On intense vortex structures in isotropic turbulence

A. Leonard
California Institute of Technology
APS DFD 2013

Johns Hopkins web-based data base
(http/turbulence.pha.jhu.edu)
DNS forced homogeneous isotropic turbulence
1024^3 $Re_\lambda = 433$

Intense vortex structures

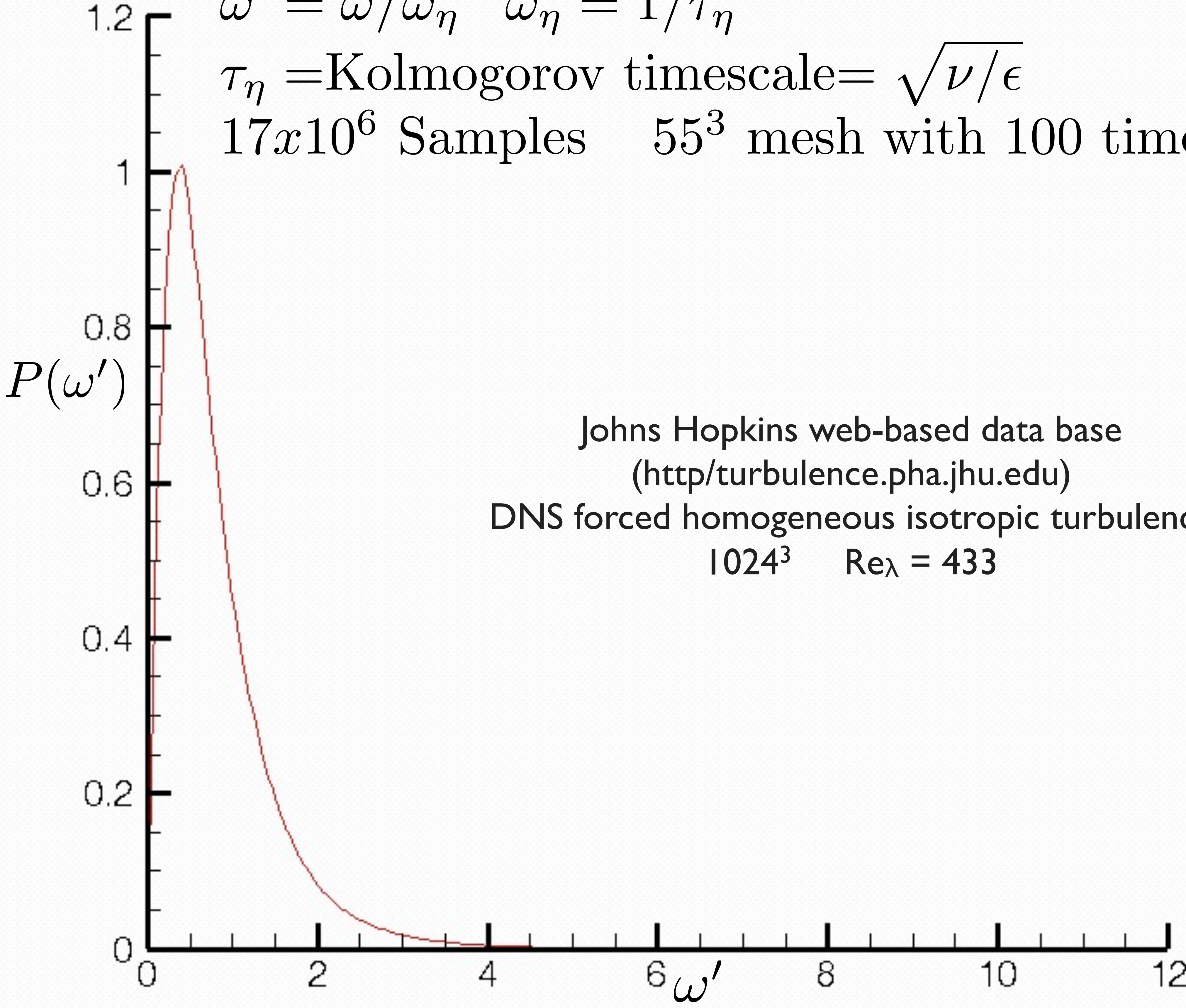
- PDF of vorticity amplitudes - asymptotics
- Geometry of intense structures, vorticity distribution
- Evolution in time

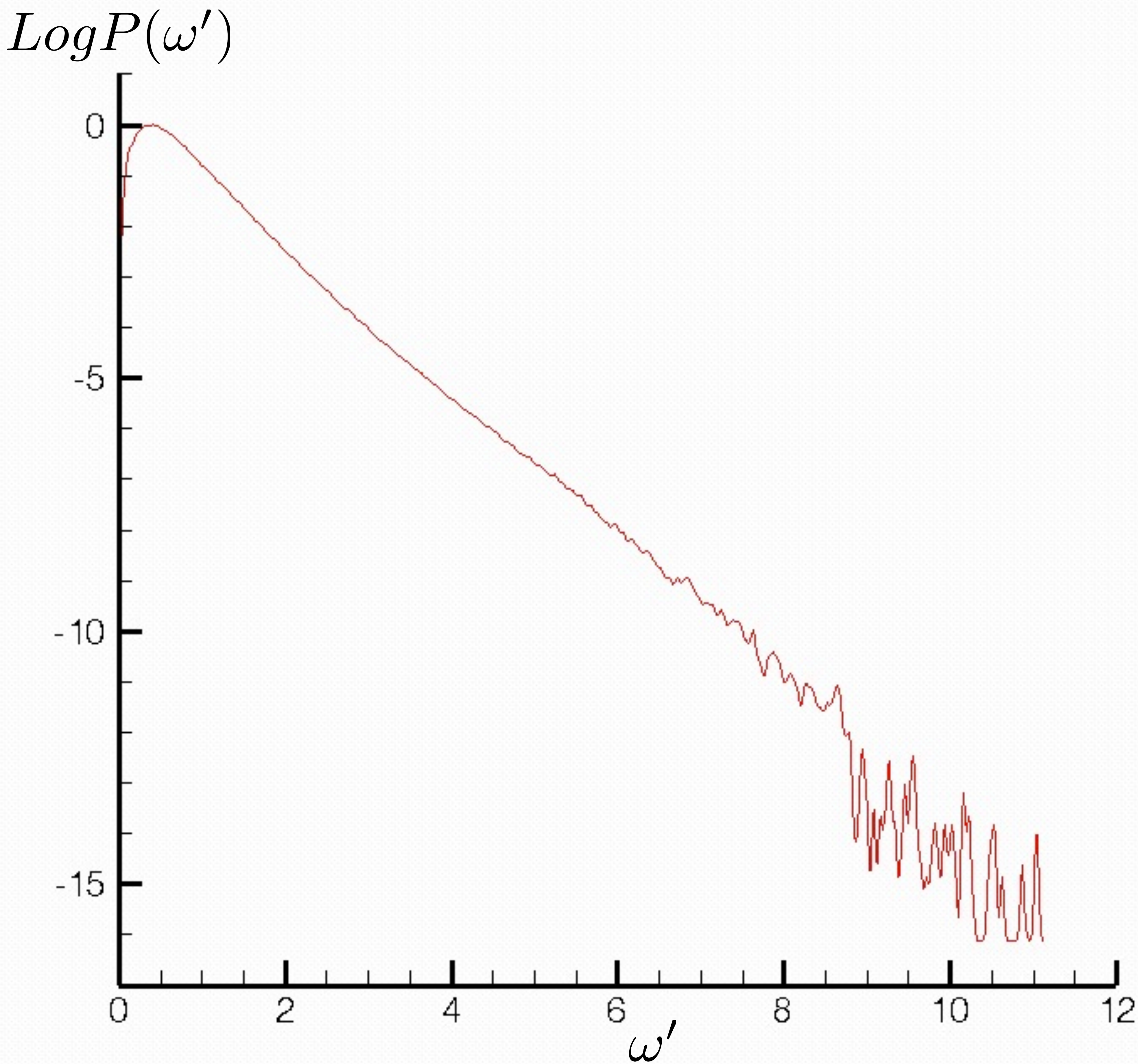
PDF of Normalized Vorticity Amplitudes

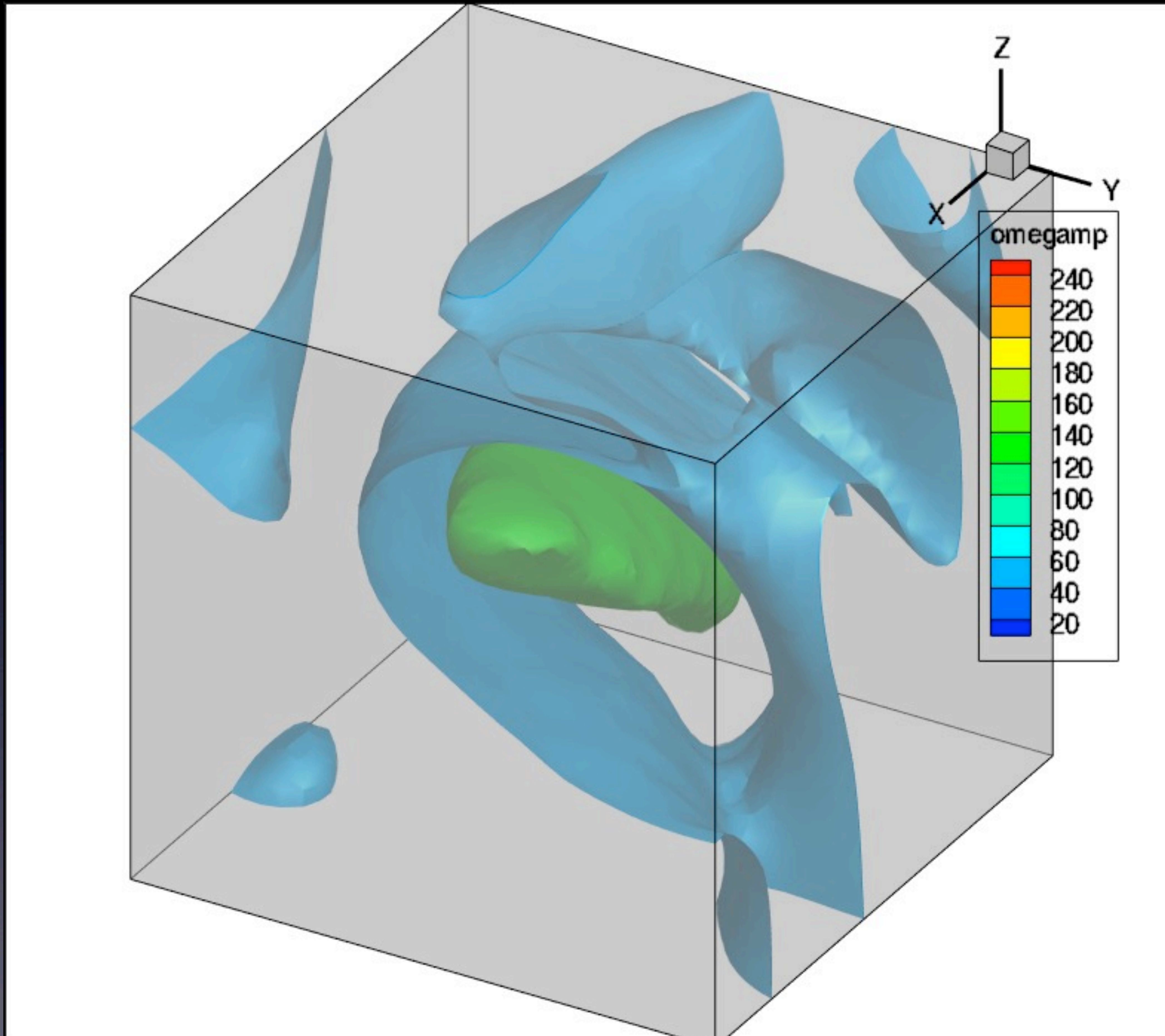
$\omega' = \omega/\omega_\eta \quad \omega_\eta = 1/\tau_\eta$

τ_η =Kolmogorov timescale= $\sqrt{\nu/\epsilon}$

$17x10^6$ Samples $\quad 55^3$ mesh with 100 timesteps

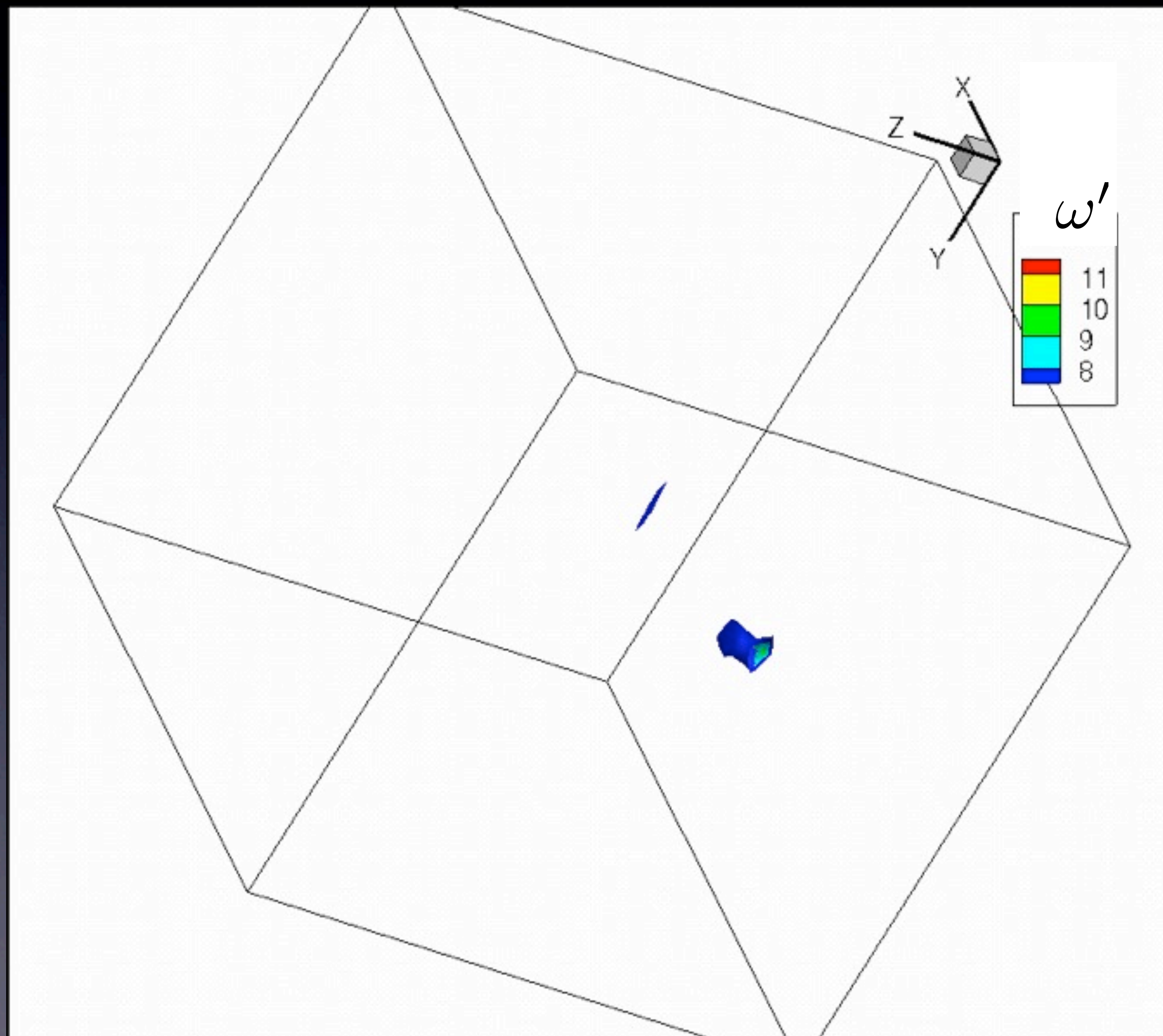



$LogP(\omega')$
0
-5
-10
-15
0
2
4
6
8
10
12
ω'


Z
Y
X
omegamp
240
220
200
180
160
140
120
100
80
60
40
20

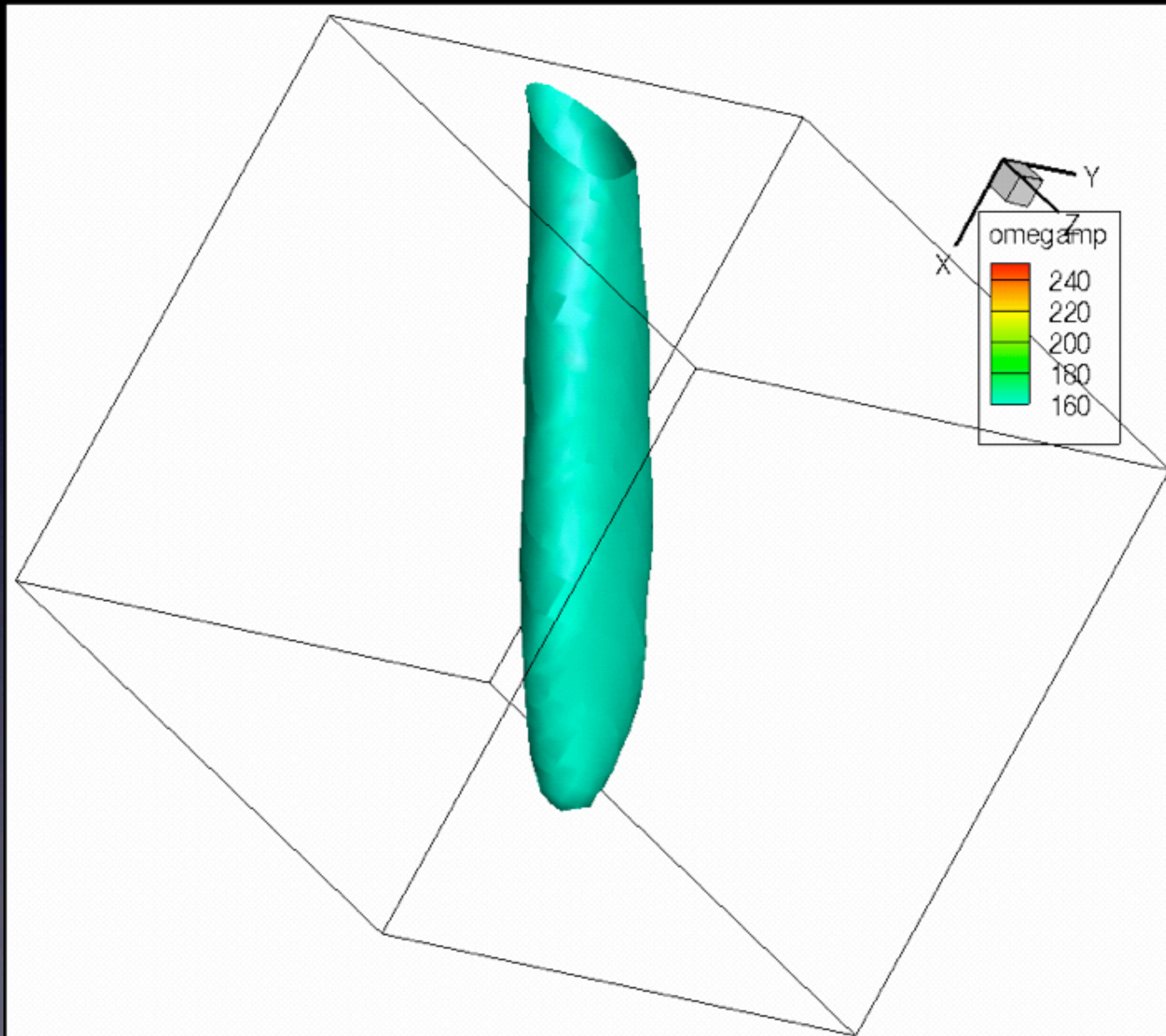
High Amplitude ω'

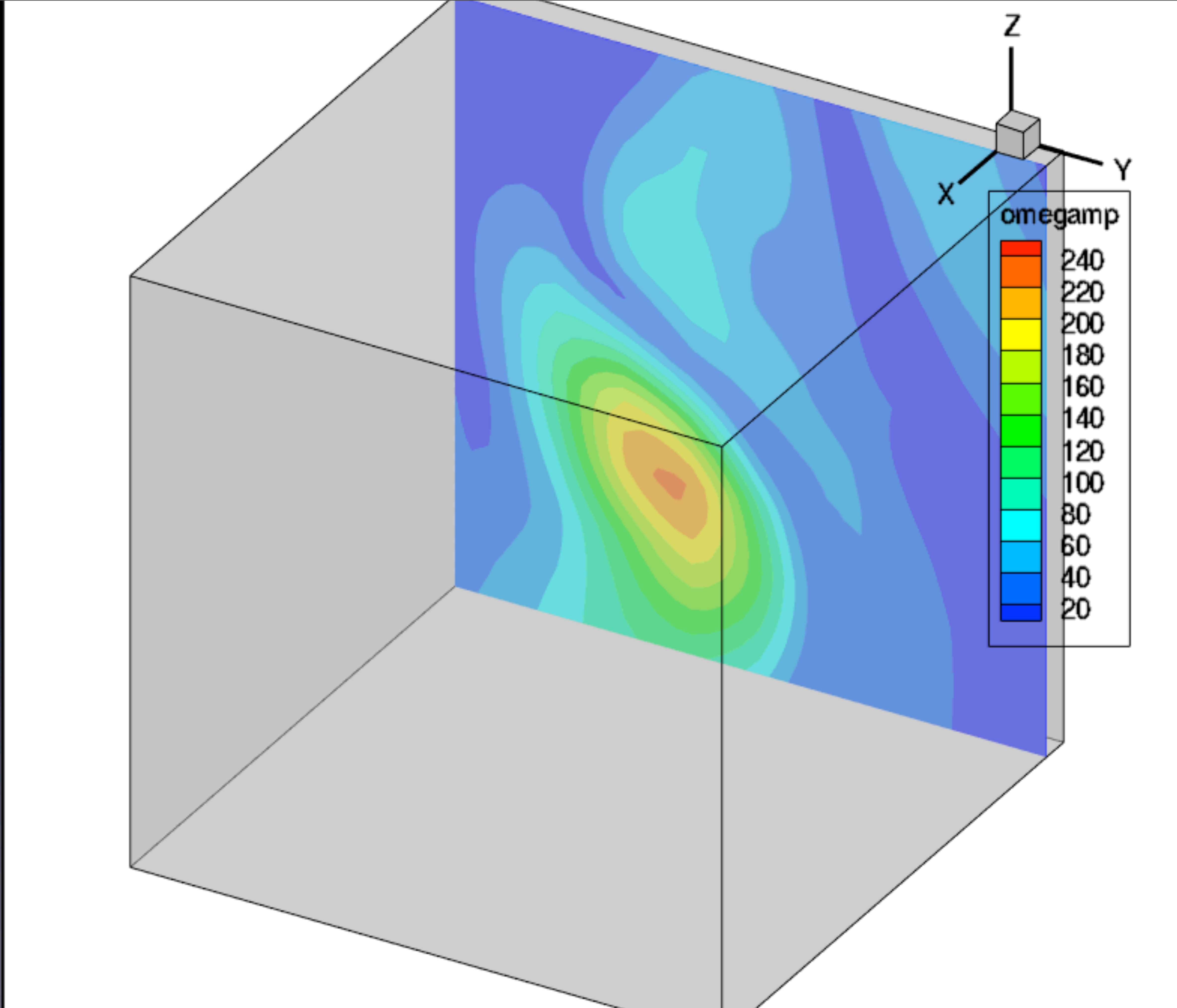
Box size $(32\Delta)^3 = (68\eta)^3 = (1.7\lambda)^3$



Intense ω' Region (zoom in)

Box size $(9\Delta)^3 = (19\eta)^3 = (0.47\lambda)^3$




Z
Y
X
omegamp
240
220
200
180
160
140
120
100
80
60
40
20

PDF for a single structure - $P_S(\omega)$

Model I - Exponential decay of circulation

$$\omega_x = \frac{\Gamma}{\pi\sigma_r^2} exp(-r^2/\sigma_r^2) exp(-x^2/\sigma_x^2)$$

Let $S = r^2/\sigma_r^2 + x^2/\sigma_x^2 \rightarrow S = log(\frac{\omega_{max}}{\omega})$

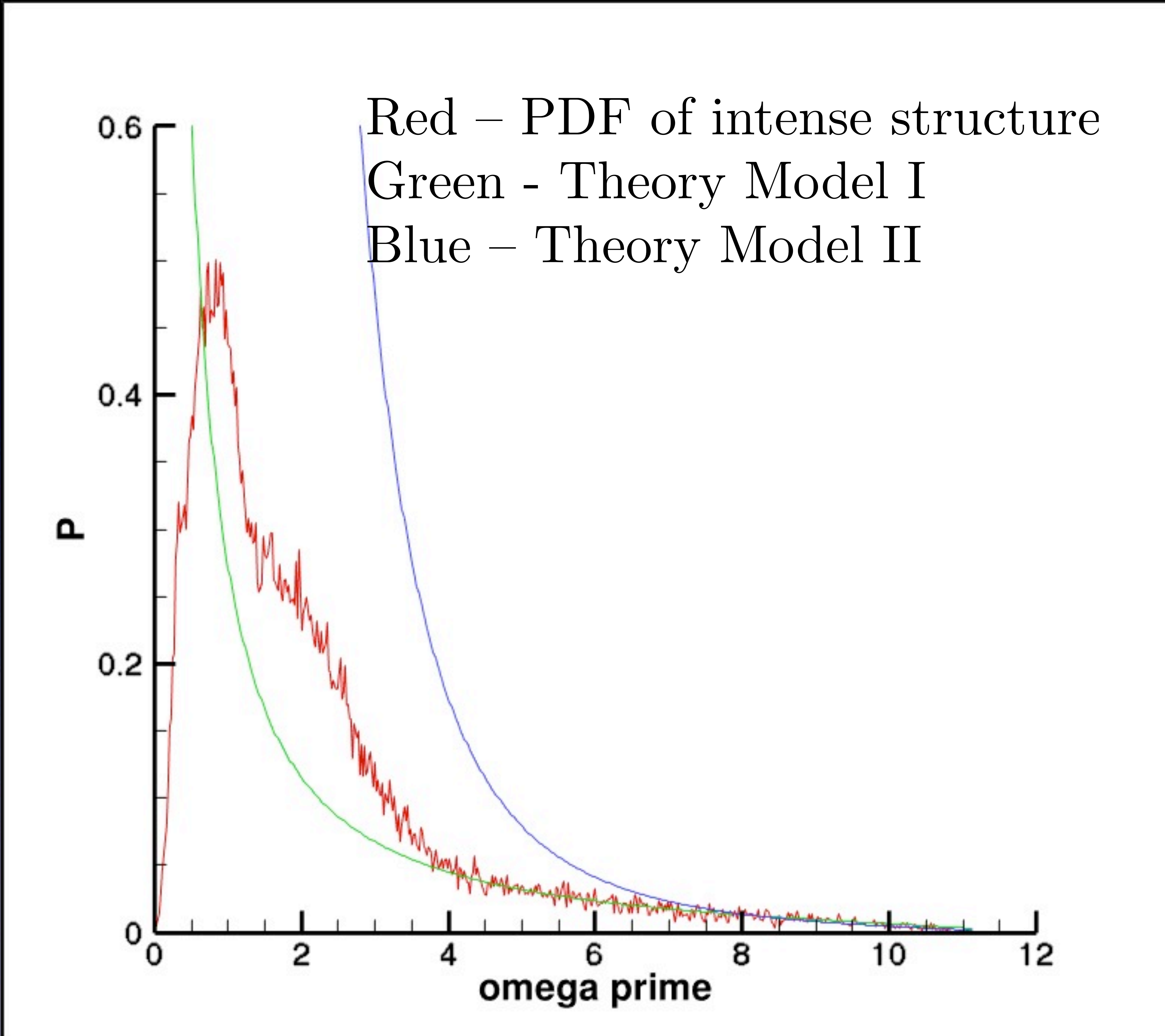
Volume inclosed by $S \rightarrow V(S) = \frac{4}{3}\pi\sigma_r^2\sigma_x S^{3/2}$

$$P_V(V) = \frac{dV}{V_T} \rightarrow \boxed{P_S(\omega)d\omega = C[log(\frac{\omega_{max}}{\omega})]^{1/2}\frac{d\omega}{\omega}}$$

Model II - Constant circulation, expanding vortex core

$$\omega_x = \frac{\Gamma}{\pi\sigma_r^2(x)} exp(-r^2/\sigma_r^2(x))$$

with $\sigma_r^2(x) = \sigma_o^2(1 + x^2/\sigma_x^2)$


Red – PDF of intense structure
Green - Theory Model I
Blue – Theory Model II
0.6
0.4
0.2
0
P
0
2
4
6
8
10
12
omega prime

Structure evolution in time
(preliminary)

$$\omega_x = \frac{\Gamma(t)}{\pi\sigma_r^2(t)} exp(-r^2/\sigma_r^2(t)) exp(-x^2/\sigma_x^2(t))$$

Consider vorticity transport equation for ω_x

Leading order straining flow -

$$\mathbf{U} = (a(t)x + b(t)(x^2 - r^2/2) + c(t)(9x^3/3 - x^2r^2/2)\mathbf{e}_x$$
$$+ \text{(-a(t)r/2 - b(t) xr -c(t) x}^2 r/2)\mathbf{e}_r$$

Collect terms of $O(1), O(x^2/\sigma_x^2)$, and $O(r^2/\sigma_r^2)$

$$\frac{d\sigma_r^2(t)}{dt} = -a(t)\sigma_r^2(t) + 4\nu - \frac{c(t)}{2}\sigma_r^4(t)$$

$$\frac{d\sigma_x^2(t)}{dt} = 2a(t)\sigma_x^2(t) + 4\nu + 3c(t)\sigma_x^4(t)$$

$$\frac{d\Gamma(t)}{dt} = -(\frac{2\nu}{\sigma_x^2(t)} + \frac{c(t)\sigma_r^2(t)}{2})\Gamma(t)$$

Quasi-steady solution

$$\frac{1}{\sigma_r^2(t)} \approx \frac{a(t)}{4\nu} + \frac{c(t)}{2a(t)}$$

$$\frac{1}{\sigma_x^2(t)} \approx -\frac{c(t)}{a(t)}$$

$$\Gamma(t) \approx Const.$$

Summary
Intense vortex structures

- Geometry of intense structures, vorticity distribution
  Cigar shaped, aspect ratio $\approx$ 6.5, gaussian distribution of circulation along axis
- Connection between geometry and PDF
  Vorticity distribution yields pdf for a single structure
- Evolution in time
  Equations for gaussian widths in radius and axial direction and for circulation interms of local strainrate field, longtime survival possible