{k}$ and phase $-(\alpha + \pi)$ under Wigner time reversal.

Nonetheless the distribution of photons in the Wigner time reversed coherent state for both QED and MC-QED obey a Poisson statistical distribution since by direct calculations we find that for both theories

$${}_{T_w} \langle a(\alpha), \mathbf{k} | a(\alpha), \mathbf{k} \rangle_{T_w} = \langle a(\alpha), \mathbf{k} | a(\alpha), \mathbf{k} \rangle = \sum_{\mathbf{n}=0,1,\dots,\infty} \prod_{\mathbf{n}} (\langle N \rangle)$$

Since reflection from phase conjugate mirrors physically creates the effects of time reversal on coherent optical beam of photons, this difference between QED and MC-QED should be observable in the context of optical interferometer experiments involving combinations of ordinary mirrors and phase conjugate mirrors.

Even though MC-QED is a non-local quantum field theory, the formal similarity between the quantum field theoretic structure of MC-QED and QED implies that the connection between the coherent photon state vectors $|a(\varphi), \mathbf{k}\rangle$ and the corresponding coherent classical electromagnetic photon fields which they represent is given by the expectation value over the coherent state $|a(\alpha), \mathbf{k}\rangle$ of the observed radiation charge-field $A_{\mu}^{(k)}(\text{rad})^{(\text{obs})}(x)$ as

$$\langle a(\alpha), \mathbf{k} | A_{\mu}^{(k)}(\text{rad})^{(\text{obs})}(x) | a(\alpha), \mathbf{k} \rangle$$

where in the above we have

$$|a(\alpha), \mathbf{k}\rangle = \sum_{n=0,1,\dots,\infty} [\exp(-\langle N \rangle / 2) (\langle N \rangle^n / n!)^{1/2} \exp(i n \alpha) |n, \mathbf{k}\rangle]$$

$$|n, \mathbf{k}\rangle = (1/n!)^{1/2} \alpha^{\mu}(\mathbf{k})^{\dagger} \alpha^{\nu}(\mathbf{k})^{\dagger} \alpha^{\eta}(\mathbf{k})^{\dagger} \dots |0\rangle$$

$$A_{\mu}^{(j)}(\text{rad})^{(\text{obs})}(x) = A_{\mu}^{(j)}(-)(x) = \int d^3j / \sqrt{[2(2\pi)^3 \mathbf{k}^2]} \{ a_{\mu}^{(j)}(-)(\mathbf{k}) e^{-i \mathbf{k} \cdot \mathbf{x}} + a_{\mu}^{(j)}(-)(\mathbf{k})^{\dagger} e^{i \mathbf{k} \cdot \mathbf{x}} \}$$

and $\mathbf{k} \cdot \mathbf{x} = k_{\nu} x^{\nu} = \mathbf{k} \cdot \mathbf{x} + k_0 x^0 = (\underline{\mathbf{k}} \cdot \underline{\mathbf{x}} - vt)$, $\mathbf{k} = \mathbf{n} / \lambda$, $k_0 = v / c$

Now since the MC-QED commutation relations imply that

$$[a_{\nu}^{(j)}(-)(\mathbf{k}'), \alpha_{\mu}(\mathbf{k})^{\dagger}] = [\alpha_{\mu}(\mathbf{k}), \alpha_{\nu}(\mathbf{k}')^{\dagger}] = -\eta_{\mu\nu} \lambda_0 \delta^3(\mathbf{k} - \mathbf{k}')$$

then the action of the $a_{\nu}^{(j)}(-)(\mathbf{k}')$ operator on $|n, \mathbf{k}\rangle$ produces the same effect as the action of the $\alpha_{\mu}(\mathbf{k})$ operator on $|n, \mathbf{k}\rangle$.

Hence $\alpha_{\nu}(\mathbf{k}) | a(\alpha), \mathbf{k}\rangle = a(\alpha) | a(\alpha), \mathbf{k}\rangle$ implies that $a_{\nu}^{(j)}(-)(\mathbf{k}) | a(\alpha), \mathbf{k}\rangle = a(\alpha) | a(\alpha), \mathbf{k}\rangle$ where $a(\alpha) = (\langle N \rangle)^{1/2} \exp(i\alpha)$. From this we see that

$$\begin{aligned} \langle a(\alpha), \mathbf{k} | a_{\mu}^{(j)}(-)(\mathbf{j}) e^{-i \lambda \cdot \mathbf{x}} | a(\alpha), \mathbf{k} \rangle &= e^{-i \mathbf{k} \cdot \mathbf{x}} \langle a(\alpha), \mathbf{k} | a_{\mu}^{(j)}(-)(\lambda) | a(\alpha), \mathbf{k} \rangle \\ &= e^{-i \mathbf{k} \cdot \mathbf{x}} a(\alpha) \langle a(\alpha), \mathbf{k} | a(\alpha), \mathbf{k} \rangle \\ &= C(\langle N \rangle) e^{-i(\mathbf{k} \cdot \mathbf{x} - \alpha)} \end{aligned}$$

where

$$C(\langle N \rangle) = (\langle N \rangle)^{1/2} \sum_{n=0,1,\dots,\infty} [\exp(-\langle N \rangle) (\langle N \rangle^n / n)]$$

Taking the hermetian conjugate of the above equations we also see that

$$\langle a(\alpha), \mathbf{k} | a_{\mu}^{(j)}(-\mathbf{k})^\dagger e^{i\mathbf{k}\cdot\mathbf{x}} | a(\alpha), \mathbf{k} \rangle = C(\langle N \rangle) e^{i(\lambda \cdot \mathbf{x} - \alpha)}$$

Hence for a coherent photon state $| a(\alpha), \mathbf{k} \rangle$ with a mean photon number $\langle N \rangle$

$$\langle a(\alpha), \mathbf{k} | A_{\mu}^{(j)}(\text{rad})^{(\text{obs})}(\mathbf{x}) | a(\alpha), \mathbf{k} \rangle = C(\langle N \rangle) \int d\mathbf{k}^3 / \sqrt{[2(2\pi)^3 \mathbf{k}^2]} \{ e^{-i(\mathbf{k}\cdot\mathbf{x} - \alpha)} + e^{i(\mathbf{k}\cdot\mathbf{x} - \alpha)} \}$$

where $C(\langle N \rangle) = (\langle N \rangle)^{1/2} \sum_{\mathbf{n}=0,1,\dots,\infty} [\exp(-\langle N \rangle) (\langle N \rangle)^{\mathbf{n}} / \mathbf{n}]$

Recalling for a Wigner Time reversed coherent photon state that

$$| a(\alpha), \mathbf{k} \rangle_{T_w} = T_w | a(\alpha), \mathbf{k} \rangle = | a(-[\alpha + \pi]), -\mathbf{k} \rangle$$

then it follows that

$$\begin{aligned} T_w \langle a(\alpha), \mathbf{k} | A_{\mu}^{(j)}(\text{rad})^{(\text{obs})}(\mathbf{x}) | a(\alpha), \mathbf{k} \rangle_{T_w} \\ = \langle a(-[\alpha + \pi]), -\mathbf{k} | A_{\mu}^{(j)}(\text{rad})^{(\text{obs})}(\mathbf{x}) | a(-[\alpha + \pi]), -\mathbf{k} \rangle \\ = C(\langle N \rangle) \int d\mathbf{k}^3 / \sqrt{[2(2\pi)^3 \mathbf{k}^2]} \{ e^{-i(-\mathbf{k}\cdot\mathbf{x} - vt - [\alpha + \pi])} + e^{i(-\mathbf{k}\cdot\mathbf{x} - vt + [\alpha + \pi])} \} \end{aligned}$$

where $C(\langle N \rangle) = (\langle N \rangle)^{1/2} \sum_{\mathbf{n}=0,1,\dots,\infty} [\exp(-\langle N \rangle) (\langle N \rangle)^{\mathbf{n}} / \mathbf{n}]$

Hence in MC-QED under Wigner time reversal we see that

$$\begin{aligned} C(\langle N \rangle) \int d\mathbf{k}^3 / \sqrt{[2(2\pi)^3 \mathbf{k}^2]} \{ e^{-i(\mathbf{k}\cdot\mathbf{x} - \alpha)} \} \\ \text{-----} \rightarrow C(\langle N \rangle) \int d\mathbf{k}^3 / \sqrt{[2(2\pi)^3 \mathbf{k}^2]} \{ e^{-i(-\mathbf{k}\cdot\mathbf{x} - vt + [\alpha + \pi])} \} \end{aligned}$$

While in QED under Wigner time reversal we see that

$$\begin{aligned} C(\langle N \rangle) \int d\mathbf{k}^3 / \sqrt{[2(2\pi)^3 \mathbf{k}^2]} \{ e^{-i(\mathbf{k}\cdot\mathbf{x} - \alpha)} \} \\ \text{-----} \rightarrow C(\langle N \rangle) \int d\mathbf{k}^3 / \sqrt{[2(2\pi)^3 \mathbf{k}^2]} \{ e^{-i(-\mathbf{k}\cdot\mathbf{x} - vt + \alpha)} \} \end{aligned}$$

Hence in a similar manner as that of QED the connection between the coherent photon state vectors $|a(\alpha), \mathbf{k}\rangle$ and the corresponding coherent classical electromagnetic photon fields which they represent is given in MC-QED by the expectation value over the coherent state $|a(\alpha), \mathbf{k}\rangle$ of the observed radiation charge-field $A_{\mu}^{(k)}{}^{(obs)}(x)$. However in contrast to QED, where a coherent photon state associated with propagation vector \mathbf{k} and phase α is predicted to transform into a coherent photon state associated with propagation vector $-\mathbf{k}$ and phase $-\alpha$ under Wigner time reversal, in MC-QED a coherent photon states associated with propagation vector \mathbf{k} and phase α is predicted to transform into a coherent photon state associated with propagation vector $-\mathbf{k}$ and phase $-(\alpha + \pi)$ under Wigner time reversal.

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