A virtual universe which looks like the Universe, but without dark matter

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ABSTRACT

In this paper we search for an alternate universe which looks like the Universe, i.e. with exactly the same (local) physics and which is sticking for most of the observational data, but where the dark matter is absent. Rather, in this parallel copy, the effects of dark matter are simulated by variable dimensional physical constants. In a suitable manner, we ask the question: could a world, where the dimensional physical constants are not universal, support live and intelligent observers? Is such a world existing within the multiverse ? The study of such trial universes could contribute to a better understanding of our own world.

Subject headings: Multiverse; extra-terrestrial life

1. Introduction

One knows that the Universe possesses a set of fundamental constants which are such that had they been very slightly different, the Universe would have been void of intelligent life. Is then our universe, the Universe, unique ? If the response is yes the Universe would have been from the beginning finely adjusted in order that the life appears (Bostrom N., 2002). In the past, the anthropic principle was very often enounced to conclude that the Universe is finely-tuned (Carr and Rees, 1979), even though ultimately its turns out that sole an insignificant zone of the Universe is potentially capable of sustaining life (Stenger, 2011). This question is still a matter of debate (Barnes, 2012).

But there exists another interpretation which is issued from the quantum mechanics : the Many-Worlds or Multiverse hypothesis (Barrow, 2002). This concept has been initially built in order to escape to the troublesome notion of wave function collapse in the Copenhagen interpretation of quantum mechanics. The Multiverse is a set (or more rigorously a class) including all universes, finite or infinite, logically (and also actually) possible universes. Can we test the Multiverse hypothesis ? The existence of the Multiverse seems supported by numerous physics theories and especially the string theories (Weinberg S., 2009) and the inflationary models (Guth, Kaiser, Nomura, 2014). Some cosmologists argue however that Multiverse could be unfalsifiable (Ellis, 2011; Ijjas, Steinhardt, Loeb, 2013).

Is the Universe the only reality or a peculiar reality among an infinity ? Our purpose here is not to speculate whether the Multiverse exists or even to discuss on its falsiability. We consider only the Multiverse as a class of virtual (with some of them potentially plausible) worlds. Virtual or possible worlds can be a very interesting and pragmatic opportunity to test hypothetical statements. More generally, the Multiverse concept, even if this entity is virtual, could be basically a productive research program (Ellis, 2011).

Live-supporting universes are undoubtedly very scarce in the Multiverse. On the other hand, it is also very difficult to imagine an universe with characteristics too remote of our own world (with, for instance, other physical laws or different fundamental physical constants). We have restricted here our search to universes with the same local physics as ours and we have assumed that the light propagates in straight line in the vacuum. On the other hand, the fundamental constants (dimensionless constants) are taken identical to ours.

In the Universe, cold dark matter has been accumulating the evidences from the flat rotational velocity curves of galaxies, gravitational lensing and collision clusters. More generally, the ΛCDM model, referred to as the standard model of cosmology, provides a good account of the characteristics of the Universe at various scales (Ostriker, Mitton, 2013). But other concurrent also exist (Milgrom, 1983; Moffat, 2006; Brownstein, Moffat, 2007). More specifically, the challenge of the present study is to search if other universes, with almost the same observational data collected by us in the Universe, are potentially existing in the Multiverse, but suppressing just an essential ingredient which does our world so well running, that is dark matter.

2. The physical constants

There exist two types of physical constants. The dimensionless physical constants, such as the fine-structure constant α, the proton-to-electron mass ratio, ... are independent of the time and location in the Universe. These constants have the same numerical value in all systems of units. Their immutability has been repeatedly tested (Yasunori, 2004; Chand et al, 2004; Srianand et al, 2004) which constitutes a cornerstone of the physics laws in our world (Uzan, 2011). On the other hand, the dimensional physical constants are expressed in a specific unit (for instance the speed of light c, the gravitational constant G, ...). But contrarily to a widespread opinion, these constants can be varied as long as the dimensionless ones remain fixed. A variation of the speed of light c, accompanied by corresponding variations in the elementary charge e and the dielectric constant ϵ_0 so that the fine-structure constant remains unchanged, produces a virtual world indistinguishable of our world (Barrow, 2002).

All objects existing in our world (the Universe), transposed in the virtual Universe (V_Universe), are labelled by V_. The V_Earth is a copy of the Earth with the same mass and the same radius, circulating around a G-type star similar to the Sun, and located at the same distance in the habitable zone. A virtual observer (called V_O in the following) present on the V_Earth postulates the relation

$$\frac{m_e}{m_{eV\_O}} = f(\frac{n}{n_{V\_O}}) \tag{1}$$

where m_e denotes the electron mass and n is the local mean particle density measured at a given point. The function f is unknown, but it appears natural to admit that f is certainly a monotonic increasing function of n. This function is assumed to be bounded $f_{min} < f < f_{max}$ and normalised $f(1) = 1$. All quantities labelled V_O are those which are locally measured by the virtual observer. In the V_Universe, the electron mass depends on its environment, the electron mass is assumed to be much higher in a dense environment than elsewhere. This idea is directly derived from the Mach's conjecture about the inertia, the latter one being considered as a kind of interaction between bodies (Bondi, Samuel, 1997; Pfister, King, 2015). Following this principle, expressed in a local form, V_O admits that the constants of physics are dynamic quantities which "feel" their environment. The mean particle density in question could be, for instance, the density of particles obtained by smoothing out the matter composing stars and planets, say over a region of the order of 10 pc. The function f is assumed to be variable within the range $10^{-24} \lesssim n \lesssim 10^{-26}$ $g\ cm^{-3}$ and constant outside this interval. This range corresponds to the variation of the mean particle density within the outskirts of the galactic disc (Table 1). This choice appears arbitrary, but the galaxy formation is seen in the V_Universe as a phase transition with some kind of broken symmetry where the dimensional constants go from a value to another one.

Solar system	10^{-23} $g\ cm^{-3}$
Galaxy (disc)	10^{-22} (core region) -10^{-26} (outer regions) $g\ cm^{-3}$
galactic halo	$10^{-26} g\ cm^{-3}$
extragalactic medium	$10^{-29} - 10^{-30} g\ cm^{-3}$

Table 1

The universal constants chosen by V_O are

- The fine-structure constant is an universal constant (e electron charge, ϵ_0 dielectric permittivity, $\hbar$ reduced Planck constant, c speed of light)[1]

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \sim 1$$

[1] In the following, the symbol ~ 1 signifies the quantity is an universal constant (f-independent constant), i.e. a constant independent of the point and over time.

- Likewise for its gravitational equivalent (linked to the Eddington luminosity)

$$\alpha_G = \frac{Gm_e^2}{\hbar c} \sim 1$$

- Mass ratio (proton/electron)

$$\frac{m_p}{m_e} \sim 1$$

- Any reference frequency ν emitted by a atom or a molecule and measured by an observer at rest, for instance the frequency of H_α line. We have

$$\nu_{H_\alpha} = \alpha^2 m_e c^2 \sim 1 \Rightarrow m_e c^2 \sim 1$$

- Electron energy (electron at rest)

$$E_e = m_e c^2 \sim 1 \quad \Rightarrow c \sim m_e^{-\frac{1}{2}} \Rightarrow c \sim f^{-\frac{1}{2}} \text{ or } c = f^{-\frac{1}{2}} c_{V\_O}$$

- From the Wien law

$$\frac{\nu_{max}}{T} = 0.2014...\frac{k_B}{h} \sim 1 \ \Rightarrow \ k_B \sim 1 \qquad\qquad T \sim 1$$

- For a star of a given spectral type (with definite luminosity L_* and temperature T_*)

$$L_* = 4\pi\sigma R_*^2 T_*^4 \sim 1, \ \ T_* \sim 1 \ \ \Rightarrow \ \ R_* \sim f^{-\frac{1}{2}}, \ \sigma = \frac{2\pi^5 k_B^4}{15c^2h^3} \sim f^{-1}$$

V_O observes the same Hertzprung-Russel diagram as us ($L_* \sim 1, \ \ T_* \sim 1$).

- All radioactive decay constants $\gamma \sim 1$. For instance, for the alpha process (R_{nucl} nuclear radius, $V(r)$ coulombian potential and E energy of a alpha particle)

$$\gamma_\alpha = \frac{c}{2R_{nucl}} exp\left\{-2\int_{R_{nucl}}^{r} \frac{\sqrt{2m\left[V\left(r\right)-E\right]}}{\hbar} dr\right\} \sim 1$$

From $\alpha_G \sim 1$ and $c \sim f^{-\frac{1}{2}}$, we deduce that $G \sim f^{-\frac{5}{2}}$.

The radius of any atom $r_{at} \sim f^{-\frac{1}{2}}$. The materialized length (meter-stick) can be simply visualised by a chain composed of a number defined of N atoms, $L = N r_{at} \sim f^{-\frac{1}{2}}$. The unit of materialized length (a meter-stick) is defined by the distance travelled by the light in a fraction of second $\Delta t = \frac{L}{c} = \frac{L_{V\_O}}{c_{V\_O}}$.

V_O assumes that the laws of physics are fully identical to ours, except that they are now restricted to their local form : the Maxwell equations, those of the standard model of particle physics, the Einstein equations of the General Relativity, etc[2]. The acception of "local" is taken here in the sense of "at the scale of a planetary system". But a connection between two distant points must be defined in the V_Universe. The choice is large, but the guide is that a new physics is acceptable at the condition that no internal self-contradiction appears within it. On the other hand V_O hypothesizes that all the dimensional physical constants present in the current equations are "parallel transported" from a point to another point. More simply, m_e, c, the materialized unit length (visualized by a physical artifact), ... are deplaced in such a way that V_O has no means to feel a difference. In its new environment, when V_O achieves any local measurement, he obtains the same results than in his old environment. A first exemple comes from the cosmic rays which are particles originating from the outside of the solar system (supernovae, active galactic nuclei,...). These particles (protons) have exactly the same properties (mass, charge) as their terrestrial equivalent. V_O thus assumes that the properties of the particles adapt instantaneously to their new environment.

3. The propagation of the light

In a live-supporting universe, we guess that the light must propagate in straight line in vacuum (excepting light bending produced by a gravitational field, but this effect is weak and not taken into account in this paragraph). Otherwise stellar radiations could be more or less strongly collimated in small regions. A lot of focused beams could then sweep the V_Earth with potentially destructive consequences on life.

In the Universe, the Fermat's principle in Classical Optics states that the path of a light ray to go from a point A to another point B, in a inhomogeneous medium, is such that the integral (defining the travel time Δt between A and B)

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[2]In fact, the range of interactions in Physics is very small for both the strong and weak nuclear forces. As for the interaction between two electric charges, the range of the force is infinite (Coulomb law), but the screening between positive and negative charges drastically reduces the range to the Debye radius. The only force which put a real challenge is the newtonian gravitation which acts at a distance and which is not screened.

$$\Delta t = \int_{A \to B} \frac{dl}{c} = \int_{A \to B} \frac{1}{c} \sqrt{\left(\frac{d\mathbf{r}}{dl}\right)^2} dl$$

is extremal. We obtain the well known eikonal equation of light rays

$$\frac{d}{dl}\left(\frac{1}{c}\frac{d\mathbf{r}}{dl}\right) = \nabla\left(\frac{1}{c}\right) \tag{2}$$

A very similar equation can be derived from the metric element in space-time

$$ds^2 = c^2\left(\mathbf{r}\right) dt^2 - dl^2$$

The geodesic equations ($i, j, k = 0, 1, 2, 3$) are in this case (with λ an arbitrary parameter)

$$\frac{d^2 x^k}{d\lambda^2} + \Gamma^k_{ij} \frac{dx^i}{d\lambda}\frac{dx^j}{d\lambda} \equiv 0$$

The calculation of the Christoffel symbols yields the equations

$$\frac{d^2 x}{d\lambda^2} + c\frac{\partial c}{\partial x}\left(\frac{dt}{d\lambda}\right)^2 = 0 \quad ... \quad \frac{d^2 t}{d\lambda^2} + 2c\frac{dLnc}{d\lambda}\left(\frac{dt}{d\lambda}\right) = 0 \tag{3}$$

which are equivalent to (1).

But if we take the Fermat's principle in its original form, we conclude that the light propagates in the V_Universe as in a inhomogeneous medium, which necessarily implies refraction, seen as an unwanted effect here. Even though the Fermat's principle is preserved at the scale of a planetary system, V_O is constrained to either give up this principle at large scale (galaxy) or to strongly modify it. The Fermat's principle is an universal law in our world, the latter one being very special with "finely tuned" universal constants of physics; but may be, in an exotic universe this principle is different. A possible, even though very formal, way to escape to this difficulty is to associate to the vector $\frac{d\mathbf{r}}{d\lambda}$ (with λ an arbitrary parameter) a corresponding co-vector $\frac{d\mathbf{r}^*}{d\lambda}$ which belongs to a dual space possessing a linear induced structure. The vector $\frac{d\mathbf{r}}{d\lambda}$ is not an unit vector but $\frac{d\mathbf{r}}{d\lambda}.\frac{d\mathbf{r}^*}{d\lambda} = 1$[3]. Dual spaces are well known (Lebedev, Cloud, Eremeyev, 2010) and very often used in Physics.

The Fermat's principle now provides that the functional

[3]We have $d\mathbf{r} = dx\mathbf{e}_1 + dy\mathbf{e}_2 + dz\mathbf{e}_3$ and $d\mathbf{r}^* = dx^*\mathbf{e}_1^* + dy^*\mathbf{e}_2^* + dz^*\mathbf{e}_3^*$. The scalar product gives $d\mathbf{r}.d\mathbf{r}^* = dxdx^* + dydy^* + dzdz^* = d\lambda^2$ ($\mathbf{e}_1.\mathbf{e}_1^* = 1, \mathbf{e}_1.\mathbf{e}_2^* = 0, ...$).

$$\int_{A\to B} L\left(\mathbf{r},\mathbf{r}^*,\dot{\mathbf{r}},\dot{\mathbf{r}}^*\right) d\lambda$$

with the splitted lagrangian $L\left(\mathbf{r},\mathbf{r}^*,\dot{\mathbf{r}},\dot{\mathbf{r}}^*\right) = 2\sqrt{(\frac{1}{c(\mathbf{r})}\frac{d\mathbf{r}}{d\lambda})(\frac{1}{\tilde{c}(\mathbf{r}^*)}\frac{d\mathbf{r}^*}{d\lambda})}$, must be extremal. The speed of light in the dual space, $\tilde{c}(\mathbf{r}^*)$, is assumed to be a constant equal to c^*[4]. The lagrangian equation

$$\frac{d}{d\lambda}\frac{\partial L}{\partial \dot{\mathbf{r}}^*} - \frac{\partial L}{\partial \mathbf{r}^*} = 0$$

leads now to the eikonal equation

$$\frac{d}{d\lambda}\left(\frac{1}{\sqrt{c}}\frac{d\mathbf{r}}{d\lambda}\right) = 0 \tag{4}$$

where $c \equiv c(\mathbf{r})$ and $\dot{c} = \frac{d\mathbf{r}}{d\lambda}.\nabla c$.

With this new formulation of the Fermat's principle, the light propagates in straight line. In fact the propagation of light in the V_Universe is codified by the physics of its dual space where the constants are universal. On the other hand, the dual equation is (the factor two disappears in the rescaled parameter λ)

$$\frac{d}{d\lambda}\left(\frac{1}{\sqrt{c}}\frac{d\mathbf{r}^*}{d\lambda}\right) = \nabla\left(\frac{1}{\sqrt{c}}\right) \tag{5}$$

Here c designates the function $c(\mathbf{r}) \equiv c \circ g^{-1}(\mathbf{r}^*)$ where g^{-1} is the reciprocal function of $\mathbf{r}^* = g(\mathbf{r})$ (g globally represents three scalar functions). In the dual space the physical constants are assumed to be universal ($\tilde{c}(\mathbf{r}^*) = c^*$), but the light rays are however curved. If a hypothetic observer is present in this space, he will see deflections of the light rays but without the presence of visible matter (an example in our world is given by the Bullet Cluster, but the problem is solved by the presence of dark matter or by other hypotheses such as the modified newtonian dynamics (Milgrom, 1983) or the nonsymmetric gravitational theory (Brownstein, Moffat, 2007)). We can however notice that the curvature is smoothed ($\sqrt{c}$ instead of c as in equation 2).

A similar reasoning can be made starting from a splitted ds^2

$$ds^2 = d\mathbf{s}.d\mathbf{s}^*$$

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[4]In fact this constant can take its value in an interval, say ranging in the present paper from c^*_{min} to c^*_{max}. The dual space is thus not an unique space but rather a continuous foliation of leaves located "above" the V_Universe. We define then the mapping $\mathbf{r} \longrightarrow \mathbf{r}^* = g(\mathbf{r})$ as a one-to-one correspondance between the V_Universe and a leave of its dual taken in the foliation.

where $d\mathbf{s} = c(\mathbf{r})\,d\mathbf{t} - d\mathbf{r}$ and $d\mathbf{s}^* = c^* d\mathbf{t}^* + d\mathbf{r}^*$ with the rules for the products : $d\mathbf{t}.d\mathbf{t}^* = (dt\mathbf{e}_0).(dt^*\mathbf{e}_0^*) = dtdt^*$, $d\mathbf{t}.d\mathbf{r}^* = 0$,..., $d\mathbf{r}.d\mathbf{r}^* = dx.dx^* + dy.dy^* + dz.dz^*$[5].

On a mathematical point of view the V_Universe with three supplementary dimensions is logically possible but could it be actual ? In our world there exists a large debate whether Mathematics is simply an useful tool for Physics or represents the fundamental reality. Adding supplementary dimensions can indeed appear very artificial, but this technique is currently used in other contexts of Physics, in superstring theories for instance. Nevertheless these extradimensions are then compactified. However proposals involving supplementary non-compact or large extradimensions have also been considered (see for instance, Randall, 2005; Shifman, 2010). Then perhaps the mathematical structure formed by the V_Universe and its foliated dual could eventually be appear as an actual component in the Multiverse.

4. Kinematics and dynamics in the V_Universe

Let us denote the position of an electron (mass m_e) by the vector $\mathbf{r}$ (arbitrary origin). Once a reference frame is chosen, the velocity and the acceleration can be specified as usually by

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} \qquad \mathbf{a} = \frac{d\mathbf{v}}{dt}$$

For the linear momentum $\mathbf{p}$, V_O postulates the relationship (by analogy with the special Relativity)

$$\mathbf{p} = \frac{m_e\mathbf{v}}{\sqrt{1-(\frac{\mathbf{v}}{c}).(\frac{\mathbf{v}^*}{c^*})}} \quad \text{or} \quad c\mathbf{p} = \frac{m_e c^2(\frac{\mathbf{v}}{c})}{\sqrt{1-(\frac{\mathbf{v}}{c}).(\frac{\mathbf{v}^*}{c^*})}}$$

and likewise for the energy

$$E = \sqrt{m_e^2 c^4 + (c\mathbf{p}).(c^*\mathbf{p}^*)} = \frac{m_e c^2}{\sqrt{1 - (\frac{\mathbf{v}}{c}).(\frac{\mathbf{v}^*}{c^*})}}$$

The time is homogeneous, which implies that the energy is conservative for a free motion. Given $m_e^2 c^4$ is an universal constant, both the scalar products $(c\mathbf{p}).(c^*\mathbf{p}^*)$ and $(\frac{\mathbf{v}}{c}).(\frac{\mathbf{v}^*}{c^*})$ must be conservative for a free motion. V_O admits that $c\mathbf{p}$ is also conservative[6]. In this case

[5] In the presence of gravitation, we must assume that the metric can still be splitted into $d\mathbf{s} = h_\mu d\mathbf{q}^\mu$ and $d\mathbf{s}^* = h_\mu^* d\mathbf{q}^{*\mu}$ in any coordinate system $\{q^\rho\}$. The lagrangian is $L(q^\rho, q^{*\rho}, \dot{q}^\rho, \dot{q}^{*\rho}) = \sqrt{h_\mu h_\nu^* \dot{\mathbf{q}}^\mu \dot{\mathbf{q}}^{*\nu}}$.

[6] This equation can be derived from a variational principle (by analogy with the splitted Fermat's principle, introduced in the paragraph 3). The splitted lagrangian for a free motion is

$$L(\mathbf{r}, \mathbf{r}^*, \mathbf{v}, \mathbf{v}^*) = -2m_e c^2 \sqrt{(\mathbf{e}_0 - \frac{\mathbf{v}}{c(\mathbf{r})}).(\mathbf{e}_0^* + \frac{\mathbf{v}^*}{c^*})}$$

$$\frac{d}{dt}(c\mathbf{p}) = \mathbf{0} \;\Rightarrow\; \frac{d\mathbf{p}}{dt} = -\frac{\dot{c}}{c}\mathbf{p}$$

and

$$\frac{d}{dt}\left(\frac{E}{c}\right) = -\frac{\dot{c}}{c}\left(\frac{E}{c}\right)$$

If the electron is submitted to a force $\mathbf{F}$, the dynamic equation is now written

$$\frac{d\mathbf{p}}{dt} = -\frac{\dot{c}}{c}\mathbf{p} + \mathbf{F} \tag{6}$$

and for the energy equation

$$\frac{d}{dt}\left(\frac{E}{c}\right) = -\frac{\dot{c}}{c}\left(\frac{E}{c}\right) + \frac{\mathbf{F}.\mathbf{v}}{c} \tag{7}$$

For a free motion, $\mathbf{F} = 0$, the linear momentum is no longer conservative contrarily to the situation encountered in our world, but $\frac{d\mathbf{p}}{dt}$ remains colinear with $\mathbf{p}$. The trajectory of a free particle is a straight line. The ratio $\frac{v}{c}$ being conservative, the vector $\beta = \frac{v}{c}$ or the rapidity w[7] identified by $\beta = \tanh w$, seems to be much more appropriate than the velocity.

For a photon the linear momentum is $\mathbf{p} = \hbar\mathbf{k}$ where $\mathbf{k}$ is the wavelength vector. The equation of $\mathbf{k}$ is

$$\frac{d\mathbf{k}}{dt} = -\frac{\dot{c}}{c}\mathbf{k} \tag{8}$$

Again we find that the propagation of a photon is in straight line (neglecting gravity), in agreement with the modified Fermat's principle proposed above. For the wavelength we have

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{c}}{c} \Rightarrow \lambda \sim f^{-\frac{1}{2}}$$

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with $m_e c^2 = m_e^* c^{*2}$. From the dual equation we can easily show that the projection of $\mathbf{v}^*$ on $\mathbf{v}$, let$\mathbf{v}^*_{//}$, is conservative. We can put $\|\mathbf{v}^*_{//}\| = \frac{v}{c}c^*$ and $\frac{\mathbf{v}^*}{c^*}.\frac{\mathbf{v}}{c} = \frac{v^2}{c^2}$. Thus this supplies again the usual relationships for both the energy and momentum of the special Relativity.

[7]The rapidity can be calculated by measurements of energy and linear momentum

$$w = \frac{1}{2}Ln\frac{E+pc}{E-pc}$$

In the Universe an inertial frame of reference is defined by three different directions (non co-planar) which are three rectilinear paths of particles in free motion. In the V_Universe this concept is much more difficult to define. Let us imagine however the V_Universe as composed of a gas of galaxies in motion in a medium of very low density. We can now define an inertial frame of reference, let R_{out}, attached to this medium, where the speed of light is assumed to be constant and equal to c_{out}.

Let $\mathbf{v}$ the peculiar velocity of a star at a point $\mathbf{r}$ of some galaxy, $\mathbf{V}$ the translation velocity of this galaxy, both being measured in the inertial frame of reference R_{out}. We can then admit the composition law for the velocities (available for non-relativistic velocities)

$$\mathbf{w} = \mathbf{v} + \mathbf{V}$$

Generally, for any observer, the velocity $\mathbf{w}$ of a star has components both along the line of sight (radial component) and perpendicular to this line (tangential component). The tangential component $w_\perp$ can be obtained directly (if detectable). On the other hand, the radial component $w_\parallel$ is obtained from the Doppler shift. A hypothetic observer located in the outer region of the galaxy carries out a measurement of the radial velocity of a star. The apparent radial velocity $w_{\parallel app}$ can be obtained from the following formula (available for low velocities)[8]

$$\frac{w_{\parallel\ app}}{c_{out}} = \frac{\mathbf{v.n}}{c(r)} + \frac{\mathbf{V.n}}{c_{out}} \tag{9}$$

where $\mathbf{n}$ denotes the unit vector along the line of sight.

However in this case, contrarily to our world where the speed of light is an universal constant, we can now note that $w_{\parallel\ app} \neq w_\parallel$.

5. The flat rotation curve of galaxies and the spiral structure

V_O observes an outer galaxy and searches to understand its structure. Gravity is most accurately described by the General Relativity, but the newtonian limit is sufficient if the gravitational field is weak enough. For the action at a distance between two particles, respectively a particle of mass M', placed at $\mathbf{r'}$, and another particle of mass m, placed at $\mathbf{r}$, V_O introduces the energetic c-potential[9]

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[8] This is the β-addition relation for non-relativistic velocities

$$\beta''_{\parallel\ app} = \beta_\parallel + \beta'_\parallel$$

where $\beta_\parallel = \frac{\mathbf{v.n}}{c(r)}$, $\beta'_\parallel = \frac{\mathbf{V.n}}{c_{out}}$, $\beta''_{\parallel app} = \frac{w_{\parallel app}}{c_{out}}$.

[9] We start from the lagrangian density by introducing the c-potential ϕ^c (thereafter defined), let

$$L = -\rho c^2 \phi^c - \frac{c^5}{8\pi G}(\nabla\phi^c)^2$$

$$U^c(\mathbf{r},\mathbf{r}') = -\frac{GMm}{c}\frac{1}{\|\mathbf{r}-\mathbf{r}'\|} = -\frac{G'M'm'}{c'}\frac{1}{\|\mathbf{r}-\mathbf{r}'\|}$$

where $G \equiv G(\mathbf{r}), c, M, m$ are measured locally at $\mathbf{r}$ and $G' \equiv G(\mathbf{r}'), c', M', m'$ at $\mathbf{r}'$. The ratio $\frac{GMm}{c}$ has the same value at $\mathbf{r}$ and $\mathbf{r}'$ (f-invariant quantity). The energetic c-potential is symmetric by exchange of $\mathbf{r}$ and $\mathbf{r}'$.

The derivation with respect to $\mathbf{r}$ gives the c-force acting on m (The ratio $\frac{GMm}{c}$ is independent of $\mathbf{r}$)

$$\mathbf{F}^c_{M\to m} = -\nabla_{\mathbf{r}} U^c = -\frac{GMm}{c}\frac{\mathbf{r}-\mathbf{r}'}{\|\mathbf{r}-\mathbf{r}'\|^3}$$

and for the usual force at $\mathbf{r}$

$$\mathbf{F}_{M\to m} = c\mathbf{F}^c_{M\to m} = -GMm\frac{\mathbf{r}-\mathbf{r}'}{\|\mathbf{r}-\mathbf{r}'\|^3} \tag{10}$$

In a symmetric manner[10]

Applying the Hamilton principle, we deduce the local Gauss law for gravity at $\mathbf{r}$

$$\triangle\phi^c = 4\pi\frac{G}{c^5}(\rho c^2)$$

($\frac{G}{c^5}$ is an universal constant in the V_Universe). For a punctual mass M' located at $\mathbf{r}'$, we have $\rho c^2 = M'c'^2\delta(\mathbf{r}-\mathbf{r}') = Mc^2\delta(\mathbf{r}-\mathbf{r}')$. The c-potential at $\mathbf{r}$ is

$$\phi^c = -\frac{GM}{c^3}\frac{1}{\|\mathbf{r}-\mathbf{r}'\|} \Rightarrow U^c(\mathbf{r},\mathbf{r}') = mc^2\phi^c$$

Let us specify that ϕ^c is not the usual potential and it is expressed in a distinct unit. The relativistic equivalent to the Gauss law is

$$R^c_{ij} - \frac{1}{2}g^c_{ij}R^c = \chi^c T_{ij}$$

where R^c_{ij} represents the components of the the c-Ricci tensor, R^c the c-curvature, obtained from $g^c_{ij} = \frac{g_{ij}}{c}$ ($g^{cij} = cg^{ij}$), T_{ij} the components of the impulsion-energy tensor (f-invariant). The constant χ^c is equal to $8\pi\frac{G}{c^5}$. A simple calculation gives then

$$R_{ij} = cR^c_{ij} + c^{\frac{3}{2}}[2\nabla^c_i\partial_j c^{-\frac{1}{2}} + (\Delta c^{-\frac{1}{2}} - 3\sqrt{c}\|\nabla c^{-\frac{1}{2}}\|^2)g^c_{ij}]$$

and

$$R = g^{ij}R_{ij}$$

where ∇^c_i designates the covariant derivatives, ∂_i the partial derivatives, Δ^c the Laplace-Beltrami operator and ∇ the gradient operator. In the extragalactic domain or, at smaller scales, for a planetary system or a black hole (stellar or galactic), we can take $R^c_{ij} \simeq \frac{R_{ij}}{c}$.

[10]We have

$$\mathbf{F}^c_{M\to m} + \mathbf{F}^c_{m\to M} = \mathbf{0}$$

$$\mathbf{F}_{m\to M} = -G'M'm'\frac{\mathbf{r}' - \mathbf{r}}{\|\mathbf{r}' - \mathbf{r}\|^{3}}$$

One might think that there exists an infinity of such laws which can still be proposed by V_O. The criterion of choice is simply here that the local laws of physics, together with the observational data, are the same as ours. At the scale of a planetary system $G'M'm' \to GMm$ and the symmetrical gravitational law is recovered.

Now the mass m is the mass of a test particle and M is identified to the total attractive mass of the galaxy, assumed to be concentrated in the central region, located at $\mathbf{r}' = 0$. This is obviously an oversimplification, but which does not change the conclusions. In the outer region of the galactic plane, at the distance r from the center, a local observer accompanies the test particle. He estimates the attractive mass by counting the number of stars of each spectral type which compose it. Then he multiplies these numbers by the corresponding stellar masses measured in his proper environment at $\mathbf{r}$. He finds by local considerations $M = M(\mathbf{r})$ for the attractive mass (neither $M(\mathbf{r}')$ and no longer M_{V_O} given the latter quantities are not measurable by the local observer. The measurements are made locally by the observer who feels the attractive force).

The dynamic equation for the test particle is (The equivalence principle is implicitely admitted by V_O)[11]

$$\frac{d\mathbf{v}}{dt} = \frac{\dot{c}}{c}\mathbf{v} - GM\frac{\mathbf{r}}{r^3}$$

Using the polar coordinates (r, θ) in the galactic plane, we have

$$\ddot{r} - r\dot{\theta}^2 = \frac{\dot{c}}{c}\dot{r} - \frac{GM}{r^2} \qquad r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{\dot{c}}{c}r\dot{\theta} \tag{11}$$

On the other hand we can note that

$$\mathbf{F}_{M\to m} + \mathbf{F}_{m\to M} = -(GMm - G'M'm')\frac{\mathbf{r} - \mathbf{r}'}{\|\mathbf{r} - \mathbf{r}'\|^{3}} \neq 0$$

but it is not very surprising, given that we know that the linear momentum is not conserved in the V_Universe. We can find a simple analogy in electrostatics, where the asymmetry in the interaction appears between two charges q and q', each of them being situated in a distinct dielectric medium (eliminating the fictitious charges) (Landau, Lifshitz,1984, p.34). How, in this case, can we avoid the self-acceleration of the barycentrer of the galaxy ? In fact the spiral galaxies are generally symmetric objects (even though not axisymmetric), where the spiral arms appear by symmetric pairs. The effect of the asymmetry in the interactions is thus canceled or at least smoothed.

[11] The gravitational term can be derived from the interaction lagrangian

$$L_{int} = -GM\frac{\mathbf{r}}{r^3}.\mathbf{r}^*$$

This term is not symmetric by exchange of $\mathbf{r}$ and $\mathbf{r}^*$, except for $f \sim r^{-2}$. In this special case $L_{int} \sim \mathbf{r}.\mathbf{r}^*$.

For an uniform circular motion, these equations reduce to

$$\frac{v^2}{r} = \frac{GM}{r^2}$$

Exploiting now the fact that $GM \sim f^{-\frac{3}{2}}$, we obtain

$$v \sim f^{-\frac{3}{4}} r^{-\frac{1}{2}} \tag{12}$$

It is important to notice that the velocity v is not directly reachable by V_O. He must necessarily go through an intermediary step which is the Doppler formula, i.e.[12]

$$\frac{\Delta\nu}{\nu} = \frac{\mathbf{v}.\mathbf{n}}{c}$$

where $\mathbf{n}$ is the unit vector along the line of view ($v \ll c$ and the non relativistic Doppler formula can be used here).

Given that $c \sim f^{-\frac{1}{2}}$, we have

$$\frac{\Delta\nu}{\nu} \sim f^{-\frac{1}{4}}\ r^{-\frac{1}{2}} \tag{13}$$

In the Universe, the observations of the surface brightness distribution of spiral galaxies lead to an exponential law for the mean density in the disc (Bovy, Rix , 2013), that is

$$n \propto e^{-\frac{r}{\delta}}$$

where δ is a e-folding scale length. This is a strong decrease indeed. But as already mentioned, the function $f(n)$ is unknown. V_O must choose this function to adapt it to the observations. He suggests a simple power law of the type $f\,(n) \sim r^{-\kappa}$ in the outskirts of galaxies. The equations (12) and (13) become

$$v \sim r^{-\frac{1}{2}\left(1-\frac{3\kappa}{2}\right)} \tag{14}$$

$$\frac{\Delta\nu}{\nu} = \frac{v}{c} \sim\ r^{-\frac{1}{2}\left(1-\frac{\kappa}{2}\right)} \tag{15}$$

The case $\kappa = 0$ leads to $v \sim r^{-\frac{1}{2}}$ and $\frac{\Delta\nu}{\nu} = r^{-\frac{1}{2}}$ which does not correspond to observations and can be rejected (assuming no dark matter in the V_ Universe). A most interesting situation is for

[12]Taking into account of the translational velocity $\mathbf{V}$ of the galaxy relative to V_O, this relation becomes (see &4)

$$\frac{\Delta\nu}{\nu} = \frac{\mathbf{v}.\mathbf{n}}{c} + \frac{\mathbf{V}.\mathbf{n}}{c_{V\_O}}$$

$\kappa = 2$ (see note 11). In this case v varies as r. No differential rotation is exhibited and the spiral form is conservative (the winding dilemma for grand-design spirals is automatically solved in the V_Universe). Moreover, $\frac{\Delta\nu}{\nu}$ is constant and flat curves for the radial velocities are found without dark matter. It is very astonishing that, with $\kappa = 2$ in the V_Universe, the observation of a flat rotation curve for the galaxies is related to no differential rotation and thus no deformation of a spiral configuration. In the Universe (our world), flat rotation curves are due to dark matter and spiral arms are not permanent features but are very well explained by density waves. Spiral arms are induced by tidal interaction of a nearby galaxy or by the presence of a rotating bar in the central region, and then stabilised by dark matter (Phillipps, 2005).

A contrario, what would be the reasoning of V_O if he was ignorant that, in the V_Universe, the constants of physics are depending on point ? His reasoning would be the following one (like us in fact, he would take his own references $G_{V\_O}, ...$): the gravitational law would be

$$\mathbf{F}_{M\to m} = -G_{V\_O} M_{V\_O} m_{V\_O} \frac{\mathbf{r}}{r^3}$$

which would lead to

$$\frac{v^2}{r} = \frac{G_{V\_O} M_{\_VO}}{r^2} \quad \Rightarrow \quad v = \sqrt{\frac{G_{V\_O} M_{V\_O}}{r}} \qquad \text{(keplerian law)}$$

and to the Doppler formula

$$\frac{\Delta\nu}{\nu} = \frac{v}{c_{V\_O}}$$

In order to explain both the flat rotation curve and the conservative spiral structure, V_O would then be forced to hypothesize the existence of the dark matter and to imagine a density wave theory (but in an erroneously manner, contrarily to us in our world where the physics constants are truly constant).

Assuming $\kappa = 2$, the test particle is no longer submitted to a inverse-square force law but to a radial harmonic force. However all bound orbits of stars are still closed (Bertrand's theorem). In this case the equations become

$$\ddot{r} - r\dot{\theta}^2 = \frac{\dot{r}^2}{r} - \frac{GM}{r^2}$$

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{\dot{c}}{c} r\dot{\theta}$$

The second equation of this system gives

$$r\ddot{\theta} + \dot{r}\dot{\theta} = 0$$

or

$$r\dot{\theta} = A = Cte$$

Replacing in the first one (and with $GM \propto r^3$), we obtain

$$\ddot{r} - \frac{A^2}{r} = \frac{\dot{r}^2}{r} - Br$$

where B is a second constant. For a circular motion $r = r_c$ and $\frac{A^2}{r_c} = Br_c$. For an elliptical motion, the equation can be rewritten (with $r(t) \to y(\tau)\,, \quad t = \frac{1}{\sqrt{B}}\tau$)

$$\ddot{y} - \frac{1}{y} = \frac{\dot{y}^2}{y} - y = 0 \qquad (16)$$

A study of this equation shows that a large range of elliptical orbits is possible. For instance with $y(0) = 1$, going from $y(0) = 0$ to $y(0) = 2$, the excentricity goes from 0 to 0.92.

Even though the value $\kappa = 2$ is assumed to be universal in the present model, the interval for $\kappa-$values comprised between 1 and 3 is still observationally acceptable. For $\kappa = 1$, v slightly increases as $r^{\frac{1}{4}}$ and $\frac{\Delta\kappa}{\nu}$ decreases as $r^{-\frac{1}{4}}$. For $\kappa = 3$, v increases as $r^{\frac{7}{4}}$ and $\frac{\Delta\nu}{\nu}$ slightly increases as $r^{\frac{1}{4}}$. In the latter situation, the strong increase of v with r must very rapidly lead to annular galaxies which are not common (Some cases of annular galaxies are however known in the Universe, even though this peculiar morphology is linked to the formation process, may be by collision of a small galaxy with a large disc-shaped galaxy).

6. The size of stars and planets in the V_Galaxy

The disc of the V_Galaxy has a radius of $r_G = 16\ kpc$ and a thickness of 0.3 kpc. The V_Sun is located at 8 kpc from the V_galactic center. The speed of light in the V_Sun environnement is $c_{V\_O} = 299,792.458\ m\ s^{-1}$ (as identically copied from our world). A law in r for the speed of light (f varying as r^{-2}) gives $c_{out} = 2c_{V\_O}$ and $G_{out} = 2^5 G_{V\_O}$ in the V_extragalactic region.

Let $\Delta r_{sol.syt} \simeq 1.5\ 10^{13}\ m$, the radius of the V_solar system, the (anisotropic) fractional shift of the speed of light is given by

$$\frac{\Delta c}{2c_{V\_O}} = \frac{\Delta r_{sol.syt}}{r_G} \simeq 3\ 10^{-8}$$

or

$$\Delta c = 17\ m\ s^{-1}$$

This is a very large difference indeed. In our world, the precision reached to date in the laboratory concerning speed of light measurements is $\frac{\Delta c}{c} = 4\ 10^{-18}$ (Nagel et al, 2015), even though the value of $299,792.458\ km\ s^{-1}$ is now adopted as an absolute reference for the speed of light since the General Conference of Weights And Measures, 1983 Oct 21. We can thus think that if V_O disposes of the same experimental techniques as us, implemented on two distant planets (V_Earth and V_Mars), he could "easily" measure this difference. In fact no, because his meter-stick would vary in the same manner throughout the V_solar system (by parallel transport of all physical constants). Likewise the techniques exposed in the paper of Nagel et al would be inoperative (In the V_Universe, the frequencies are assumed to be invariable for objects at rest). Unfortunately, there is no possibility to test the theory of V_O at the scale of the V_solar system.

But how to test the theory of V_O ? It suffices to him to measure the diameter D of a star of a given spectral type (a G-type star like our Sun for instance), located at a very large distance d[13] in the direction of the galactic center. The stellar radius (materialized length) is reduced by a factor

$$\Delta D = D - D_{VO} = D_{V\_O}\left(\frac{r_{\_V_O}}{r_{V\_O} + d} - 1\right) \simeq\ - D_{V_O}\frac{d}{r_{V\_O}} \tag{17}$$

and likewise in the anticenter direction (but with an inversed sign). With $D_{V\_O} = 1.4\ 10^8\ m$ and $d = 10\ pc$, we find $\left|\frac{\Delta D}{D_{V\_O}}\right| = 0.001$ (In fact in our world, only very large and very close stars can be imaged directly using speckle interferometry, but may be V_O possesses some technical devices much more performing than us, for instance a large network of telescopes distributed throughout his solar system).

7. The superluminal velocities

Another astonishing fact can also be mentioned. Highly energetic jets emanating from the core of active V_galaxies can now possess true velocities v more larger than c_{V_O}. For instance, we can observe $v = kc_{V\_O}$ with $k > 1$ (impossible in our world without a conflict with the Relativity). But in the V_Universe, the problem of faster-than-light motion does not arise while $kv_{V\_O} < c_{out}$, that is if $k < 2$ ($c_{out} = 2v_{V\_O}$). If, however, velocities larger than $2v_{V\_O}$ occur in the V_Universe, then some part of the phenomenon has to be again attributed to optical illusions. For instance the

[13]Measurements of distances in the V_Universe can be achieved in the same way that in the Universe. Given that a star with a well-defined spectral type has properties (Luminosity and temperature) which are universal in the V_Universe (see &1), the estimation of the apparent brightness of a calibrated star supplies the distance. A trigonometric method (parallax) can also be used. Another procedure will consist to evaluate the time of transit of a laser impulse from two remote points, but this method is technologically impractical.

highly collimated jet of matter, emerging from the core of the galaxy $M87$, is $6c$ (Biretta, Sparks, Macchetto, 1999).

Let us imagine now that a particle of such a jet reaches V_O. Is a velocity measurement contradicts the Relativity ? In fact we have seen above that $\frac{v}{c}$ is conservative for a free motion (neglecting gravity). The velocity of this particle, when arriving in the detector of V_O, is then $v = kv_{V\_O}\frac{c_{V\_O}}{c_{out}} < c_{V\_O}$ while $k < 2$ and the Relativity is saved.

In our world the totality of the superluminal motions seen in some radiogalaxies is attributed to optical illusions. The problem has been skillfully solved by relativistic considerations (Rees, 1966).

8. The cosmological implications

What are the consequences of depending-on-point physics constants for the expansion of the V_Universe ?

In the Universe, the expansion parameter $a(t)$ is given by the well known fundamental equation (see for instance : Peebles, 1993; Cheng, 2005)

$$\frac{\dot{a}^2}{a^2} - H_0^2\left(\Omega_{m0}\frac{a_0^3}{a^3} + \ \Omega_{r0}\frac{a_0^4}{a^4} + \Omega_V\right) = 0$$

with $\Omega_m = \frac{\rho_B + \rho_{DM}}{\rho_{0crit}}$ (B for baryonic and DM for dark matter), $\Omega_r = \frac{\rho_{rad}}{\rho_{0crit}}$, $\Omega_\Lambda = \frac{\rho_V}{\rho_{0crit}}$ (At the present time $t = 0$ and $\Omega_{m0} + \ \Omega_{r0} + \Omega_V = 1$). The numerical constants are given in the following table (issued from the Planck 2015 results. XIII)

Age de l'Univers T	$13.813 \pm 0.038 \quad Gyears$
Hubble constant H_0	$67.31 \pm 0.96 \ \ km/s/Mpc$
Baryonic density	0.02222 ± 0.00023
dark matter density	0.1197 ± 0.0022
Radiation density Ω_{r0}	$(8.4 \pm 0.5) \ \times \ 10^{-5}$
Ω_Λ	0.685 ± 0.013

In the V_Universe, the cosmology is very similar to that of the Universe, but simply with another parametrization, let $c_{V\_O} \ \to c_{out} = 2c_{V\_O}$, $G_{V\_O} \to G_{out} = 2^5 G_{V\_O}$ and $\rho_{0crit,V\_O} \to \rho_{0crit,out} =$

$2^{-2}\rho_{0crit,V\_O}$[14]. We have thus

$$\frac{\dot{a}^2}{a^2} - \frac{8\pi G_{out}}{3}\left(\rho_{0m,out}\frac{a_0^3}{a^3} + \rho_{0r,out}\frac{a_0^4}{a^4} + \ \rho_{V,out}\right) = 0$$

or

$$\frac{\dot{a}^2}{a^2} - H_{0,V\_O}^2\left(\frac{8\rho_{0m,V\_O}}{\rho_{0crit,V\_O}}\frac{a_0^3}{a^3} + \frac{8\rho_{0r,V\_O}}{\rho_{0crit,V\_O}}\frac{a_0^4}{a^4} + \frac{8\rho_{V,V\_O}}{\rho_{0crit,V\_O}}\right) = 0$$

with $H_{0,V\_O}^2 = \frac{8\pi}{3}\, G_{V\_O}\rho_{0crit,V\_O}$. Once again the dark matter is not needed (From the Table above, we deduce that the proportion $\frac{\rho_B}{\rho_B+\rho_{DM}} \sim 6.39$). We obtain

$$\frac{\dot{a}^2}{a^2} - H_{0,V\_O}^2\left(\frac{6.39\left(\frac{8}{6.39}\rho_{0B,V\_O}\right)}{\rho_{0crit,V\_O}}\frac{a_0^3}{a^3} + \frac{8\rho_{0r,V\_O}}{\rho_{0crit,V\_O}}\frac{a_0^4}{a^4} + \frac{8\rho_{V,V\_O}}{\rho_{0crit,V\_O}}\right) = 0$$

In order that V_O measures the same numerical value for the Hubble constant that us, we must put $H_{0,V\_O} = H_0$. On the other hand $\rho_{0B,V\_O} = 0.80\ \rho_{0B}$. Eventually, we can write

$$\frac{\dot{a}^2}{a^2} - H_0^2\left(\frac{6.39(1.25\rho_{0B,V\_O})}{\rho_{0crit}}\frac{a_0^3}{a^3} + \frac{(8\rho_{0r,V\_O})}{\rho_{0crit}}\frac{a_0^4}{a^4} + \frac{(8\rho_{V,V\_O})}{\rho_{0crit}}\right) = 0$$

We find $\rho_{0B,V\_O} = 0.8\rho_{0B}$. A clear observational difference with the Universe is found for the radiation field. The radiative density energy ($u = \frac{4\sigma}{c}T^4$) has to be divided by 8 in the V_Universe. The Cosmic Microwave Background temperature is then $T_{0,out} = T_{0,V\_O} = 6.39^{-\frac{1}{4}}\ \times 2.7 = 1.7\ K$. Finally the vacuum energy density is divided by a factor 8 in the V_Universe. Another significant difference with our world is the radius of the observable domain in the V_Universe twice of ours ($\frac{c_{out}}{H_0} = 2\frac{c_{V\_O}}{H_0}$).

9. Conclusion

We have succeeded in building a virtual supporting-live universe which looks like ours under the very restrictive conditions that the local laws of Physics are fully the same (with identical adimensional physical constants) and that the light propagates in straight line. The difference is for the dimensional physical constants which are assumed to be variable at larger scales (the reference being the mean size of a galaxy). It follows that in order to reconcile depending-on-point dimensional physical constants and both propagation of the light in straight line and rectilinear motion for free particles, a dual counterpart must be associated to this universe. The depending-on-point variation of dimensional physical constants is furthermore specified by an unique scale

[14] At a very large scale, outside the galaxies, the extragalactic medium is assumed to be homogeneous and the physical constants are again independent of the point.

factor. With an adjusted parametrization of the latter quantity, plenty of observational facts are then found to be identical, in appearance, to those observed in the Universe. In some sense, this universe is a kind of plausible (or may be actual) copy of ours in the Multiverse. The interpretation of phenomena is however distinct. In the virtual universe, contrarily to ours, there are no dark matter, no winding galaxy problem and no need of density wave theory. Besides, true superluminal velocities for particles are possible without conflict with the Relativity. But some clear differences are also existing between both the Universe and the virtual universe. First the baryonic density is a little bit smaller in the latter one. The temperature of the Cosmic Microwave Background is found to be equal to 1.7 K against $2.7K$ in ours. The vacuum energy is drastically reduced by a factor 8. Eventually the Hubble radius is twice that of the Universe. It will be interesting to go further in the present analysis, and especially to propose a scenario for the formation process and the evolution of galaxies, in this universe by comparison with ours.

10. Bibliography

Barnes, L, 2012, Pub. Astr.Soc. of Australia, **29**, 529

Barrow J.D., 2002, The Constants of Nature; From Alpha to Omega - The Numbers that Encode the Deepest Secrets of the Universe, Pantheon Books

Biretta, J. A., Sparks, W. B., Macchetto, F., 1999, ApJ, **520** , 621

Bondi H., Samuel J., 1997, Phys. Lett., **A228**, 3121

Bouvier, Wadhwa, 2010, Nature Geoscience 3, **637**

Bovy, J., Rix H.W. 2013, ApJ., **779**, 115

Brownstein J.R., Moffat J. W., 2007, Monthly Notices of the Royal Astronomical Society, **382**, 29

Carr B.J., Rees M.J. 1979, Nature, **278**, 605

Chand H., Srianand R., Petitjean P., Aracil B., 2004, A & A, **417** , 853

Cheng T.P., 2004, Relativity, Gravitation and Cosmology, Oxford University Press

Ellis G.F. R.,2011, Scientific_American,**305**, 38

Guth A.H., Kaiser D.I., Nomura, 2014, Y. Phys. Lett., **B733**, 112

Ijjas A., P. J. Steinhardt P.J., Loeb A., Phys. Lett., 2013, **B723**, 261

Landau D., Lifshitz, E. 1984, Electrodynamics of continuous media, Pergamon Press, p.34

Lebedev L.P.; Cloud M.J., Eremeyev V.A., 2010, Tensor Analysis With Applications to Mechanics. World Scientific

Maldacena J., 2005, Scientific American, **293**, 56

Milgrom M, 1983, ApJ, **270**, 365

Moffat J. W., 2006, Journal of Cosmology and Astroparticle Physics **3**, 4

Nagel M., Parker S.R., Kovalchuck E.V., Stanwix P.L., Hartnett J.G., Ivanov E.N., Peters A., Tobar M.E., 2015, Nature Communications, **6**, 8174

Ostriker, J. P., Mitton, S., 2013, Heart of Darkness: Unraveling the mysteries of the invisible universe. Princeton, NJ: Princeton University Press

Peebles J., 1993, Principles of Physical Cosmology, Princeton Series Press

Pfister H., King M., 2015, Inertia and Gravitation, The fundamental Nature and space-time. The lecture notes in Physics Heidelberg, vol. 897

Phillipps S., 2005, The Structure & Evolution of Galaxies, Wiley, p.132

Planck Collaboration, 2016,A&A, **587**

Randall, L., 2005, Warped Passages: Unraveling the Mysteries of the Universe's Hidden Dimensions. New York: HarperCollins

Rees M.J., 1966, Nature, **211**, 468.

Shifman, M., 2010, International Journal of Modern Physics A, **25**,199.

Srianand R., Chand H., Petitjean P., Aracil B., 2004, Phys._Rev._Lett. **92** , 121302

Stenger V., 2011, The Fallacy of Fine-Tuning: Why the Universe is Not Designed for Us, Prometheus Books.

Uzan J.P., "Varying Constants, Gravitation and Cosmology", 2011, Living Rev. Relativity, **14**, 2.

Yasunori F., 2004,"Oklo Constraint on the Time-Variability of the Fine-Structure Constant". Astrophysics, Clocks and Fundamental Constants. Lecture Notes in Physics.Berlin, p.167

Weinberg S., 2009, Universe or Multiverse?, Cambridge University Press, ed. B. Carr