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Observational Evidence Favors a Static Universe
Part 3

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ABSTRACT

This is the third part a three part report. It covers the complete model for curvature cosmology and includes the topics of entropy, Olber's paradox, black holes, astrophysical jets and large-number coincidences that are particularly relevant for curvature cosmology but but are not decisive in testing cosmologies. Curvature pressure can explain the de ciency of solar neutrinos and curvature redshift can explain the anomalous acceleration of Pioneer 10.The preceding report (Part 2) covered the topics: X-ray background radiation, cosmic background microwave radiation, dark matter, Sunyaev–Zel'dovich effect, gravitational lensing, Lyman- forest, nuclear abundances, galactic rotation curves, redshifts in our Galaxy, anomalous redshifts and voids. An analysis of the best raw data for these topics shows that, in general, they are consistent with both Big Bang cosmology and curvature cosmology. Whereas the conclusions in Part 1 would be valid for any reasonable static cosmology the analysis in Part 2 requires specific characteristics of curvature cosmology. Part 3, presents and covers the complete model.

Keywords: Big Bang, Infinite Universe, Steady State Universe, Static Universe, Expanding Universe

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1 Introduction

Part 3, provides a complete description of CC and its two major hypotheses: curvature redshift and curvature pressure. Although it is not a new idea it is argued that gravitation is an acceleration and not a force. This idea is used to justify the averaging of accelerations rather than forces in deriving curvature pressure.

The next section (Section 3) includes the topics of entropy, Olber's paradox, black holes, astrophysical jets and large-number coincidences that are particularly relevant for CC but are not important for choosing between BB and CC.

Although the explanation for the deficiency in observed neutrinos from the sun can be explained by neutrino oscillations it is include here because curvature pressure makes excellent estimates of the expected numbers without any free parameters. The heating of the solar corona is a very old problem and still not fully explained. It is treated here simply to show that curvature redshift offers no help.

Finally it is shown the *Pioneer* 10 anomalous acceleration can be explained by the effects of curvature redshift that is produced by interplanetary dust provided the density of the dust is a little higher than current estimates.

2 Curvature Cosmology Theory

Curvature cosmology (CC) is a static tired-light cosmology where the Hubble redshift (and many other redshifts) is produced by an interaction of photons with curved spacetime called curvature redshift. It is a static solution to the equation of general relativity that is described by the Friedmann equations with an additional term that stabilizes the solution. This term called curvature pressure is a reaction of high speed particles back on the material producing the curved spacetime. This sense of this reaction is to try and reduce the curvature. The basic cosmological model is one in which the cosmic gas dominates the mass distribution and hence the curvature of spacetime. In this first order model, the gravitational effects of galaxies are neglected. The geometry of this CC is that of a three-dimensional surface of a four-dimensional hypersphere. It is almost identical to that for Einstein's static universe. For a static universe, there is no ambiguity in the definition of distances and times. One can use a universal cosmic time and define distances in light travel times or any other convenient measure. In a statistical sense CC obeys the perfect cosmological principle of being the same at all places and at all times.

CC makes quite specific predictions that can be refuted. Thus, any observations that unambiguously show changes in the universe with redshift would invalidate CC. In CC, there is a continuous process in which some of the cosmic gas will aggregate to form galaxies and then stars. The galaxies and stars will evolve and eventually all their material will be returned to the cosmic gas. Thus, a characteristic of CC is that although individual galaxies will be born, live and die, the overall population will be statistically the same for any observable characteristic.

This paper is the culmination of many years of work and is a complete re-synthesis of many approaches that I have already published. Because hypotheses and notations have changed and evolved, direct references to these earlier versions of the theory would be misleading. Table 1 (all with author D. F. Crawford) is provided briefly stating each reference and the major topic in each paper. In nearly all cases, the data analyzed in the papers has been superseded by the more recent data that are analyzed in this paper.

2.1 Derivation of curvature redshift

The derivation of curvature redshift is based on the fundamental hypothesis of Einstein's general theory of relativity that spacetime is curved. As a consequence, the trajectories of initially-parallel point particles, geodesics, will move closer to each other as time increases. Consequently in space with a positive curvature, the cross sectional area of a bundle of geodesics will slowly decrease. In applying this idea to photons, we assume that a photon is described in quantum mechanics as a localized wave where the geodesics correspond to the rays of the wave. Note that this wave is quite separate from an electromagnetic wave that corresponds to the effects of many photons. It is fundamental to the hypothesis that we can consider the motion in spacetime of individual photons. Because the curvature of spacetime causes the focussing of a bundle of geodesics, this focussing also applies to a wave. As the photon progresses, the cross sectional area of the wave associated with it will decrease. However, in quantum mechanics properties such as angular momentum are computed by an integration of a radial coordinate over the volume of the wave. If the cross sectional area of the wave decreases, then the angular momentum will also decrease. However, an-

Table 1: Published papers

Year	Reference	Major topic
1975	Nature, 254, 313	First mention of photon extent and gravity
1979	Nature, 277, 633	Photon decay near the sun: limb effect[a]
1987	Aust. J. Phys.40, 440	First mention of curvature redshift[b]
1987	Aust. J. Phys., 40, 459	Application to background X-rays
1991	Astrophysical Journal, 377, 1	More on curvature redshift and applications
1993	Astrophysical Journal, 410, 488	A static stable universe: Newtonian cosmology
1995	Astrophysical Journal, 440, 466	Angular size of radio sources
1995	Astrophysical Journal, 441, 488	Quasar distribution
1999	Aust. J. Phys., 52, 753	Curvature pressure and many other topics
2006	Book (Crawford, 2006)	"Curvature Cosmology"
2008	Web site [c]	Major update[d] of the book

[a]Not only is the theory discredited but also the observations have not stood the test of time.

[b]This gives the equation for photons but not for non-zero rest mass particles.

[c]http://www.davidcrawford.bigpondhosting.com

[d]Superseded by this paper.

gular momentum is a quantized parameter that has a fixed value. The solution to this dilemma is that the photon splits into two very low-energy photons and a third that has the same direction as the original photon and nearly all the energy. It is convenient to consider the interaction as a primary photon losing a small amount of energy into two secondary photons. Averaged over many photons this energy loss will be perceived as a small decrease in frequency. Since in quantum mechanics electrons and other particles are considered as waves, a similar process will also apply. It is argued that electrons will interact with curved spacetime to lose energy by the emission of very low-energy photons.

2.1.1 Photons in Curved Spacetime

Einstein's general theory of relativity requires that the metric of spacetime be determined by the distribution of mass (and energy). In general this spacetime will be curved such that in a space of positive curvature nearby geodesics that are initially parallel will come closer together as the reference position moves along them. This is directly analogous to the fact that on the earth lines of longitude come closer together as they go from the equator to either pole. In flat spacetime, the separation remains constant. For simplicity, let us consider geodesics in a plane. Then the *equation for geodesic deviation* can be written Misner, Thorne & Wheeler (1973), p 30 as

$$\frac{d^2\xi}{ds^2} = -\frac{\xi}{a^2},$$

where ξ is normal to the trajectory and s is measured along the trajectory. The quantity $1/a^2$ is the Gaussian curvature at the point of consideration. For a surface with constant curvature, that is the surface of a sphere, the equation is easily integrated to get (ignoring a linear term) $\xi = \xi_0 \cos(s/a)$. Note that this equation also describes the separation of lines of longitude as we move from the equator to either pole. Now geodesics describe the trajectories of point particles. Null-geodesics are associated with mass-less particles. However, photons are not point particles. The experiment of using single photons in a two-slit interferometer shows that individual photons must have a finite size. Quantum mechanics requires that all particles are described by wave functions and therefore we must consider the propagation of a wave in spacetime. Because photons are bosons, the usual quantum mechanical approach is to describe the properties of photons by creation and destruction operators. The emphasis of this approach is on the production and absorption of photons with little regard to their properties as free particles. Indeed because photons travel at the speed of light, their lifetime in their own reference frame between creation and destruction is zero. However, in any other reference frames they behave like normal particles with definite trajectories and lifetimes. Havas (1966) has pointed out that the concept of a single photon is rather tenuous. There is no way we can tell the difference between a single photon and a bundle of photons with the same energy, momentum, and spin. However, it is an essential part of this derivation that a single photon has an actual existence.

Assume that a photon can be described by a localized wave packet that has finite extent both along and normal to its trajectory. This economic description is sufficient for the following derivation. We define the frequency of a photon as $\nu = E/h$ and its wavelength as $\lambda = hc/E$ where E is its energy. These definitions are for convenience and do not imply that we can ascribe a frequency or a wavelength to an individual photon; they are properties of groups of photons. The derivation requires that the wavelength is short compared to the size of the wave packet and that this is short compared to variations in the curvature of spacetime. Furthermore, we assume that the rays of any wave follow null geodesics and therefore any deviations from flat spacetime produce change in shape of the wave packet. In other words, since the scale length of deviations from flat space are large compared to the size of the wave packet they act as a very small perturbation to the propagation of the wave packet.

Consider a wave packet moving through a spacetime of constant positive curvature. Because of geodesic deviation, the rays come closer together as the wave packet moves forward. They are focussed. In particular the direction θ, of a ray (geodesic) with initial separation ξ_0 after a distance s is (assuming small angles)

$$\theta = -\frac{s\xi_0}{a^2},$$

where a is the local radius of curvature. Since the central geodesic is the direction of energy flow, we can integrate the wave-energy-function times the component of θ normal to the trajectory, over the dimensions of the wave packet in order to calculate the amount of energy that is now travelling normal to the trajectory. The result is a finite energy that depends on the average lateral extension of the wave packet, the local radius of curvature, and the original photon energy. The actual value is not important but rather the fact that there is a finite fraction of the energy that is moving away from the trajectory of the original wave packet. This suggests a photon interaction in which the photon interacts with curved spacetime with the hypothesis that the energy flow normal to the trajectory goes into the emission of secondary photons normal to its trajectory. From a quantum-mechanical point of view, there is a strong argument that some interaction must take place. If the spin of the photon is directly related to the angular momentum of the wave packet about its trajectory then the computation of the angular momentum is a similar integral. Then because of *focussing* the angular momentum clearly changes along the trajectory, which disagrees with the quantum requirement that the angular momentum, that is the spin, of the photon is constant. The Heisenberg uncertainty principle requires that an incorrect value of spin can only be tolerated for a finite time before something happens to restore the correct value. We now consider the consequences.

Consider motion on the surface of a three dimensional sphere with radius r. As described above, two adjacent geodesics will move closer together due to focussing. Simple kinematics tells us that a body with velocity v associated with these geodesics has acceleration v^2/r, where r is the radius of curvature. This acceleration is directly experienced by the body. In addition, it experiences a tidal acceleration within itself. This tidal acceleration is equivalent to the focussing of the geodesics. Although the focussing and acceleration are closely linked, we need to consider whether the occurrence of one implies the occurrence of the other. Does the observation of focussing (tidal acceleration) imply acceleration in the orthogonal direction? It is true in two and three dimensions, but it needs to be demonstrated for four dimensions.

The geometry of a three dimensional surface with curvature in the fourth dimension is essentially the same as motion in three dimensions except that the focussing now applies to the cross-sectional area and not to the separation. Does this acceleration have the same physical significance? Assuming it does, a wave packet that is subject to focussing has acceleration in an orthogonal dimension. For instance if we could constrain a wave packet (with velocity c) to travel on the surface of a sphere in three dimensions it would not only show a focussing effect but also experience an acceleration of c^2/r normal to the surface of the sphere. Then a wave packet (and hence a photon) that has its cross-sectional area focussed by curvature in the fourth dimension with radius r would have an energy loss rate proportional to this acceleration. The essence of the curvature-redshift hypothesis is that the tidal distortion causes the photon to interact and that the energy loss rate is proportional to c^2/r. For a photon with energy E the loss rate per unit time is cE/r, and per unit distance it is E/r.

In general relativity the crucial equation for the focussing of a bundle of geodesics was derived by Raychaudhuri (1955), also see Misner et al. (1973) and Ellis (1984) and for the current context we can assume that the bundle has zero shear and zero vorticity. Since any change in geodesic deviation along the trajectory will not alter the direction of the geodesics we need consider only the cross-sectional area A of the geodesic bundle to get the equation

$$\frac{1}{A}\frac{d^2A}{ds^2} = -\mathbf{R}_{\alpha\beta}\mathbf{U}^{\alpha}\mathbf{U}^{\beta} = -\frac{1}{a^2},$$

where $\mathbf{R}$ is the Ricci tensor (it is the contraction of the Riemann-Christoffel tensor), $\mathbf{U}$ is the 4-velocity of the reference geodesic and a is the local radius of curvature. This focussing can be interpreted as the second order rate of change of cross-sectional area of a geodesic bundle that is on the three-dimensional surface in four-dimensional space. Then if we consider that a photon is a wave packet we find that the rate at which the photon loses energy per unit distance is E/a or more explicitly

$$\frac{1}{E}\frac{dE}{ds} = -\frac{1}{a} = -\left(\mathbf{R}_{\alpha\beta}\mathbf{U}^{\alpha}\mathbf{U}^{\beta}\right)^{1/2},$$

What is interesting about this equation is that, for the Schwarzschild (and Kerr) solutions for the external field for a mass, the Ricci tensor is zero; hence, there is no focussing and no energy loss. A geodesic bundle passing a mass such as the sun experiences a distortion but the wave packet has not changed in area. Hence, this model predicts that photons passing near the limb of the sun will not suffer any energy loss due to curvature redshift.

The field equation for Einstein's general theory of gravitation is

$$\mathbf{R}_{\alpha\beta} = 8\pi G \left(\mathbf{T}_{\alpha\beta} - \frac{1}{2} T \mathbf{g}_{\alpha\beta} \right) + \Lambda \mathbf{g}_{\alpha\beta},$$

where T is the contracted form of $\mathbf{T}_{\alpha\beta}$ the stress-energy-momentum tensor, $\mathbf{g}$ is the metric tensor, G is the Newtonian gravitational constant and Λ is the cosmological constant. It states that the Ricci tensor describing the curvature of spacetime is determined by the distribution of mass (and energy). Direct application of the field equations (without the cosmological constant) in terms of the stress-energy-momentum tensor $\mathbf{T}_{\alpha\beta}$, the metric tensor $\mathbf{g}$ and with the material having a 4-velocity $\mathbf{V}$ gives

$$\frac{1}{a^2} = 8\pi G \left(\mathbf{T}_{\alpha\beta} \mathbf{U}^\alpha \mathbf{U}^\beta - \frac{1}{2} T \mathbf{g}_{\alpha\beta} \mathbf{V}^\alpha \mathbf{V}^\beta \right). \tag{1}$$

For null geodesics $\mathbf{g}_{\alpha\beta} \mathbf{V}^\alpha \mathbf{V}^\beta$ is zero which leaves only the first term. For a perfect fluid the stress-energy-momentum tensor is

$$\mathbf{T}_{\alpha\beta} = \frac{p}{c^2} \mathbf{g}_{\alpha\beta} + \left(\rho + \frac{p}{c^2} \right) \mathbf{U}_\alpha \mathbf{U}_\beta, \tag{2}$$

where p is the proper pressure and ρ is the density. Combining Eq. 1 with Eq. 2 gives for null geodesics

$$\frac{1}{a^2} = \frac{8\pi G}{c^2} \left(\rho + \frac{p}{c^2} \right).$$

For cases where the proper pressure is negligible compared to the density we can ignore the pressure and get

$$\begin{aligned}\frac{1}{E}\frac{dE}{ds} &= -\frac{1}{a} = -\left(\frac{8\pi G\rho}{c^2}\right)^{1/2} \\ &= -1.366\times 10^{-13}\sqrt{\rho}\,\mathrm{m}^{-1}. \end{aligned} \tag{3}$$

For many astrophysical types of plasma, it is useful to measure density by the equivalent number of hydrogen atoms per cubic metrae: that is we can put $\rho = Nm_H$ and get

$$\frac{1}{E}\frac{dE}{ds} = -\left(\frac{8\pi GNM_H}{c^2}\right) = -5.588\times 10^{-27}\sqrt{N}\,\mathrm{m}^{-1}. \tag{4}$$

The rate of energy loss per distance travelled depends only on the square root of the density of the material, which may consist of gas, plasma, or gas and dust. This equation can be integrated to get

$$\ln(E/E_0) = \left(\frac{8\pi GM_{\mathrm{H}}}{c^2}\right)^{1/2}\int_0^x \sqrt{N(x)}dx. \tag{5}$$

2.1.2 Curvature redshift secondary photons

The above derivation does not define the form of energy loss. The most realistic model is that the photon decays into three secondary photons, one of which takes nearly all the energy and momentum and two very low-energy secondary photons. It is convenient (although not strictly correct) to think of the high-energy secondary as a continuation of the primary but with slightly reduced energy. Two secondary photons are required to preserve spin and, by symmetry, they are emitted in opposite directions with the same energy This assumption that the two secondary photons have the same energy is made without proper justification. What can be said is that if they are not, they will still have nearly equal energies because the probability of having one with a much longer relative wavelength is very low. From symmetry they are ejected at right angles to the original trajectory. Thus, the primary photon is not deflected. We can get an estimate of how often these interactions occur and hence what the secondary energies are by using the Heisenberg uncertainty principle applied to the primary. For linear momentum and distance it is $\Delta p\Delta x \cong h/4\pi$, and putting $X = \Delta x$ we get $\Delta E = hc/4\pi X$. Now after the photon with energy E_0 has travelled a distance X the energy-loss is $\Delta E = E_0 X/a$, and hence

$$X^2 = \frac{ahc}{4\pi E_0} = \frac{a\lambda_0}{4\pi} = \frac{c\lambda_0}{4\pi\sqrt{8\pi G\rho}}. \tag{6}$$

If each secondary photon takes half the energy-loss, we find

$$\Delta E = \frac{1}{2}\frac{E_0 X}{a}. \tag{7}$$

Therefore the secondary photons have a wavelength of

$$\lambda = \frac{2a\lambda_0}{X} = 8\pi X \ = 4\sqrt{\pi a\lambda_0}. \tag{8}$$

For example consider a visible photon with wavelength 600 nm travelling in gas with density N, then $X = 2.93 \times 10^9 N^{-1/4}$ m and the wavelength is $\lambda = 7.36 \times 10^{10} N^{-1/4}$ m which corresponds to a frequency of $\nu = 4.07 N^{1/4}$ mHz Now for fully ionized plasma the plasma frequency is

$$\nu_p = \left(\frac{Ne^2}{\pi m_e}\right)^{1/2} = 8.975 N^{1/2}\,\mathrm{Hz},$$

and the ratio is

$$\frac{\nu}{\nu_p} = 4.55 \times 10^{-4} N^{-1/4}.$$

Thus, for optical photons and all plasmas with densities greater than $N = 0.14\,\mathrm{m}^{-3}$ the secondary photons have frequencies well below the plasma frequency and therefore cannot propagate but will be quickly absorbed by the plasma. The energy lost by the primary photon is dissipated into heating the plasma.

2.1.3 Inhibition of curvature redshift

From the discussion above it is clear that the process of curvature redshift requires a gradual focussing to a critical limit, followed by the emission of secondary photons. It is as if the photon gets slowly excited by the focussing until the probability of secondary emission becomes large enough for it to occur. If there is any other interaction the excitation due to focussing will be nullified. That is, roughly speaking, curvature-redshift interaction requires an undisturbed path length of at least X (Eq. 6) for significant energy loss to occur. A suitable criterion for inhibition to occur is that the competing interaction has an interaction length less than X. Although Compton or Thompson scattering are possible inhibitors there is another interaction that has a much larger cross-section. This is the coherent multiple scattering that produces refractive index.

In classical electro-magnetic theory, the refractive index of a medium is the ratio of the velocity of light in vacuum to the group velocity in the medium. However, in quantum mechanics photons always travel at the velocity of light in vacuum. In a medium, a group of photons appears to have a slower velocity because the individual photons interact with the electrons in the medium and each interaction produces a time delay. Because the interaction is with many electrons spread over a finite volume, the only possible result of each interaction is the emission of another photon with the same energy and momentum. Now consider the absorption of a wave. In order to cancel the incoming wave a new wave with the same frequency and amplitude but with opposite phase must be produced. Thus, the outgoing wave will be delayed by half a period with respect to the incoming wave. For example if the phase difference was not exactly half a period for an electro-magnetic wave incident on many electrons, the principle of conservation of energy would be violated. This simple observation enables us to compute the interaction length for refractive index n. If L is this interaction length then it is

$$L = \frac{\lambda_0}{2\,|n-1|},$$

where n is the refractive index and the modulus allows for plasma and other materials where the refractive index is less than unity. Note that L is closely related to the extinction length derived by Ewald and Oseen (see (Jackson, 1975) or Born & Wolf (1999)) which is a measure of the distance needed for an incident electromagnetic wave with velocity c to be replaced by a new wave. For plasmas the refractive index is

$$n \cong 1 - \frac{N_e \lambda_0^2}{2\pi r_0},$$

where N_e is the electron density and r_0 is the classical electron radius. We can combine these two equations to get (for a plasma)

$$L = (N_e r_0 \lambda_0)^{-1}. \tag{9}$$

Thus, we would expect the energy loss to be inhibited if the average curvature-redshift interaction distance is greater than that for refractive-index interactions, i.e. if $X > L$. Therefore, we can compute the ratio (assuming a plasma with $N \cong N_e$) and using Eq. 6 to get

$$X/L = 0.0106 N^{3/4} \lambda_0^{3/2} \tag{10}$$

This result shows that curvature redshift will be inhibited if this ratio is greater than one, which is equivalent to $\lambda_0 \geqslant 20.7 N^{-1/2}$ m. For example, curvature redshift for the 21 cm hydrogen line will be inhibited if the electron density is greater than about $10^4\,\mathrm{m}^{-3}$.

2.1.4 Possible laboratory tests

It is apparent from the above analysis that to observe the redshift in the laboratory we need to have sufficient density of gas (or plasma) to achieve a measurable effect but not enough for there to be inhibition by the refractive index. The obvious experiment is to use the Mössbauer effect for γ-rays that enables very precise measurement of their frequency. Simply put, the rays are emitted by nuclei in solids where there is minimal recoil or thermal broadening of the emitted ray. Since the recoil-momentum of the nucleus is large compared to the atomic thermal energies and since the nucleus is locked into the solid so that the recoil momentum is precisely defined, then the γ-ray energy is also precisely defined. The absorption process is similar and has a very narrow line width. Such an experiment has already been done by Pound & Snyder (1965). They measured gravitational effects on 14.4 keV γ-rays from ^{57}Fe being sent up and down a vertical path of 22.5 m in helium near room pressure. They found agreement to about 1% with the predicted fractional redshift of 1.5×10^{-15}, whereas fractional curvature redshift predicted by Eq. 4) for this density is 1.25×10^{-12}. Clearly, this is much larger. At γ-ray frequencies, the electrons in the helium gas are effectively free and we can use Eq. 9 to compute the refractive index interaction length. For helium at STP, it is $L = 0.077$ m, which is much less than curvature-redshift interaction length which for these conditions is X=11 m. Hence, we do not expect to see any significant curvature redshift in their results. Pound and Snyder did observe one-way frequency shifts but they were much smaller than curvature redshift and could be explained by other aspects of the experiment. However, the Pound and Snyder experiment provides a guide to a possible test for the existence of curvature redshift. Because curvature redshift has a different density variation to that for the inhibiting refractive index it is possible to find a density for which curvature redshift is not inhibited. Although there is a slight advantage in using heavier gases than helium due to their higher atomic number to atomic weight ratio, their increased absorption to γ-rays rules them out. Hence, we stay with helium and from Eq. 9 we can compute curvature-redshift interaction length to be

$$X = 10.8 \left(\frac{p_0}{p} \right)^{1/4} \text{ m},$$

where p is the pressure and p_0 is the pressure at STP. For the same gas the refractive index interaction length is

$$L = 0.077 \left(\frac{p_0}{p} \right) \text{ m}.$$

It follows that the curvature redshift will not be inhibited if $X < L$ or in this case, the pressure is less that $0.0014p_0$ which is about 1 mm of Hg. For this pressure, we find that $X = 57$ m which requires that the apparatus must be much longer than 57 m. For argument let us take the length to be 100 m then the fractional redshift expected is 2.1×10^{-13} which is detectable. The experimental method would use a horizontal (to eliminate gravitational redshifts) tube filled with helium and with accurately controlled temperature. Then we would measure the redshift as a function of pressure. The above theory predicts that if it is free of inhibition then the redshift should be proportional to the square root of the pressure.

Alternatively, it may be possible to detect the secondary photons. For helium with a pressure of 1 mm Hg the expected frequency of the secondary radiation is about 100 kHz. The expected power from a 1 Cu source is about 5×10^{-22} W. Unfortunately, the secondary radiation could be spread over a fairly wide frequency band which makes its detection somewhat difficult but it may be possible to detect the radiation with modulation techniques.

Another possibility is to use γ-rays of much shorter wavelength where it may be possible to detect the secondary radiation in an experiment that did not try to measure the redshift. For example consider the passage of keV to Mev gamma rays from radioactive elements or synchrotron sources in air. For air at a density of 1.20 kg m^{-3} and with the γ-ray energy E_0 in keV the frequency of the secondaries is derived from Eq. 8 to be

$$\lambda = 0.465 \left(\frac{E_0}{\mathrm{keV}} \right)^{1/2} \text{ Mhz},$$

and the gravitational interaction length is

$$X = 25.66 \left(\frac{E_0}{\mathrm{keV}} \right)^{1/2} \text{ m}.$$

Now for there to be no inhibition the gravitational interaction length must be less than the refractive index interaction length (L) which from Eq. 9 and for air has the equation

$$L = 0.7905 \left(\frac{E_0}{\mathrm{keV}} \right) \text{ m}.$$

In addition the gamma rays must have a path length greater than X. An appropriate measure of this path length is the distance over which the number of γ-rays have been attenuated to half the original number. Table 2 shows these quantities for a range of primary energies.

Note that the curvature redshift will be inhibited by the attenuation length until the γ-rays have an energy a bit less than 20 keV. There is no inhibition from either cause for energies larger than 20 keV. The expected power per gamma ray per meter of path length is given by

$$\Delta P = 7.24 \times 10^{-21} \left(\frac{E_0}{\mathrm{keV}} \right) \text{ W m}^{-1}.$$

Table 2: Curvature redshift in air.

Energy/keV	X^a	L^b	attn. length^c	ν^d
10	8.11	7.9	1.1	1.47
20	5.74	15.8	7.4	2.08
50	3.63	39.5	27.8	3.29
100	2.57	79.1	37.4	4.65
200	1.81	158.1	44.7	6.58
500	1.15	395.2	66.1	10.4

[a]Gravitational interaction length in metros

[b]Refractive index interaction length in metros

[c]Distance to halve beam intensity in metros

[d]Secondary frequency in Mhz

Clearly a powerful γ-ray source with energies greater than about 50 keV is required.

Yet another possible test is to measure the frequency from a spacecraft at two receivers as a function of the differential distance between the receivers and the spacecraft. For example in the analysis of the *Pioneer* 10 acceleration anomaly (section (3.7) it was shown that the interplanetary dust density could contribute a measurable frequency shift. Comparison of this frequency shift at the same time at two receivers at different distances would remove most other causes of frequency shifts. One advantage of this test is that it does not require very accurate frequency generation on the satellite. Typically the two receivers would be two ground stations. The major problem is the uncertainty and indeed large variation in the density of the exosphere and any other frequency shifts due to earth rotation that cannot be accurately modelled. Note that at the typical X-band frequencies inhibition will prevent the neutral atmosphere showing any curvature-redshift effects.

2.1.5 Interactions for other particles

Since the focussing due to spacetime curvature applies to the quantum wave, it is expected that electrons and other particles would interact with curved spacetime in a manner similar to photons. The argument is the same up to Eq. 4 but now we have to allow for nonzero mass. The problem (not solved here) is to find a covariant expression that properly describes the energy-momentum loss to secondary particles and yet preserves the correct normalization of the energy-momentum 4-vector. An alternate approach is to consider the motion in a local Minkowskian reference frame. In this case the loss equations (with P_0 denoting the energy component) are

$$\begin{aligned}\frac{dP^0}{dx} &= \frac{\beta^2 P^0}{a_e} \\ \frac{dP^j}{dx} &= \frac{P^j}{a_e}, \qquad j = 1, 2, 3\end{aligned}$$

where β is the usual velocity ratio, a_e is the local radius of curvature for electrons and as required by normalization and the conservation of proper mass, we have from Eq. 11

$$\frac{dP^\alpha}{dx} P^\alpha = 0.$$

Noting that for a nonzero rest mass particle $\mathbf{V}^\alpha \mathbf{V}_\alpha = -1$. The radius of curvature a_e can be evaluated for the simple case of a uniform gas (or plasma) using equations (1) and (2) to get

$$a_e = \left\{ \frac{8\pi G}{c^2} \left[\left(\gamma^2 - \frac{1}{2} \right) \rho + \frac{p}{c^2} \left(\gamma^2 + \frac{1}{2} \right) \right] \right\}^{-1/2},$$

where $\gamma = 1/\sqrt{1-\beta^2}$. Then with the further simplification of negligible pressure and with the material at rest and where $T = (\gamma - 1)\, mc^2$ is the kinetic energy, the energy loss rate is

$$\frac{1}{T}\frac{dT}{dx} = -\frac{1}{a_e} \quad = - \left\{ \frac{8\pi G \rho \left(\gamma^2 - \frac{1}{2} \right)}{c^2} \right\}^{1/2} \beta^2. \tag{11}$$

It shows that for nonzero rest mass particles, the energy loss rate has a strong dependence on velocity, and for extreme relativistic velocities, the fractional energy-loss rate is proportional to γ. Because of the strong velocity dependence, the energy loss rate for electrons will be much higher than that for nuclei in any plasma near thermal equilibrium. In addition, Eq. 11 shows that the energy loss rate has the same square root dependence on density as the energy loss rate for photons.

Since an electron interacts without being absorbed and re-emitted, we do not expect the same type of inhibition that applies to photons. Instead the electron slowly gets excited with the addition of energy which it releases as low-energy photons when it interacts with some other particle. The need to preserve spin and momentum prevents it from emitting photons without the presence of another particle. In the cosmic medium, the most likely interactions are electro-magnetic scattering off other charged particles and the inverse-Compton effect off 3K background radiation photons. In high temperature plasma the electromagnetic (Rutherford) scattering is probably dominant since there will be many small angle deflections with large impact parameters. Thus the model for curvature redshift of non-photon particles is one in which an excited electron emits most of its excitation energy as a low-energy photon during the scattering off another photon, electron or nucleus.

2.2 Derivation of curvature pressure

The hypothesis of curvature pressure is that for moving particles there is a pressure generated that acts back on the matter that causes the curved spacetime. In this case, curvature pressure acts on the matter (plasma) that is producing curved spacetime in such a way as to try to decrease the curvature. In other words, the plasma produces curved spacetime through its density entering the stress-energy tensor in Einstein's field equations. The magnitude of the curvature is an increasing function of the plasma density.

2.2.1 Gravitation is not a force

The phrase *gravitational force* is not only a popular expression but is endemic throughout physics. In particular, gravitation is classified as one of the four fundamental forces with its heritage going back to Newton's law of gravitation. I argue that the formulation of gravitation as a force is a misconception. In both Newtonian theory and general relativity, gravitation is acceleration. To begin let us examine the original Newtonian gravitation equation

$$m_{\rm I}\mathbf{a} = \mathbf{F} \quad = -\frac{GMm_{\rm G}}{r^3}\mathbf{r}, \tag{12}$$

where (following Longair (1991) we identify M_I as the inertial mass of the test object, M as the active gravitational mass of the second object and $m_{\rm G}$ as the passive gravitational mass of the test object. The vector $\mathbf{a}$ is its acceleration and $\mathbf{r}$ is its displacement from the second object. This equation is usually derived in two steps: first, the derivation of a gravitational field and second, the force produced by that field on the test mass. By analogy with Coulombs law, the passive gravitational mass has a similar role to the electric charge.

However many experiments by Eötvös, Pekar & Fekete (1992), Dicke (1964), and Braginskiĭ & Panov (1972) have shown that the passive gravitational mass is equal to the inertial mass to about one part in 10^{12}. The usual interpretation of the agreement is that they are fundamentally the same thing. However, an alternative viewpoint is that the basic equation is wrong and that the passive gravitational mass and the inertial mass should not appear in the equation. In this case the correct equation is

$$\mathbf{a} = -\frac{GM}{r^3}\mathbf{r}. \tag{13}$$

Thus, the effect of gravitation is to produce accelerations directly; there is no force involved. Some might argue that since the two masses cancel the distinction is unimportant. On the other hand, I would argue that the application of Ockham's razor dictates the use of Eq. 13 instead of Eq. 12.

The agreement of the inertial mass with the passive gravitational mass is the basis of the weak equivalence principle in that it applies regardless of the composition of the matter used. Carlip (1998) Shows that it applies to both the potential and the kinetic energy in the body. The theory of general relativity is based on the principle of equivalence as stated by Einstein: *All local, freely falling, non-rotating laboratories are fully equivalent for the performance of physical experiments.* The relevance here is that it is impossible to distinguish between acceleration and a uniform gravitational field. Thus when gravitation is considered as acceleration and not a force the passive gravitational mass is a spurious quantity that is not required by either theory.

2.2.2 A Newtonian model

A simple cosmological model using Newtonian physics in four-dimensional space illustrates some of the basic physics subsequently used to derive the features of curvature pressure. The model assumes that the universe is composed of gas confined to the three-dimensional surface of a four-dimensional hypersphere. Since the visualization of four dimensions is difficult let us suppress one of the normal dimensions and consider the gas to occupy the two-dimensional surface of a normal sphere. From Gauss's law (i.e. the gravitational effect of a spherical distribution of particles with radial symmetry is identical to that of a point mass equal in value to the total mass situated at the center of symmetry) the gravitational acceleration at the radius r of the surface is normal to the surface, directed inward and it has the magnitude

$$\ddot{r} = -\frac{GM}{r^2},$$

where M is the total mass of the particles and the dots denote a time derivative. For equilibrium, and assuming all the particles have the same mass and velocity we can equate the radial acceleration to the gravitational acceleration and get the simple equation from celestial mechanics of

$$\frac{v^2}{r} = \frac{GM}{r^2}.$$

If there is conservation of energy, this stable situation is directly analogous to the motion of a planet about the sun. When there is a mixture of particles with different masses, there is an apparent problem. In general, particles will have a distribution of velocities and the heavier ones can be expected to have, on average, lower velocities. Thus, equilibrium radii will vary with the velocity of the particles. However, the basis of this model is that all particles are constrained to have the same radius regardless of their mass or velocity with the value of the radius set by the average radial acceleration. Thus for identical particles with a distribution of velocities we average over the squared velocities to get

$$\langle v^2 \rangle = \frac{GM}{r}. \tag{14}$$

If there is more than one type of particle with different masses then we invoke the precepts of Section 2.2.1 and average over the accelerations to get the same result as Eq. 14. The effect of this balancing of the accelerations against the gravitational potential is seen within the shell as a curvature pressure that is a direct consequence of the geometric constraint of confining the particles to a shell. If the radius r decreases then there is an increase in this curvature pressure that attempts to increase the surface area by increasing the radius. For a small change in radius in a quasi-equilibrium process where the particle velocities do not change the work done by this curvature pressure (two-dimensions) with an incremental increase of area dA is $p_{\rm c}dA$ and this must equal the gravitational force times the change in distance to give

$$p_{\rm c}dA = \frac{GM^2}{r^2}\,dr,$$

where $M = \sum m_i$ with the sum going over all the particles. Therefore, using Eq. 14 we can rewrite the previous equation in terms of the velocities as

$$p_{\rm c}dA = \frac{M\left\langle v^2\right\rangle}{r}\,dr.$$

Now $dA/dr = 2A/r$, hence the two-dimensional curvature pressure is

$$p_{\rm c} = \frac{M\left\langle v^2\right\rangle}{2A}.$$

Thus in this two-dimensional model the curvature pressure is like the average kinetic energy per unit area. This simple Newtonian model provides a guide as to what the curvature pressure would be in the full general relativistic model. The essential result is that there is a curvature pressure that is due to the constraint of requiring all the particles to stay within the two-dimensional surface.

2.2.3 General relativistic model

In deriving a more general model in analogy to the Newtonian one, we first change $dA/dr = 2A/r$ to $dV/dr = 3V/r$ and secondly we include the correction γ^2 needed for relativistic velocities. The result is

$$p_{\rm c} = \frac{M\left\langle \gamma^2\beta^2\right\rangle c^2}{3V} = \frac{\left\langle \gamma^2 - 1\right\rangle Mc^2}{3V}.$$

In this case the constraint arises from the confinement of all the particles within a three-dimensional hyper-surface. Now we expect to be dealing with fully ionized high temperature plasma with a mixture of electrons, protons, and heavier ions where the averaging is done over the accelerations. Define the average density by $\rho = M/V$ then the cosmological curvature pressure is

$$p_{\mathrm{c}} = \frac{1}{3}\left\langle \gamma^2 - 1 \right\rangle \rho c^2. \tag{15}$$

In effect, my hypothesis is that the cosmological model must include this curvature pressure as well as thermodynamic pressure. Note that although this has a similar form to thermodynamic pressure it is quite different. In particular, it is proportional to an average over the squared velocities and the thermodynamic pressure is proportional to an average over the kinetic energies. This means that, for plasma with free electrons and approximate thermodynamic equilibrium, the electrons will dominate the average due to their much larger velocities. From a Newtonian point of view, curvature pressure is opposed to gravitational mutual acceleration. In general relativity, the plasma produces curved space-time through its density entering the stress-energy tensor in Einstein's field equations. Then the constraint of confining the particles to a three-dimensional shell produces a pressure whose reaction is the curvature pressure acting to decrease the magnitude of the curvature and hence decrease the density of the plasma.

For high temperature plasma in equilibrium, the Jüttner distribution can be used to evaluate the curvature pressure. For a gas with temperature T and for molecules with mass m, de Groot et al. (1980) showed that

$$\gamma^2\left(\alpha\right) = 3\alpha K_3(1/\alpha)/K_2(1/\alpha), \tag{16}$$

where $\alpha = kT/mc^2$ and $K_n(1/\alpha)$ are the modified Bessel functions of the second kind Abramowitz & Stegun (1972). For small, α this has the approximation

$$\gamma^2(\alpha) = 1 + 3\alpha + 152\alpha^2 + 458\alpha^3 + \ldots. \tag{17}$$

For a Maxwellian (non-relativistic) distribution, the first two terms are exact and the α^2 term is the first term in the correction for the Jüttner distribution.

2.2.4 Local curvature pressure

For the universe, the calculation of curvature pressure is simple because of the constant curvature and homogeneous medium. However, for a localized region such as a star with inhomogeneous medium and curvature the calculation is much more difficult. We start with the premise that it is the motion of particles that reacts back on the material producing the curvature by producing a pressure that tends to reduce the curvature. The problem is that the calculation of the curvature at any point requires the integration of Einstein's equations of general relativity. Then if the particles' motion produces a reaction force, the problem is to determine how that reaction force is apportioned amongst the matter that produces the curvature. One approach that is valid for most astrophysical applications where the spacetime curvature is small is to use the Newtonian approximation. Let a, be the effective radius of curvature of the four dimensional space where the particles' are constrained. Then the premise is that this constraint produces an acceleration due to curvature (assuming for the moment that there is only one type of particle) of

$$g_{\rm c} = \frac{\langle v^2 \rangle}{a},$$

where the angular brackets denote an averaging over all the velocities. Now consider a spherically symmetric distribution of gas. If the distribution is static, the central gravitational attraction is balanced by some pressure p_g, so that

$$\frac{dp_{\rm g}}{dr} = -\rho(r)g(r),$$

where $\rho\,(r)$ is the density at radius r and $g(r)$ is the gravitational acceleration at r. Similarly, we define a curvature pressure by

$$\frac{dp_{\rm c}}{dr} = -\rho(r)g_{\rm c}(r). \tag{18}$$

However, if there is a mixture of particles there is an important difference. Because electrons have a much lighter mass than ions the velocity average for mixed particles (provided the gas is ionized) will be dominated by the electrons and the appropriate density to use in Eq. 18 is that for the electrons. Now the curvature radius a, is given by Eq. 3, and for a gas with relativistic particles we put

$$\left\langle v^2 \right\rangle = \left\langle \gamma^2 - 1 \right\rangle c^2.$$

We need to include a factor of one third because only the velocity component orthogonal to the direction of the acceleration is relevant. Then the curvature pressure acceleration is

$$g_{\rm c}(r) = \frac{1}{3}\left\langle \left(\gamma^2 - 1\right)\sqrt{\rho(r)} \right\rangle c^2 \sqrt{\frac{8\pi G}{c^2}},$$

and

$$\frac{dp_{\rm c}}{dr} = -\frac{1}{3}\left\langle (\gamma^2 - 1)\sqrt{\rho(r)} \right\rangle c^2 \sqrt{\frac{8\pi G}{c^2}}\rho(r). \tag{19}$$

Since the hypothesis is that this curvature pressure is a reaction to the accelerations produced by the gas at radius r, the averaging over velocities must be over all the gas that is being accelerated. By Gauss's law and symmetry this is the gas with radii greater than r thus we get

$$\left\langle \left(\gamma^2 - 1\right) \sqrt{\rho(r)} \right\rangle = \frac{\int_r^\infty N(\hat{r})\hat{r}^2(\gamma^2 - 1)\sqrt{\rho(\hat{r})}\, d\hat{r}}{\int_r^\infty N(\hat{r})\hat{r}^2\, d\hat{r}},$$

where $N(r)$ is the particle number density. Now for plasmas where the temperatures less than about 10^8 K we can use Eq. 17 to get

$$\frac{1}{3}\left\langle \gamma^2 - 1 \right\rangle = \frac{kT}{m_e c^2}.$$

Hence the working equation for local curvature pressure is

$$\frac{dp_c}{dr} = -k\left\langle T(r)\sqrt{\rho(r)} \right\rangle \sqrt{\frac{8\pi G}{c^2}}\rho(r),$$

where the function in angular brackets is

$$\left\langle T(r)\sqrt{\rho(r)} \right\rangle = \frac{\int_r^\infty N_e(\hat{r})\hat{r}^2 T(\hat{r})\sqrt{\rho(\hat{r})}\, d\hat{r}}{\int_r^\infty N_e(\hat{r})\hat{r}^2\, d\hat{r}},$$

and $N_e(r)$ is the electron number density.

A theory of curvature pressure in a very dense medium where quantum mechanics dominates and where general relativity may be required is needed to develop this model. Nevertheless, without such a theory, we expect the pressure to be proportional to the local gravitational acceleration and an increasing function of the temperature of the particles. Thus, we might expect a curvature pressure that would resist a hot compact object from collapsing to a black hole. Because of the energy released during collapse, it is unlikely for a cold object to stay cold enough to overcome the curvature pressure and collapse to a black hole.

2.3 The curvature cosmological model

Curvature cosmology can now be derived by including curvature redshift and curvature pressure into the equations of general relativity. This is done by using homogeneous isotropic plasma as a model for the real universe. The general theory of relativity enters through the Friedmann equations for a homogeneous isotropic gas. Although such a model is simple compared to the real universe, the important characteristics of CC can be derived by using this model. The first step is to obtain the basic relationship between the density of the gas and the radius of the universe. The inclusion of curvature pressure is not only important in determining the basic equations but it also provides the necessary means of making the solution static and stable. Then it is shown that the effect of curvature redshift is to produce a redshift that is a function of distance, and the slope of this relationship is (in the linear limit of small distances) the Hubble constant.

The first-order model considers the universe to be a gas with uniform density and complications such as density fluctuations, galaxies, and stars are ignored. In addition, we assume (to be verified later) that the gas is at high temperature and is fully ionized plasma. Because of the high symmetry, the appropriate metric is the one that satisfies the equations of general relativity for a homogeneous, isotropic gas. This metric was first discovered by A. Friedmann and fully investigated by H. P. Robertson and A. G. Walker. The Robertson-Walker metric for a space with positive curvature can be written (Rindler, 1977) as

$$ds^2 = c^2dt^2 - [R(t)]^2\left[\frac{dr^2}{1-r^2} + r^2\left(d\theta^2 + \sin^2(\theta)d\varphi^2\right)\right]$$

where ds is the interval between events, dt is time, $R(t)dr$ is the comoving increment in radial distance, $R(t)$ is the radius of curvature and R_0 is the value of $R(t)$ at the present epoch.

2.3.1 The Friedmann equations

Based on the Robertson-Walker metric, the Friedmann equations for the homogeneous isotropic model with constant density and pressure are (Longair, 1991)

$$\ddot{R} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)R + \frac{1}{3}\Lambda R, \tag{20}$$

$$\dot{R}^2 = \frac{8\pi G}{3}\rho R^2 - c^2 + \frac{1}{3}\Lambda R^2. \tag{21}$$

where R is the radius, ρ is the proper density, p is the thermodynamic pressure, G is the Newtonian gravitational constant, Λ is the cosmological constant, c is the velocity of light and the superscript dots denote time derivatives. Working to order of $m_\mathrm{e}/m_\mathrm{p}$ thermodynamic pressure may be neglected but not curvature pressure. How to include curvature pressure is not immediately obvious. The thermodynamic pressure appears only as a relativistic correction to the inertial mass density whereas curvature pressure is closer in spirit to the cosmological constant. My solution is to include curvature pressure (with a negative sign) with the thermodynamic pressure and to set the cosmological constant to zero. This is an ad hoc variation to general relativity and its only justification is that it provides sensible equations and show good agreement with observations. Including curvature pressure from Eq. 15 and from Eq. 20 the modified Friedmann equations are

$$\ddot{R} = -\frac{4\pi G\rho}{3}\left[1 - \left\langle \gamma^2 - 1 \right\rangle\right]R, \tag{22}$$

$$\dot{R}^2 = \frac{8\pi G\rho}{3}R^2 - c^2. \tag{23}$$

(24)

Clearly there is a static solution if $< \gamma^2 - 1 >= 1$, in which case $\ddot{R} = 0$. The second equation, with $\dot{R} = 0$ provides the radius of the universe which is given by

$$R = \sqrt{\frac{3c^2}{8\pi G\rho}} = \sqrt{\frac{3c^2}{8\pi G M_{\rm H} N}}. \tag{25}$$

Thus, the model is a static cosmology with positive curvature. Although the geometry is similar to the original Einstein static model, this cosmology differs in that it is stable. The basic instability of the static Einstein model is well known (Tolman, 1934; Ellis, 1984). On the other hand, the stability of CC is shown by considering a perturbation ΔR, about the equilibrium position. Then the perturbation equation is

$$\Delta\ddot{R} = \frac{3c^2}{4\pi R_0}\left(\frac{d\langle\gamma^2 - 1\rangle}{dR}\right)\Delta R. \tag{26}$$

For any realistic equation of state for the cosmic plasma, the average velocity will decrease as R increases. Thus the right hand side is negative, showing that the result of a small perturbation is for the universe return to its equilibrium position. Thus, CC is intrinsically stable. Of theoretical interest is that Eq. 26 predicts that oscillations could occur about the equilibrium position.

2.3.2 Temperature of the cosmic plasma

One of the most remarkable results of CC is that it predicts the temperature of the cosmic plasma from fundamental constants. That is the predicted temperature is independent of the density and independent of any other characteristic of the universe. For a stable solution to Eq. 22 we need that $< \gamma^2 - 1 >= 1$, (i.e. $< \gamma^2 >= 2$) where the average is taken over the electron and nucleon number densities, that is for equal numbers of electrons and protons

$$\left\langle \gamma^2 \right\rangle \cong 0.5 \left\langle \gamma_{\rm e}^2 + \gamma_{\rm p}^2 \right\rangle,$$

where the terms on the right are for electrons and protons. Provided the temperatures are small enough for the proton's kinetic energy to be much less than its rest mass energy, we can put $\left\langle \gamma_{\rm p}^2 \right\rangle = 1$ and thus for pure hydrogen, the result is $\left\langle \gamma_{\rm e}^2 \right\rangle = 3$. Using a more realistic composition that has 8.5% by number (Allen, 1976) of helium we find that $\left\langle \gamma_{\rm e}^2 \right\rangle = 2.927$. Hence using Eq. 16 the predicted electron temperature is 2.56×10^9 K. For this temperature $\left\langle \gamma_{\rm p}^2 \right\rangle = 1.0007$. This shows that the temperature is low enough to justify the assumption made earlier, that the proton's kinetic energy is much smaller than its rest mass energy.

To recapitulate the stability of CC requires that $\ddot{R} = 0$. This requires that the plasma has the precise temperature that makes $< \gamma^2 - 1 >= 1$. The basis for this result is that curvature pressure exists and critical to its derivation is the averaging over accelerations and not over forces. This is where the assertion that gravitation is acceleration and is not a force is important.

2.3.3 Hubble constant: theory

The Hubble constant is proportional to the local energy loss rate given by Equation(4 which gives

$$\begin{aligned} H = \frac{c}{E}\frac{DE}{des} &= (8\pi G M_{\rm H} N)^{1/2} \\ &= 1.671 \times 10^{-18} N^{1/2}\,{\rm m}^{-1} \\ &= 51.69 N^{1/2}\,{\rm kms}^{-1}\,{\rm Mpc}^{-1}. \end{aligned} \tag{27}$$

The usual redshift parameter z is defined in terms of the wavelengths, frequencies and energies as

$$z = \frac{\lambda_0}{\lambda_e} - 1 \qquad = \frac{\nu_e}{\nu_0} - 1 \qquad = \frac{E_e}{E_0} - 1. \tag{28}$$

If the plasma density is constant then we can integrate the energy loss along the path to get

$$z = \exp\left(\frac{Hr}{c}\right) - 1, \tag{29}$$

where r is the distance travelled.

2.3.4 Geometry of CC

The Robertson-Walker metric shown in Eq. 20 is not in the simplest form that explicitly shows the geometry. Following D'Inverno (1992) we can introduce a new variable χ, where $r = R\sin\chi$ and the new metric is

$$ds^2 = c^2dt^2 - R^2\left[d\chi^2 + \sin^2\chi\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right].$$

In this metric the distance travelled by a photon is $R\chi$, and since the velocity of light is a universal constant the time taken is $R\chi/c$. There is a close analogy to motion on the surface of the earth with radius R. Light travels along great circles and χ is the angle subtended along the great circle between two points. The geometry of this CC is that of a three-dimensional surface of a four-dimensional hypersphere. For this geometry the area of a sphere with radius R is given by

$$A(r) = 4\pi R^2\sin^2(\chi).$$

The surface is finite and χ can vary from 0 to π. Integration of this equation with respect to χ gives the volume V, namely,

$$V(r) = 2\pi R^3\left[\chi - \frac{1}{2}\sin(2\chi)\right].$$

Clearly the maximum volume is $2\pi^2 R^3$ and we can using Eq. 25 to get R we have

$$\begin{aligned} R &= \sqrt{\frac{3c^2}{8\pi G M_{\rm H} N}} \\ &= 3.100 \times 10^{26} N^{1/2}\,\mathrm{m} \\ &= 10.05 N^{1/2}\,\mathrm{Gpc}. \end{aligned} \tag{30}$$

Examination of Equations (27) and (30) shows that there is a simple relationship between R and H, namely

$$H = \frac{\sqrt{3}c}{R}. \tag{31}$$

The next step is to replace r in Eq. 29 with $r = R\chi$ to get

$$z = \exp(\sqrt{3}\chi) - 1,$$

and

$$\chi = \frac{\ln(1+z)}{\sqrt{3}}. \tag{32}$$

This is the fundamental relationship between z and χ. Since the geometry of CC does not involve a time coordinate, it is much simpler than that for BB. The key equations define the CC geometry are Eq. 31 which defines the radius of the universe in terms of the Hubble constant and Eq. 32 which defines the distance variable χ in terms of the redshift parameter z. We now examine some topics that are relevant practical applications.

2.3.5 Luminosities and magnitudes

Let a source have a luminosity $L(\nu)$ ($\mathrm{W\,Hz^{-1}}$) at the emission frequency ν. Then if energy is conserved, the observed flux density ($\mathrm{W\,m^{-2}\,Hz^{-1}}$) at a distance parameter χ is the luminosity divided by the area, which is

$$S(\nu)d\nu = \frac{L(\nu)\,d\nu}{4\pi\,(R\sin(\chi))^2}.$$

However, because of curvature redshift there is an energy loss such that the received frequency ν_0 is related to the emitted frequency ν_e by Eq. 28. Including this effect the result is

$$S(\nu_0)d\nu_0 = \frac{L(\nu_\mathrm{e})\,d\nu_\mathrm{e}}{4\pi\,(R\sin(\chi))^2\,(1+z)}.$$

The apparent magnitude is defined as $m = -2.5\log(S)$ where the base of the logarithm is 10 and the constant 2.5 is exact. Since the absolute magnitude is the apparent magnitude when the object is at a distance of 10 pc (3.0857×10^{17} m), the flux density at 10 pc is

$$S_{10}(\nu_0)\,d\nu_0 = \frac{L(\nu_0)d\nu_0}{2\pi(10pc)^2},$$

where because 10 pc is negligible compared to R, approximations have been made. The flux density ratio is

$$\frac{S(\nu_0)}{S_{10}(\nu_0)} = \left\{\frac{10pc}{R\sin(\chi)}\right\}^2 \left\{\frac{L(\nu_\mathrm{e})d\nu_\mathrm{e}}{L(\nu_0)d\nu_0}\right\} \left\{\frac{1}{1+z}\right\}.$$

Defining M as the absolute magnitude and putting $\nu_\mathrm{e} = (1+z)\nu_0$ we get for $(m-M)$

$$\begin{aligned} m - M &= -2.5\log\left(\frac{S(\nu_0)}{S_{10}(\nu_0)}\right) \\ &= 5\log\left\{\frac{R\sin(\chi)}{10pc}\right\} + \mathrm{K}_z(\nu_0) + 2.5\log(1+z), \end{aligned}$$

where the K-correction (Rowan-Robertson, 1985; Peebles, 1993; Hogg et al., 2002) is described in Part 1. Furthermore, we can use Eq. 31 to replace R by H since

$$\frac{R}{\sqrt{3}} = \frac{c}{H} = \frac{2.998}{h}\,\mathrm{Gpc},$$

where h is the reduced Hubble constant. Hence, we get the distance modulus

$$\mu_{\mathrm{CC}} = 5\log\left[\frac{\sqrt{3}\sin(\chi)}{h}\right] + 2.5\log(1+z) + 42.384. \tag{33}$$

3 Application of Curvature Cosmology

These topics are relevant to CC but are not part of the comparison of CC with BB. However they are either very important for any cosmology or offer further observational support for CC.

3.1 Entropy

Consider a stellar cluster or an isolated cloud of gas in which collisions are negligible or elastic. In either case the virial theorem states that the average kinetic energy K, is related to the average potential energy V, by the equation $V = V_0 - 2K$ where V_0 is the potential energy when there is zero kinetic energy. Let U be the total energy then $U = K + V = V_0 - K$. Thus, we get the somewhat paradoxical situation that since V_0 is constant; an increase in total energy can cause a decrease in kinetic energy. This happens because the average potential energy has increased by approximately twice as much as the loss in kinetic energy. Since the temperature is proportional to (or at the least a monotonic increasing function of) the average kinetic energy it is apparent that an increase in total energy leads to a decrease in temperature. This explains the often-quoted remark that a self-gravitationally bound gas cloud has a negative specific heat capacity. Thus, when gravity is involved the whole construct of thermodynamics and entropy needs to be reconsidered. One of the common statements of the second law of thermodynamics is that (Longair, 1991): *The* *energy of the universe is constant: the entropy of the Universe tends to a maximum*, (Feynman et al., 1965): *the entropy of the universe is always increasing* or from Wikipedia *the second law of thermodynamics is an expression of the universal law of increasing entropy, stating that the entropy of an isolated system which is not in equilibrium will tend to increase over time, approaching a maximum value at equilibrium.*

Now the normal proof of the second law considers the operation of reversible and non-reversible heat engines working between two or more heat reservoirs. If we use a self-gravitating gas cloud as a heat reservoir then we will get quite different results since the extraction of energy from it will lead to an increase in its temperature. Thus if the universe is dominated by gravity the second law of thermodynamics needs reconsideration. In addition, it should be noted that we cannot have a shield that hides gravity. To put it another way there is no adiabatic container that is beyond the influence of external gravitational fields. Thus we cannot have an isolated system.

This discussion shows that in a static finite universe dominated by gravity simple discussions of the second law of thermodynamics can be misleading. The presence of gravity means that it is impossible to have an isolated system. To be convincing any proof of the second law of thermodynamics should include the universe and its gravitational interactions in the proof.

3.2 Olber's Paradox

For CC, Olber's Paradox is not a problem. Curvature redshift is sufficient to move distant starlight out of the visible band. Visible light from distant galaxies is shifted into the infrared where it is no longer seen. Of course, with a finite universe, there is the problem of conservation of energy and why we are not saturated with very low frequency radiation produced by curvature redshift. These low-energy photons are eventually absorbed by the cosmic plasma. Everything is recycled. The plasma radiates energy into the microwave background radiation and into X-rays. The galaxies develop from the cosmic plasma and pass through their normal evolution. Eventually all their material is returned to the cosmic plasma. Note that very little, if any, is locked up into black holes. Curvature pressure causes most of the material from highly compact objects to be returned to the surrounding region as high-velocity jets.

3.3 Black holes and Jets

The existence of curvature pressure provides a mechanism that could prevent the collapse of a compact object into a black hole. A theory of curvature pressure in a very dense medium where quantum mechanics dominates is needed to develop this model. Nevertheless, without a full theory we can assume that curvature pressure will depend on the local gravitational acceleration and it will be an increasing function of the temperature of the particles. Thus, we might expect a curvature pressure that would resist a hot compact object from collapsing to a black hole. Because of the energy released during collapse it is unlikely for a cold object to stay cold long enough to overcome the curvature pressure and collapse to a black hole.

What is expected is that the final stage of gravitational collapse is a very dense object, larger than a black hole but smaller than a neutron star. This compact object would appear very much like a black hole and would have most of the characteristics of black holes. Such objects could have large masses and be surrounded by accretion discs. Thus, many of the observations that are thought to show the presence of black hole could equally show the presence of these compact objects. However, there is one observational difference in that many of the mass estimates of black holes come from observations of redshifts from nearby stars. Since part or most of these redshifts may be due to curvature redshift in the surrounding gas, these mass estimates may need to be revised.

If the compact object is rotating there is the tantalizing idea that curvature pressure may produce the emission of material in two jets along the spin axis. This could be the 'jet engine' that produces the astrophysical jets seen in stellar-like objects and in many huge radio sources. Currently there are no accepted models for the origin of these jets. The postulate here is that the jets are a property of the compact object and do not come from the accretion disk. The spinning object provides the symmetry necessary to generate two jets and curvature pressure provides the force that drives the jets. This mechanism is applicable to both stellar and galactic size structures.

3.4 Large number coincidences

It is appropriate to have a brief discussion of famous numerical coincidences in cosmology (Sciama, 1971). First, however we need the results for the size parameters for the CC universe which are shown in Table 3 where the N_{H} is the density divided by the mass of a hydrogen atom. The first large number coincidence is the ratio of the radius of the universe to the classical electron radius (R/r_0). The result is 9.49×10^{40} which is to be compared with the ratio of the electrostatic force to that of the gravitation force between and electron and a proton. This is 4.3×10^{38} which being about 200 times smaller than R/r_0

Table 3: Size of CC universe.

Quantity	Value	SI units
Radius, R	12.5 Gpc	3.86×10^{26} m
Volume, V	2.46×10^{31} pc^3	1.14×10^{81} m^3
Density, N	1.55 m^{-3}	2.58×10^{-27} kg m^{-3}
Mass, M	2.94×10^{54} kg	2.94×10^{54} kg
$N_{\text{total}} = NV$	1.77×10^{81}	1.77×10^{81}

shows that it is hardly a coincidence and although interesting probably has little physical significance.

Sciama (1953, 1971) investigated the use of Mach's principle and the role of inertia in general relativity. By direct analogy to Maxwell's equations, he derived *for rectilinear motion a combination of Newton's laws of motion and of gravitation, with the inertial frame determined by Mach's principle* (his italics). In effect, there is an acceleration term added to Newton's gravitational equation. The consequence is that the total energy (inertial plus gravitational) of a particle at rest in the universe is zero. He further assumed that matter receding with a velocity greater than that of light makes no contribution. The equivalent distance in CC is the radius, R. The implication of his theory is that

$$\frac{2\pi G\rho R^2}{c^2} \approx 1.$$

Now using Eq. 30 we get the actual value for the left hand side to be 3/4 and this value does not depend on the size of the universe. The closeness of this value to unity suggests that Sciage's ideas are worthy of further investigation.

3.5 Solar neutrino production

Since the Homestead mine neutrino detector started operation in the late 1960's, its observations have shown a deficiency in the observed intensity of solar neutrinos compared to accurate theoretical calculations. This has led to an enormous activity in the development and testing of solar models. Currently the standard explanation for the deficiency in the arrival rate of solar neutrinos is that it is due to neutrino oscillations. Basically the electron neutrinos produced near the center of the sun are converted into a mix of muon and tau neutrinos by the time they reach the earth. Because of the high densities the matter oscillation as well as vacuum oscillations are important. Although there are several free parameters that must be estimated the most convincing evidence comes from the Sudbury Neutrino Observatory, where the solar neutrino problem was finally solved. There it was shown that only 34% of the electron neutrinos (measured with one charged current reaction of the electron neutrinos) reach the detector, whereas the sum of rates for all three neutrinos (measured with one neutral current reaction) agrees well with the expectations. The only reason that I include the following, alternative explanation is that I was surprised at how accurate were the results predicted by curvature pressure with no additional parameters. Since this is the only place where local curvature pressure is used it is feasible that its derivation in section is flawed. Nevertheless Because it provides accurate predictions and the neutrino oscillation model has to fit several parameters it is worth examination.

The solar model used here is based on that described by Bahcall (1989). For a local context, curvature pressure is given by Eq. 19. What was done is to use the tables (for solution BS05) generously provided by Bahcall in his

Table 4: Computed production rates for solar neutrinos for the standard model including curvature pressure.

Reaction	Relative rate	Rate $/\mathrm{cm}^2\,\mathrm{s}^{-1}$	Rate/SNU for ^{37}Cl	Rate/SNU for ^{71}Ga
pp	0.829	4.93×10^{10}	0.0	57.8
pep	0.767	1.07×10^{8}	0.17	2.15
$_7$Be	0.537	2.56×10^{9}	0.64	18.4
$_8$B	0.288	1.45×10^{6}	1.67	3.48
$_{13}$N	0.503	2.76×10^{8}	0.045	1.71
$_{15}$O	0.349	1.68×10^{8}	0.115	1.91
$_{17}$F	0.318	1.79×10^{8}	0.0	0.03
hep	0.905	8.42×10^{3}	0.036	0.09
Totals			2.66 ± 0.42	85.6 ± 5.4

web site and used them to calculate curvature pressure. It was then assumed that the thermodynamic pressure was reduced by the value of the curvature pressure and then we used the thermodynamic pressure as an index into the same tables to get the temperature. This largely avoids all the complications of equations of state and changing compositions. Naturally, this will only work if the corrections, as they are here, are small. Then this temperature was used as an index into the neutrino production table to get the production rate for each of the eight listed reactions. As a calibration and a check, the same program was used to compute the rates with no curvature pressure. In this test, the maximum discrepancy from the expected rates was 1.3%.

At a radius of 0.1 solar radii, the reduction in thermodynamic pressure was

12.5% and the reduction in temperature was 4.1%. The computed rates with curvature pressure included in the solar model are shown in Table 4. The standard rates are from Bahcall, Pinsonneau & Basu (2001); Bahcall (1989). The solar neutrino unit (SNU) is a product of the production rate times the absorption cross section and has the units of events per target atom per second and one SNU is defined to be 10^{-36} s^{-1}. For example for each ^{71}Ga target atom in the detector the expected event rate due to solar neutrinos for the pp reaction would be 57.7×10^{-36} s^{-1}. The last row shows the expected event rates for ^{37}Cl and ^{71}Ga target atoms where the uncertainties are proportional to those provided by Bahcall et al. (2001). Another type of detector uses Cherenkov light from the recoiling electron that is scattered by the neutrino. Because this electron requires high-energy neutrinos to give it enough energy to produce the Cherenkov light this type of experiment is essentially sensitive only to the ^{8}B neutrinos.

McDonald (2004) provides a list of recent observational results and they are compared with the predictions in Table 5. The columns show the name of the experiment, the type of detector, the unit, the predicted rate (with curvature pressure), the observed rate, and the χ^2 of the difference from the predicted value. The statistical and systematic uncertainties have been added in quadrature to get the observed uncertainty. The result in the last row from SNO is from the charged current reaction (ν_e+d→p+p+e) that is the expected rate if there are no neutrino oscillations. The agreement is excellent. However, there may be some biases that could be either theoretical or experimental in origin. The crucial test requires computation with a solar model that includes curvature pressure so that the more subtle effects are properly handled. The benefit

Table 5: Comparison of predicted and observed solar neutrino production rates.

Experiment	Unit	Predicted	Observed	χ^2
Homestead	SNU	2.66 ± 0.42	2.56 ± 0.23	0.04
GALLEX+GNO	SNU	85.6 ± 5.4	70.8 ± 5.9	3.42
SAGE	SNU	85.6 ± 5.4	70.9 ± 6.4	3.08
Kamiokande	^{a}	1.45 ± 0.26	2.8 ± 0.38	8.60
Super-Kamiokande	^{a}	1.45 ± 0.26	2.35 ± 0.08	10.95
SNO (e+d)	^{a}	1.45 ± 0.26	1.76 ± 0.10	0.25

$^a 10^6\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$

of this agreement is that it gives very strong support for curvature pressure in a non-cosmological context.

3.6 Heating of the solar corona

For over fifty years, astrophysicists have been puzzled by what mechanism is heating the solar corona. Since the corona has a temperature of about 2×10^6 K and lies above the chromosphere that has a temperature of about 6000K, the problem is where the energy comes from to give the corona this high temperature. Let us consider whether curvature redshift due to the gas in the corona can heat the corona via the energy loss from the solar radiation. Aschwanden (2004) quotes the number distribution of electrons in the corona to be

$$N_{\mathrm{e}} = 2.99 \times 10^{14} r^{-16} + 1.55 \times 10^{14} r^{-6} + 3.6 \times 10^{12} r^{-1.5}\,\mathrm{m}^{-3}, \qquad (34)$$

where r is the distance from the solar center in units of solar radii. If we assume spherical symmetry then all the radiation leaving the sun must pass through a shell centrad on the sun and we can use Eq. 4 and Eq. 34 to compute the fractional energy loss in that shell. To the accuracy required, we can also assume that the hydrogen number density is the same as the electron density and then the integration of Eq. 34 from the solar surface to 4 solar radii above the surface gives a total fractional energy loss of 1.32×10^{-11}. Thus with a solar power output of 3.83×10^{26} W the total energy loss to the solar corona by curvature redshift is 5.1×10^{15} W which is equivalent to 8.3×10^{-4} $\mathrm{W\,m^{-2}}$ at the surface of the sun. This may be compared with the energy losses from the corona to conduction, solar wind and radiation. The total loss rates are quoted by Aschwanden (2004) to be 8×10^{2} $\mathrm{W\,m^{-2}}$ for coronal holes, 3×10^{3} $\mathrm{W\,m^{-2}}$ for the quiet corona and 10^{4} $\mathrm{W\,m^{-2}}$ for an active corona. Since these are about seven magnitudes larger than the predicted loss, curvature redshift is not important in the inner corona. Although it is not pursued here, there is a similar problem in that the Milky Way has a corona with a high temperature. It is intriguing to speculate that curvature redshift may explain the high temperature of the galactic halo.

3.7 Pioneer 10 acceleration

Precise tracking of the *Pioneer* 10/11, *Galileo* and *Ulysses* spacecraft (Anderson et al., 1998a, 2002) have shown an anomalous constant acceleration for *Pioneer* 10 with a magnitude $(8.74 \pm 1.55) \times 10^{-10}$ $\mathrm{m\,s^{-2}}$ directed towards the sun. The major method for monitoring *Pioneer* 10 is to measure the frequency shift of the signal returned by an active phase-locked transponder. These frequency measurements are then processed using celestial mechanics in order to get the spacecraft trajectory. The simplicity of this acceleration and its magnitude suggests that *Pioneer* 10 could be a suitable candidate for investigating the effects of curvature redshift. There is a major problem in that the direction of the acceleration corresponds to a blue shift whereas curvature redshift predicts a redshift. Nevertheless, we will proceed, guided by the counter-intuitive observation that a drag force on a satellite actually causes it to speed up. This is because the decrease in total energy makes the satellite change orbit with a redistribution of kinetic and potential energy.

The crucial point of this analysis is that the only information available that can be used to get the *Pioneer* 10 trajectory is Doppler shift radar. There is no direct measurement of distance. Thus the trajectory is obtained by applying celestial mechanics and requiring that the velocity matches the observed frequency shift. Since the sun produces the dominant acceleration we can consider that all the other planetary perturbations and know drag effects have been applied to the observations and the required celestial mechanics is to be simple two body motion. If the observed velocity (away from the sun) is increased (in magnitude) by an additional apparent velocity due to curvature redshift the orbit determination program will compensate by assuming that the spacecraft is closer to the sun than its true distance. It will be shown that this distance discrepancy produces an extra apparent acceleration that is directed towards the sun. The test of this model is whether the densities required by curvature redshift agree with the observed densities.

Let the actual velocity of *Pioneer* 10 at a distance r, be denoted by $v(r)$, then since the effect of curvature redshift is seen as an additional velocity, $\Delta v(r)$ where from Eq. 4 it is given by

$$\Delta v(r) = 2\sqrt{8\pi G}\int_0^r \sqrt{\rho(r)}\,dr \tag{35}$$

where the factor of 2 allows for the two-way trip and the density at the distance r from the sun is $\rho(r)$. Since *Pioneer* 10 has a velocity away from the sun this redshift shows an increase in the magnitude of its velocity. We will assume that all the perturbations and any other accelerations that may influence the *Pioneer* 10 velocity have been removed as corrections to the observed velocity and the remaining velocity, $v(r)$, is due to the gravitational attraction of the sun. In this case the energy equation is

$$v(r)^2 = v_\infty^2 + \frac{2\mu}{r}, \tag{36}$$

where $\mu = GM$ is the gravitational constant times the mass of the sun ($\mu = 1.327 \times 10^{20}\,\mathrm{m}^3\,\mathrm{s}^{-2}$) and v_∞ is the velocity at infinity. The essence of this argument is that the tracking program is written to keep energy conserved so that an anomalous change in velocity, $\Delta v(r)$, will be interpreted as a change in radial distance we get

$$\Delta r = -\sqrt{\frac{2r^3}{\mu}}\,\Delta v(r).$$

Thus an increase in magnitude of the velocity will be treated as a decrease in radial distance which, in order to keep the total energy constant, implies an increase in the magnitude of the acceleration. Either by using Newton's gravitational equation or by differentiating Eq. 36 the acceleration $a(r)$ is given by

$$a(r) = -\frac{\mu}{r^2}. \tag{37}$$

Hence with $v_\infty = 0$ and therefore $v(r) = \sqrt{2\mu}/r$ we get

$$\Delta a(r) = \frac{2\mu}{r^3}\,\Delta r = \sqrt{\frac{8\mu}{r^3}}\,\Delta r$$

and then to the first order an increase in velocity of $\Delta v(r)$ will produce an apparent decrease in acceleration of $\Delta a(r)$, and

$$\begin{aligned}
\Delta a &= 8\sqrt{\pi\mu G}\, r^{-3/2} \int_0^r \sqrt{\rho(r)}\, dr \\
&= 16\sqrt{\pi\mu G}\, r^{-1/2} < \sqrt{\rho(r)} > \\
&= 6.90 R^{-1/2} < \sqrt{\rho(r)} > \qquad (38)
\end{aligned}$$

where for the last equations we measure the distance in AU so that $r = 1.496 \times 10^{11} R$ and the angle brackets show an average value. Now fig. 7 from (Anderson et al., 2002) shows that after about 20 AU the anomalous acceleration is essentially constant. The first step is to get an estimate of the required density and see if is feasible. Using the observed acceleration of $a_{\mathrm{P}} = 8.74 \times 10^{-10}\ \mathrm{m\,s^{-2}}$ the required average density for the two-way path is $1.60 \times 10^{-20} R\,\mathrm{kg\,m^{-3}}$ and for R=20 it is $3.21 \times 10^{-19}\,\mathrm{kg\,m^{-3}}$.

The only constituent of the interplanetary medium that approaches this density is dust. One estimate by Le Sergeant D'Hendecourt & Lamy (1980) of the interplanetary dust density at 1 AU is $1.3 \times 10^{-19}\,\mathrm{kg\,m^{-3}}$ and more recently, Grün (1999) suggests a value of $10^{-19}\,\mathrm{kg\,m^{-3}}$ which is consistent with their earlier estimate of $9.6 \times 10^{-20}\,\mathrm{kg\,m^{-3}}$ (Grün, Zook & Giese, 1985). Although the authors do not provide uncertainties it is clear that their densities could be in error by a factor of two or more. The main difficulties are the paucity of information and that the observations do not span the complete range of grain sizes. The meteoroid experiment on board *Pioneer* 10 measures the flux of grains with masses larger than 10^{-10} g. The results show that after it left the influence of Jupiter the flux (Anderson et al., 1998b) was essentially constant (in fact there may be a slight rise) out to a distance of 18 AU. It is thought that most of the grains are being continuously produced in the Kuiper belt. As the dust orbits evolve inwards due to Poynting-Robertson drag and planetary perturbations, they achieve a roughly constant spatial density. The conclusion is that interplanetary dust could provide the required density to explain the anomalous acceleration by a frequency shift due to curvature redshift.

Anderson et al. (2002) also reports a annual velocity variation of $(1.053 \pm 0.107) \times 10^{-4}\,\mathrm{m\,s^{-1}}$ with a phase angle relative to conjunction of $5°.7 \pm 1°.7$. The cause of this variation is the changing path length through the dust at about 1 AU as the earth cycles the sun. However if this annual variation is due to curvature redshift it cannot be easily distinguished from a position displacement in the plane of the ecliptic: for example this anomalous velocity corresponds to a position shift of about 5×10^{-3} arcsec. From Eq. (35 and a density of $10^{-19}\,\mathrm{kg\,m^{-3}}$ the predicted curvature-redshift velocity is $3.9 \times 10^{-3}\,\mathrm{m\,s^{-1}}$ which is an order of magnitude larger than the reported anomalous diurnal velocity. Clearly most of the predicted velocity could have been interpreted by the orbit determination program as a very small angular displacement. This could also explain the phase angle. The predicted phase angle is 90° from conjunction, whereas the observed phase angle is very close to the line of conjunction.

Finally Anderson et al. (2002) reports a diurnal component. Reading from their fig. 18 the diurnal velocity amplitude is about $1.4 \times 10^{-4}\,\mathrm{m\,s^{-1}}$. Note that due to inhibition there is no curvature redshift to be expected from the atmosphere. The major redshift will come from the inter-planetary dust. Then using the earths radius and a density of $10^{-19}\,\mathrm{kg\,m^{-3}}$ the expected diurnal velocity amplitude due to curvature redshift is $1.7 \times 10^{-7}\ \mathrm{m\,s^{-1}}$ which is three orders of magnitude too small. The average density that is needed is about $7.2 \times 10^{-14}\,\mathrm{kg\,m^{-3}}$. Unless there is such a density it is unlikely that curvature redshift could explain the diurnal velocity effect. Note a critical test would be to compare the simultaneous observation of the *Pioneer* 10 velocity from two tracking stations as a function of their different distances from *Pioneer* 10.

Overall, this analysis has shown that it is possible to explain the acceleration anomaly of *Pioneer* 10 but that a more definitive result requires curvature redshift to be included in the fitting program and more accurate estimates of the dust density are certainly needed. Subject to the caveat about the dust density, curvature redshift could explain the anomaly in the acceleration of *Pioneer* 10 (and by inference other spacecraft).

4 Conclusion

Curvature cosmology is a well defined tired-light cosmology which obeys the perfect cosmological principle. Part 1 and Part 2 showed that is has good agreement with a wide range of cosmological observations. There are problems with galactic rotation curves and a lack of a quantitative exposition for the abundances of light elements. There are possible laboratory tests of the theory but, at present, there is no definitive observations that would refute the theory.

Curvature pressure can explain the non-cosmological topic of solar neutrino production but since this already explained by neutrino oscillations it must remain a curiosity. The explanation of the *Pioneer* 10 anomalous acceleration is feasible if the inter-planetary dust density is a little larger than current estimates.

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