Can a Complex Scalar Field Accelerate The Universe?

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Abstract

Considerable work has been done by taking ϕ as a real scalar field in cosmology. We present here a five-dimensional cosmological model with a complex scalar field ϕ and a constant deceleration parameter q. The physical significance of ϕ is provided by its modulus. Exact solutions of the field equations are obtained. The decay of the extra dimension with the evolution of the universe for a five- dimensional model is exhibited. The physical properties of the model are examined.

Keywords: Higher dimensions, Chaplygin gas, Scalar field, Accelerated Universe, Deceleration parameter.

1.Introduction

Observational evidences point towards an accelerated expansion of the universe. Researches are going on for finding out the origin of accelerated expansion of the present universe. The dark energy occupies about 73% of the energy of our universe, while dark matter occupies about 23% and the usual baryonic matter 4%.The dark energy is responsible for cosmic acceleration. The present day universe in which we live is four-dimensional. Attempts are being made by a section of workers to recast the theory of relativity in a higher-dimensional space-time. The existence of extra dimensions is necessary in any attempt to unify gravity with other forces of nature. Also modern developments of superstring theory and Yang-Mills super-gravity in its field theory limit need higher dimensional space-time. In recent years there has been considerable interest in theories with higher- dimensional space-time, in which extra dimensions are eventually to a very small size, contracted apparently beyond our ability for measurement. A model of higher dimensions was proposed by Kaluza and Klein [2, 3] who tried to unify gravity with electromagnetic interaction by introducing an extra dimension which is an extension of Einstein's General Relativity in five dimensions. The activities in extra dimensions also stem from the Space-Time-Matter (STM) theory proposed by Wesson et al. [4]. In recent years, a number of authors [5, 6, 7, and 8] have considered multidimensional cosmological models. Also many more authors have studied the Kaluza-Klein inhomogeneous cosmological models with and without cosmological constant. A scalar field ϕ with a potential $V(\phi)$ which is known as quintessence and decreases slowly with time, may be another candidate for dark energy. Quintessence exerts negative pressure and it (quintessence) is a slowly rolling scalar field ϕ and has potential $V(\phi)$ [9]. It has been a natural choice to try to understand the present acceleration of the universe by also using scalar fields [10, 11]. A complex scalar field is not an unfamiliar idea in physical science. The wave function, which is an essential concept in quantum mechanics, is a complex quantity subject to the interpretation that its physical significance is by its modulus.

Another type of dark energy, the so-called pure chaplygin gas model which obeys an equation of state like [12] $p=-\frac{A}{\rho}$,where A is a positive constant, p and ρ respectively the pressure and density of the fluid can be taken for study as it possesses the negative pressure. The above equation was modified to the form $p=-\frac{A}{\rho^{\delta}}$ with $0\le\delta\le1$. This model has been studied previously by V.Gorini et.al and M.C Bento et.al [13, 14]. Some further work has been done on modified chaplygin gas obeying an equation of state [15, 16] $p = \gamma\rho - \frac{A}{\rho^{\delta}}$ with $0 \le \delta \le 1$ where γ and A are positive constants.

In section 2, we present the field equations for the five-dimensional spatially flat, homogeneous and anisotropic cosmological model. In section 3 of this paper we obtain the solution of the field equations by assuming a relation between the scale factors a (t) and b(t). In section 4, we study the physical character of the model. Lastly the paper concludes with a short discussion in section 5.

2. The Field Equations and The Cosmological Dynamics of The Scalar Field Dark Energy Model

We consider a spatially-flat, homogeneous and anisotropic five dimensional space-time model described by the line-element

$$ds^2 = dt^2 - a^2(t)[dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\chi^2] - b^2(t)dy^2 \qquad (1)$$

where $a(t)$ and $b(t)$ being functions of time represent the scale factors of the four-dimensional FRW cosmological space-time and the extra dimension respectively.

We consider that the universe is filled with scalar field ϕ having potential $V(\varphi)$ and normal matter. The Einstein field equations are given by

$$R_{ij} - \frac{1}{2} g_{ij} R = T^{(M)}{}_{ij} + T^{(\phi)}{}_{ij} \qquad (2)$$

where $8\pi G = c = 1$ and R_{ij} are the Ricci tensors and $T^{(M)}{}_{ij}, T^{(\phi)}{}_{ij}$ are the energy-momentum tensors for normal matter and scalar field respectively and

$$T^{(M)}{}_{00} = \rho_m,\ T^{(M)}{}_{11} = T^{(M)}{}_{22} = T^{(M)}{}_{33} = T^{(M)}{}_{44} = -p_m \qquad (3)$$

$$T^{(\phi)}{}_{ij} = \phi_{,i}\,\phi^*{}_{,j} - g_{ij}[\frac{1}{2} g^{kl} \phi_{,k}\,\phi^*{}_{,l} - V(|\varphi|)] \qquad (4)$$

where ϕ^* denotes the conjugate complex of ϕ.

The Einstein field equations for the metric (1) are

$$3\frac{\dot{a}^2}{a^2}+3\frac{\dot{a}\dot{b}}{ab}=\rho_m+\rho_\phi \tag{5}$$

$$2\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}+2\frac{\dot{a}\dot{b}}{ab}+\frac{\ddot{b}}{b}=-p_m-p_\phi \tag{6}$$

$$3\frac{\ddot{a}}{a}+3\frac{\dot{a}^2}{a^2}=-p_m-p_\phi \tag{7}$$

where dot denotes time-derivatives and p_ϕ and ρ_ϕ represents the pressure and density of the scalar field and p_m and ρ_m denotes the pressure and energy density of the ordinary matter respectively. p_ϕ and ρ_ϕ are given by

$$p_\phi=\frac{|\dot{\varphi}|^2}{2}-V(|\varphi|) \tag{8}$$

$$\rho_\phi=\frac{|\dot{\varphi}|^2}{2}+V(|\varphi|) \tag{9}$$

In this case we assume that there is no interaction between scalar field and normal matter, hence they are separately conserved.

The energy conservation equation for normal matter

$$\dot{\rho}_m+\left(3\frac{\dot{a}}{a}+\frac{\dot{b}}{b}\right)(p_m+\rho_m)=0 \tag{10}$$

Also the evolution equation for scalar field is

$$|\ddot{\varphi}| + 3\frac{\dot{a}}{a}|\dot{\varphi}| + \frac{\dot{b}}{b}|\dot{\varphi}| + V^{/}(|\varphi|) = 0 \qquad (11)$$

where dash denotes derivatives with respect to $|\varphi|$.

3. Solutions of Field Equations

The field equations (5)-(7) are a system of three equations with four unknown parameters $a, b, p_m, and \rho_m$ .Therefore to obtain exact solutions of the field equations we need one more equation. The 5th dimension dies away as the physical four dimensions evolve. Hence a choice has to be adopted keeping this fact in view. We assume that the relation between the metric coefficients reads as

$$b = k_1 a^{\eta} \qquad (12)$$

where k_1 and η are constants.

Substituting equation (7) into equation (6) and using equation (12) we obtain

$$\frac{\ddot{a}}{a} + (2+\eta)\frac{\dot{a}^2}{a^2} = 0 \qquad (13)$$

Integrating equation (13) we get

$$a = Ct^{\frac{1}{3+\eta}} \qquad (14)$$

where C is the constant of integration and $-3 < \eta$.

For $-3 < \eta < -2.5$, $\dot{a} > 0$ and $\ddot{a} > 0$ i.e. the universe accelerates.

From equation (12) and (14) we have

$$b = k_2 t^{\frac{\eta}{3+\eta}} \tag{15}$$

where $k_2 = k_1 C^{\eta}$

We define H and the deceleration parameter q in terms of 4 dimensions as they only are relevant to cosmological observations.

Then Hubble parameter H for 4-dimensional model is given by

$$H = \frac{\dot{a}}{a} = \frac{1}{(3+\eta)t} \tag{16}$$

The 4D deceleration parameter q is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = 2 + \eta = \text{constant} \tag{17}$$

4. Physical Nature of The Model

The scale factor or the cosmic scale factor of the Friedmann equation is a function of time which represents the relative expansion of the universe. The evolution of scale factor is a dynamical question, determined by the equation of general relativity, which are presented in the case of a locally isotropic, locally homogeneous universe by the Friedmann equations.

From equation (14) we see that the scale factor $a(t)$ and $\dot{a}(t)$ increases with the increase in cosmic time t as shown in the Fig. 1 and Fig.2 respectively

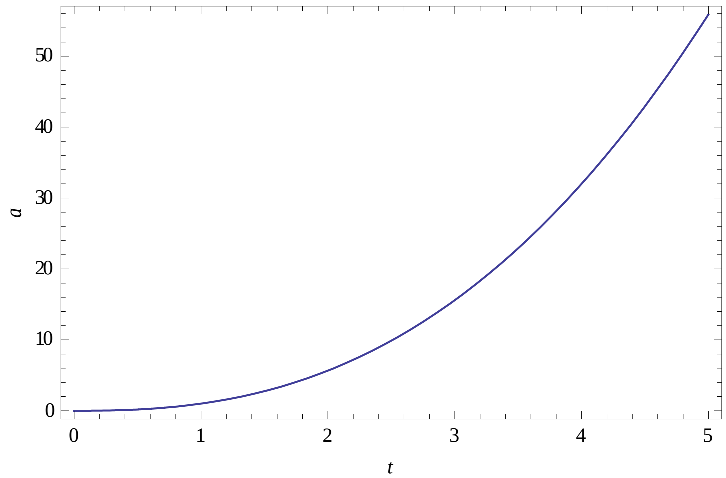

Fig. 1

Fig. 1 Variation of scale factor $a(t)$ with cosmic time t for $C = 1$, $\eta = -2.6$.

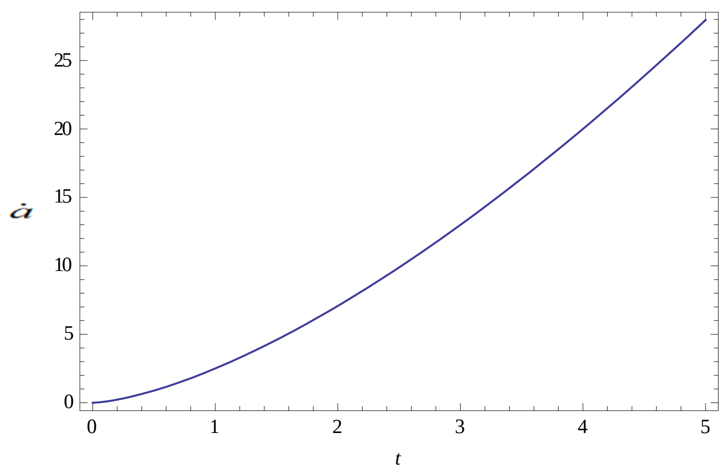

Fig.2

Fig.2 Variation of $\dot{a}(t) = \frac{da}{dt}$ with time t.

From fig.2 it is found that $\dot{a}(t)$ increases with the increases in cosmic time t, $C = 1$, $\eta = -2.6$.

From equation (16) we observe that the Hubble parameter H decreases as the cosmic time t increases as shown in the Fig.3

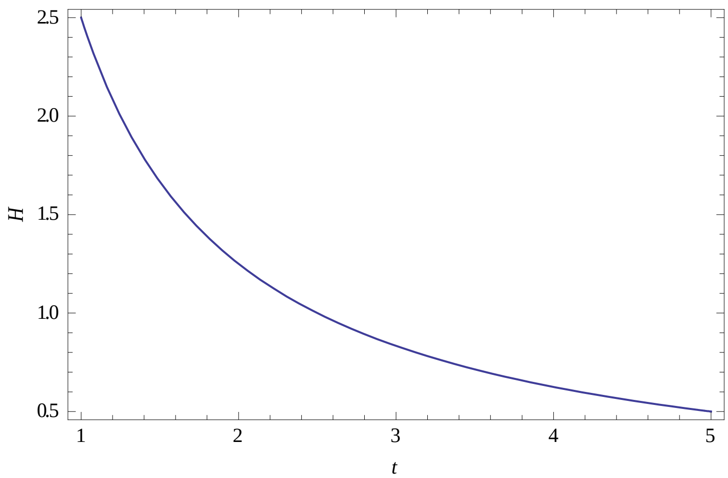

Fig.3

Fig.3 Variation of Hubble parameter H with cosmic time t for $\eta = -2.6$.

From equation (16) we see that the scale factor $b(t)$ decreases with cosmic time t as shown in Fig. 4.

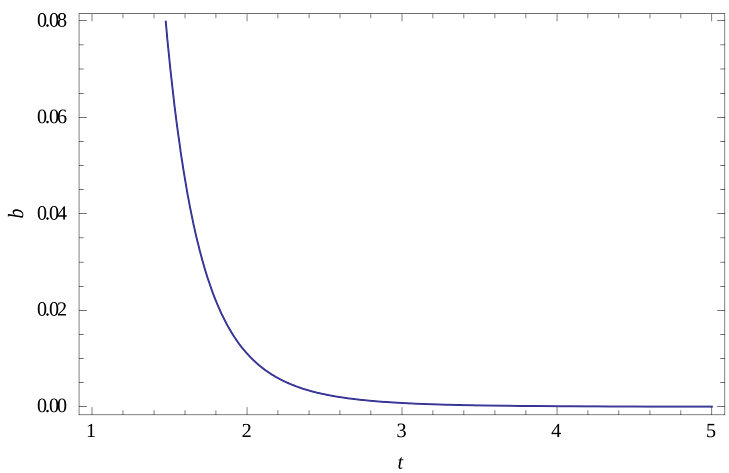

Fig.4

Fig. 4 Variation of scale factor $b(t)$ with cosmic time t for $\eta = -2.6$ and $k_2 = 1$

The barotropic equation of state for normal matter is

$$p_m = \omega \rho_m \tag{18}$$

Substituting equation (12) and equation (18) in equation (10) and integrating we obtain

$$\rho_m = \rho_0 a^{-(i+\omega)(3+\eta)} \tag{19}$$

ρ_0 being positive constant

Substituting equation (12), (14), (18) and (19) in equations (5) and (6) or (7) and solving these equations we obtain

$$\dot{\phi} = \left(6\frac{(1+\eta)}{(3+\eta)^2} t^{-2} - (1+\omega)\rho_1 t^{-(1+\omega)} \right)^{\frac{1}{2}} \tag{20}$$

where $\rho_1 = \rho_0 (C)^{-(1+\omega)(3+\eta)} > 0$

For $-3 < \eta < -2.5$, while the universe accelerates $(\because \dot{a} > 0, \ddot{a} > 0)$, the scalar field φ becomes complex. The physical significance of φ is provided by its modulus. We have taken $|\varphi|$ for accelerating universe.

From equation (20) we see that $|\varphi|$ increases with cosmic time t, which is shown in Fig.5.

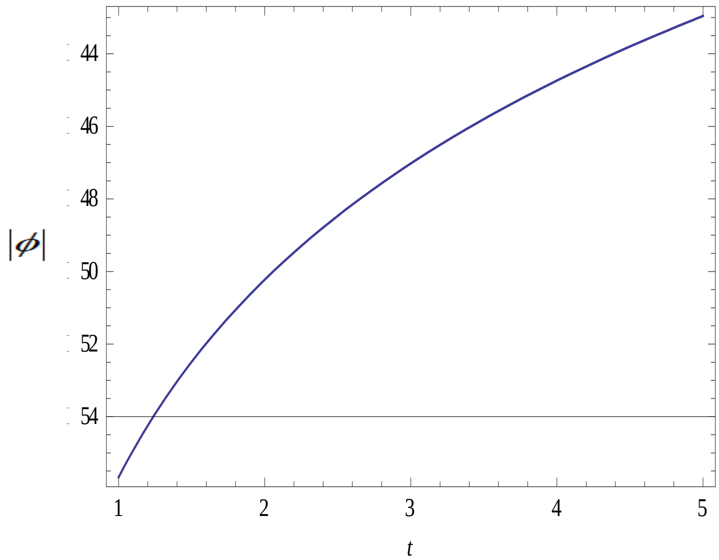

Fig.5

Fig.5.shows the variation of $|\varphi|$ against cosmic time t for barotropic fluid by taking

$\eta = -2.6,\ \omega = \frac{1}{3}, \rho_1 = 0.2$

and

$$V(|\varphi|) = \frac{1}{2}(\omega - 1)\rho_1 \frac{1}{t^{(1+\omega)}} \tag{21}$$

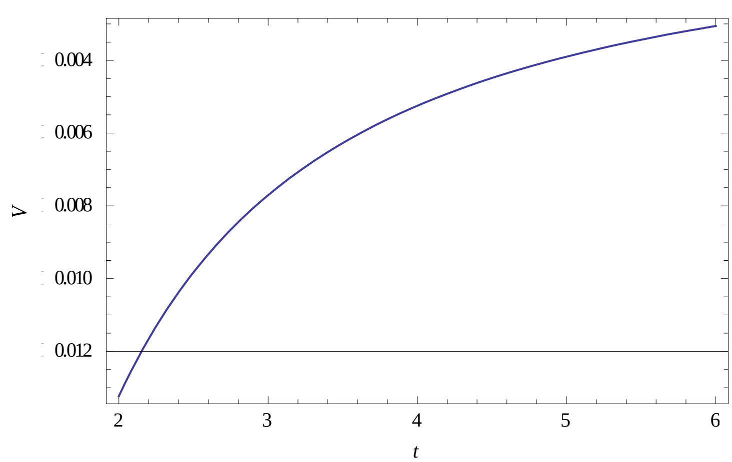

Fig.6

Fig.6. shows the variation of $V(|\varphi|)$ against cosmic time t for barotropic fluid by choosing $\omega = \frac{1}{3}, \rho_1 = 0.2$

We take potential as a function of $|\varphi|$.

From equations (20) and (21), it is seen that $V(|\varphi|)$ cannot be expressed in terms of $|\varphi|$ explicitly. For physical investigation, we have plotted $V(|\varphi|)$ against $|\varphi|$ for some particular values of arbitrary constants $\eta = -2.6,\ \omega = \frac{1}{3}, \rho_1 = 0.2$ in Fig.7. This figure shows the variation of $V(|\varphi|)$ with $|\varphi|$.

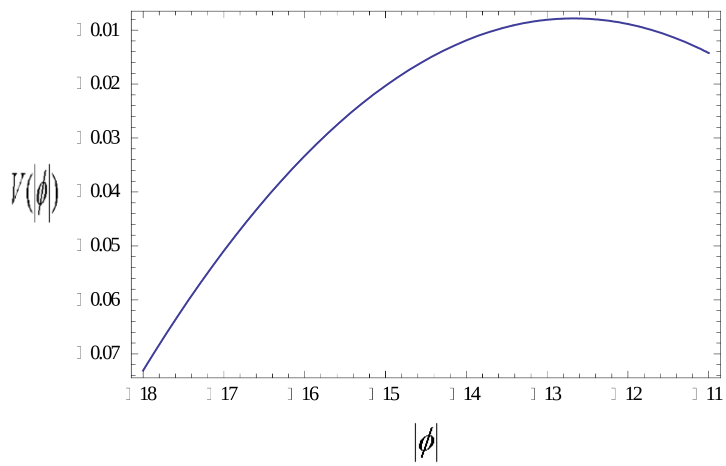

Fig.7

Fig.7 shows the variation of $V(|\varphi|)$ with $|\varphi|$ for barotropic fluid using the constants $\eta = -2.6,\ \omega = \frac{1}{3}, \rho_1 = 0.2$

From equations (14) and (20) we obtain the graphical representation of $|\varphi|$ with the scale factor $a(t)$.

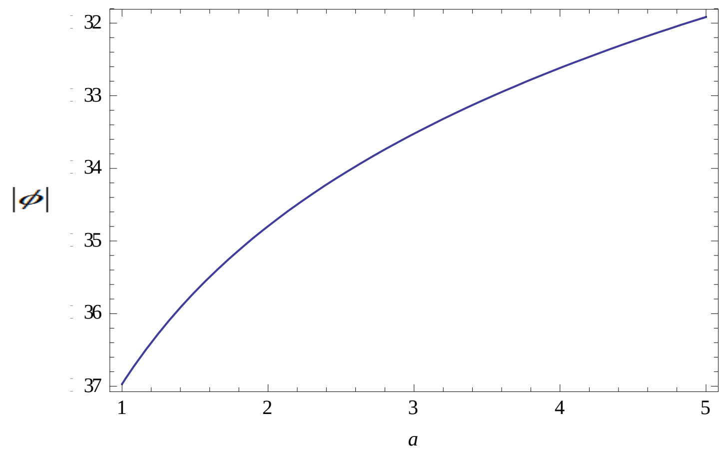

Fig.8

Fig.8 shows the variation of $|\varphi|$ with scale factor $a(t)$ for barotropic fluid by choosing $\eta = -2.6,\ \omega = \frac{1}{3}, \rho_1 = 0.2$. It is observed that $|\varphi|$ increases with the increase in the scale factor a(t) .

The equation of state for modified chaplygin gas [15, 16] is given by

$$p_m = \gamma\rho_m - \frac{A}{\rho_m{}^{\delta}}, \quad 0 \le \delta \le 1 \tag{22}$$

where γ and A are positive constants.

Then from (10) we get the expression for energy density as

$$\rho_m = [H_1 a^{-(1+\gamma)(3+\eta)(\delta+1)} + F]^{\frac{1}{\delta+1}} \tag{23}$$

where $H_1(>0)$ and $F = \frac{A}{1+\gamma} > 0$ are constants.

Substituting equations (12), (14), (22) and (23) in equation (5) and (6) or (7) and solving these equations we obtain

$$\dot{\phi} = \left(6\frac{(1+\eta)}{(3+\eta)^2} t^{-2} - (1+\omega)\{H_2 t^{-(1+\gamma)(1+\delta)} + F\}^{\frac{1}{\delta+1}} \right)^{\frac{1}{2}} \tag{24}$$

where $H_2 = \dfrac{H_1}{C^{(1+\gamma)(1+\delta)}} > 0$

For $-3 < \eta < -2.5$, while the universe accelerates $\left(\because \dot{a} > 0, \ddot{a} > 0\right)$, the scalar field φ becomes complex. We have taken modulus of φ ,i.e. $|\varphi|$ for its physical significance. From equation (24) we see that $|\varphi|$ increases with cosmic time t, which is shown in Fig.9.

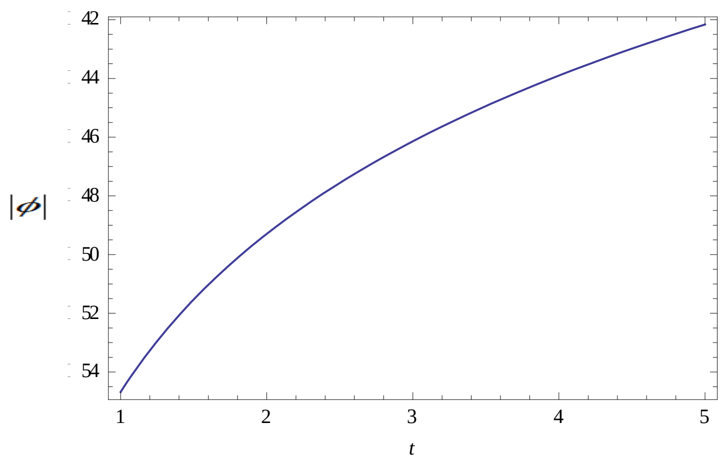

Fig.9

Fig.9 shows the variation of $|\varphi|$ against cosmic time t for modified chaplygin gas model by taking $\eta = -2.6, \omega = \dfrac{1}{3}, F = 0.1, H_2 = 0.2 \quad \delta = 0.1, \gamma = 0.2$

It is observed that $|\varphi|$ increases with time t.

and

$$V(|\varphi|) = \frac{1}{2}(\omega - 1)\left(H_2\, t^{-(1+\gamma)(\delta+1)} + F\right)^{\frac{1}{\delta+1}} \tag{25}$$

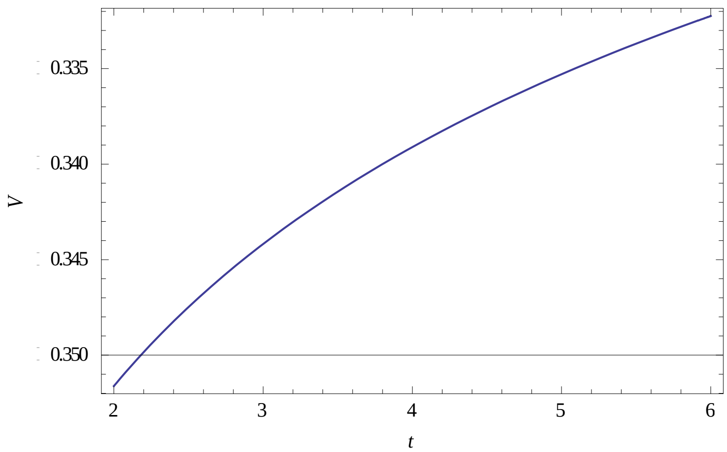

Fig.10

Fig.10 shows the variation of $V(|\varphi|)$ against cosmic time t for modified chaplygjn gas model by taking $\eta = -2.6, \omega = \frac{1}{3}, \ F = 0.1, \ H_2 = 0.2 \quad \delta = 0.1, \ \gamma = 0.2$

From equations (24) and (25), it is seen that $V(|\varphi|)$ cannot be expressed in terms of $|\varphi|$ explicitly. Graphically we have plotted $V(|\varphi|)$ against scalar field $|\varphi|$ for some particular values of arbitrary constants $\eta = -2.6, \ \omega = \frac{1}{3}, \ F = 0.1, \ H_2 = 0.2, \ \delta = 0.1, \ \gamma = 0.2$ in Fig.11.

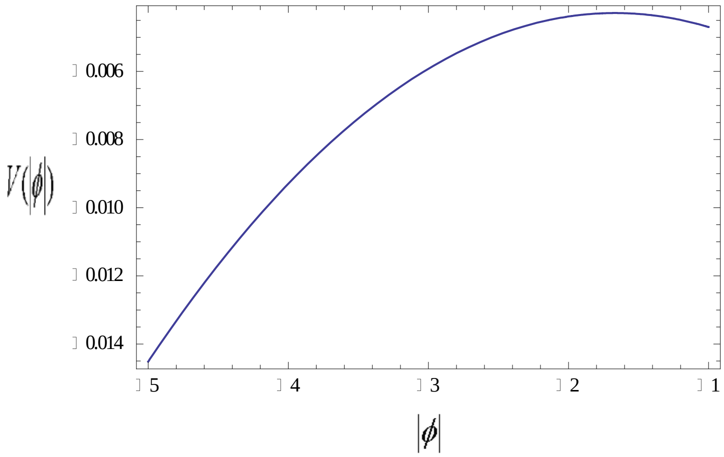

Fig.11

Fig.11 shows that the variation of $V(|\varphi|)$ against $|\varphi|$ for modified chaplygin gas by choosing the constants $\eta = -2.6, \omega = \frac{1}{3}$, $F = 0.1$, $H_2 = 0.2$, $\delta = 0.1$, $\gamma = 0.2$

Also from equation (14) and (24) we obtain the graphical representation of $|\varphi|$ with the scale factor $a(t)$

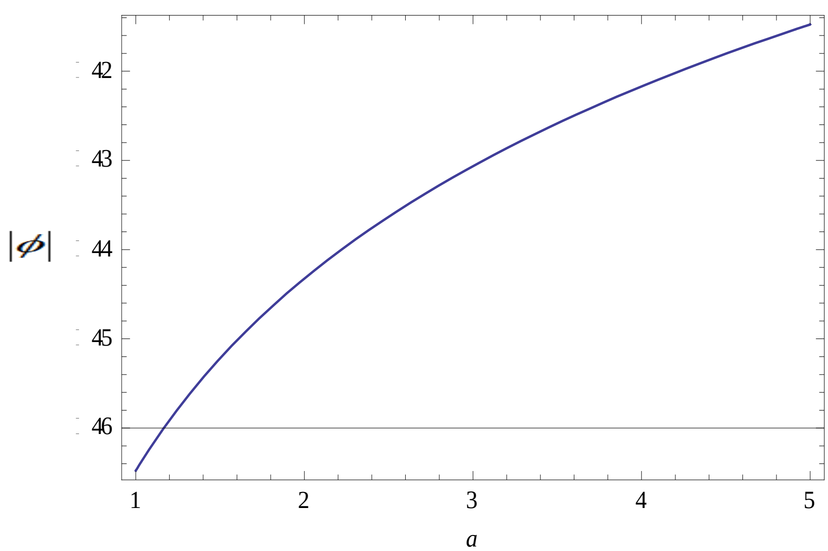

Fig.12

Fig.12 shows the variation of $|\varphi|$ with the scale factor $a(t)$ for modified chaplygin gas by taking $\eta = -2.6, \omega = \frac{1}{3}, F = 0.1, H_2 = 0.2, \delta = 0.1, \gamma = 0.2$. It is observed that $|\varphi|$ increases with the increase in the scale factor $a(t)$.

5. Discussions

We have presented (4+1) dimensional Einstein field equations where 4-dimensional space-time is described by FRW metric and that of extra dimension by a Euclidean metric. This model is meant for $k = 0$ (flat). Also we have considered the anisotropic model of the universe filled with normal matter and scalar field. For $-3 < \eta < -2.5$, from equation (14) and (15) we see that the scale factors $a(t)$ increases while $b(t)$ decreases and q, the deceleration parameter becomes negative. Thus we see that the extra dimension becomes insignificant as the time proceeds after the creation and we are left with the real four-dimensional world. We have two types of fluids viz.barotropic and chaplygian. For the sake of simplicity, the fluids are taken to be non-interacting. We intend to consider interacting fluids in a future communication.

A complex scalar field is not an unfamiliar idea in physical science. The wave function, which is an essential concept in quantum mechanics, is a complex quantity subject to the interpretation that its physical significance is given by its modulus. Here we have found that the scalar field $|\varphi|$ increases with the increase in the scale factor $a(t)$. Also 4D Hubble parameter $H(t)$ decreases with the increase in cosmic time t and the scale factor $a(t)$ increases with the increase in cosmic time t and with the increase in the scalar field $|\varphi|$, which is the scenario of accelerating universe.

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