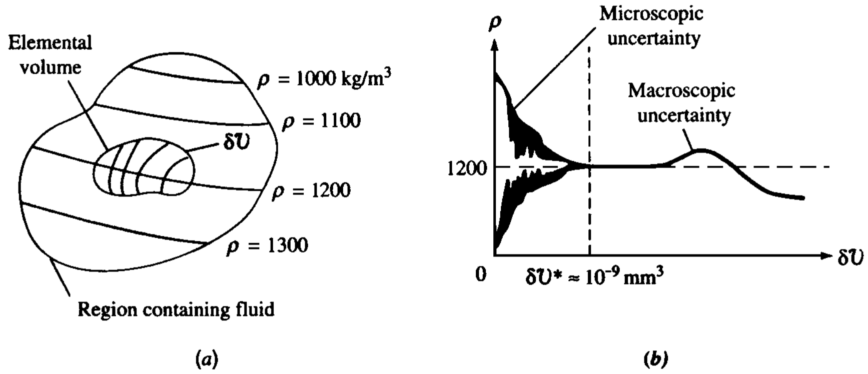
Physical (fluid) properties are assumed to vary continuously in space and each property is essentially a point function. Discontinuities may only occur across interfaces separating two phases and across shock waves. Differential calculus is applicable. (*L. Euler*, ~ 1755).[1]

Fig. 1.4 The limit definition of continuum fluid density: (*a*) an elemental volume in a fluid region of variable continuum density; (*b*) calculated density versus size of the elemental volume.



$$\text{Density (mass per unit volume):}\quad \rho = \lim_{\delta\mathcal{V}\to\delta\mathcal{V}^*} \frac{\delta m}{\delta\mathcal{V}}$$

For all gases at around sea-level conditions and for all liquids the *continuum limit* $\delta\mathcal{V}^*$ is at around 10^{-9} mm^3, corresponding to a cubic micron. For air at (1 atm, 20°C) this volume contains about 30×10^6 molecules. For extremely rarified gases, e.g. air in the outer parts of the atmosphere, the continuum limit might be so large (in relation to length dimensions of an assumed immersed body) that the continuum hypothesis is simply not valid. Another area where the macroscopic continuum approach might not be valid is gas flow in micro- and nano-tubes/channels. Such areas are not included in this course.

Fluid particle (fluid element) = a small but macroscopic accumulation of fluid of certain mass with a volume that is of the order $\delta\mathcal{V}^*$.

[1] Leonhard Euler, *April 15, 1707 (Basel, Switzerland) †September 18, 1783 (St. Petersburg, Russia).

A *fluid* is a substance (medium) that deforms continuously when being acted on by an arbitrarily small shearing stress.

$\Rightarrow$ No shear forces are possible for a fluid at rest.

$$\boxed{\textit{fluid = gas or liquid}}$$

As opposed to a fluid, a solid body can resist a certain amount of shearing action by static deformation. In a solid the molecules are always close to their neighbors; in a fluid the molecules can move about much more freely, and if being acted on by a shearing force action, whatever small this action may be, the fluid responses with a time-dependent deformation/movement, a fluid motion, a flow.

NEWTONIAN FLUID

Sir Isaac Newton[2] *1687 (Principia, Book II, Section IX):*
The resistance arising from the want of lubricity in the parts of a fluid is, other things being equal, proportional to the velocity with which the parts of the fluid are separated from one another.

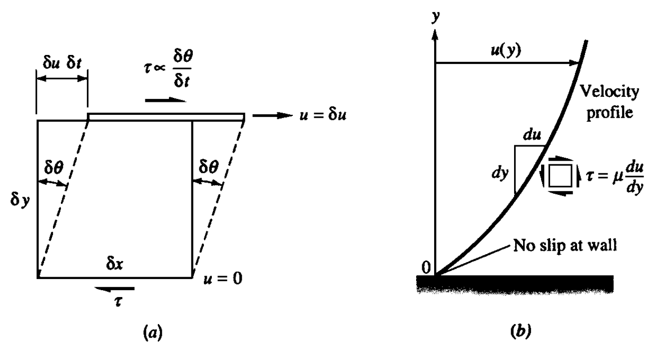


Fig. 1.6 Shear stress causes continuous shear deformation in a fluid: (*a*) a fluid element straining at a rate $\delta\theta/\delta t$; (*b*) newtonian shear distribution in a shear layer near a wall.

$$\tau \propto \frac{\delta u}{\delta y} \Rightarrow \tau = \mu \frac{du}{dy} = \mu \frac{d\theta}{dt} \quad (\mu = \text{dynamic viscosity})$$

A *Newtonian fluid* is a fluid in which the viscous stresses acting in a plane are proportional to the corresponding strain rates within the same plane. Shear stresses are proportional to shear strain rates (rates of angular deformation), normal stresses to normal strain rates (rates of linear deformation).

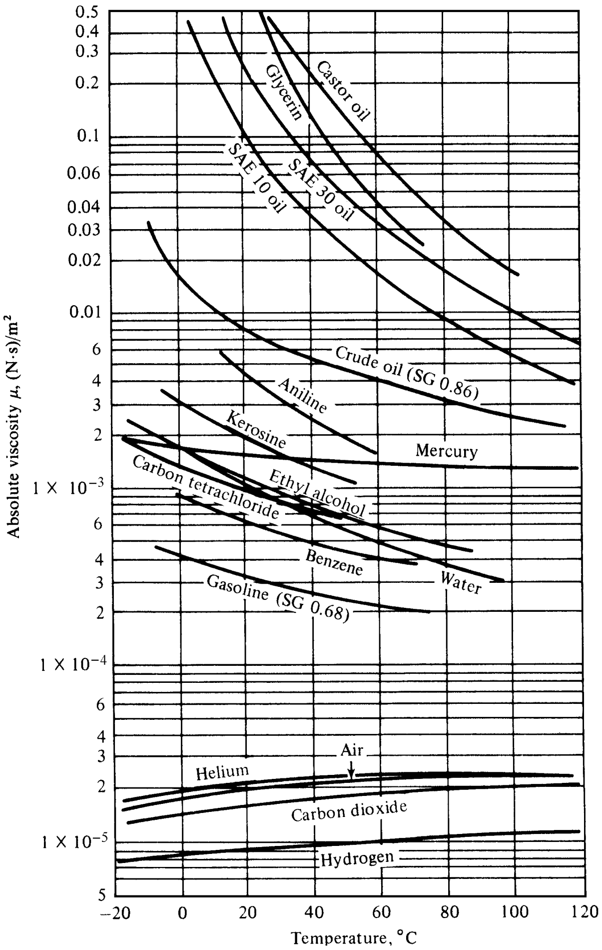
[2]*Dec. 25, 1642 (OS), Woolsthorpe, Lincolnshire; †March 20, 1727 (OS), Kensington, London.

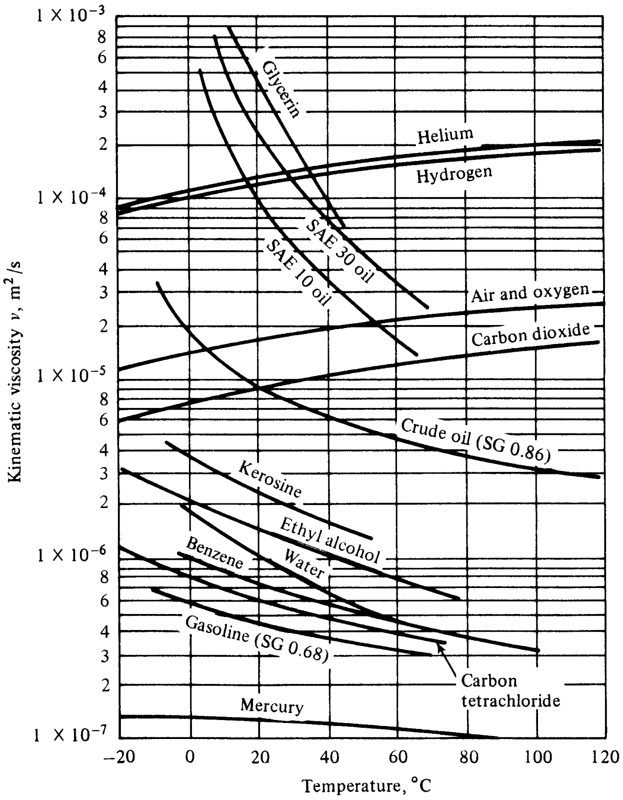
FLUID DENSITY AND VISCOSITY

Pure substances, one phase (gas or liquid):

$$\rho = \rho(p, T),\ \mu = \mu(p, T),\ \nu = \mu/\rho = \nu(p, T)$$

The influence of pressure on μ can often be neglected, $\mu \simeq \mu(T)$





- Liquid water ($0°\mathrm{C} \le T \le 100°\mathrm{C}$)

  $\rho = 1000. - 0.0178\,|\,T(°\mathrm{C}) - 4.0\,|^{1.7} \quad [\mathrm{kg/m^3}]$

  $\ln \mu/\mu_0 = -1.704 - 5.306\,z + 7.003\,z^2,$

  $z = 273.15/T(\mathrm{K}),\ \mu_0 = 1.788 \times 10^{-3}\ \mathrm{Pa\,s}$

- Dry air ($-40°\mathrm{C} \le T \le 250°\mathrm{C},\ p < 1.5\ \mathrm{MPa}$)

  $\rho = p/(RT),\ R = 287.0\ \mathrm{J\,kg^{-1}\,K^{-1}}$

  $\mu/\mu_0 = (T/T_0)^{3/2}(T_0 + C)/(T + C),\ T_0 = 273.15\ \mathrm{K},$

  $C = 110.4\ \mathrm{K},\ \mu_0 = 17.23 \times 10^{-6}\ \mathrm{Pa\,s}$

Reynolds number: $\mathrm{Re} = \dfrac{\rho V L}{\mu} = \dfrac{V L}{\nu}$

V = characteristic velocity, e.g. the average velocity in a pipe

L = characteristic length, e.g. the pipe diameter

ν = kinematic viscosity ($\nu = \mu/\rho$)

Reynolds number determines the flow character

Re *sufficiently low* ⇒ **laminar flow**

Ordered and layered motion; low mixing capability

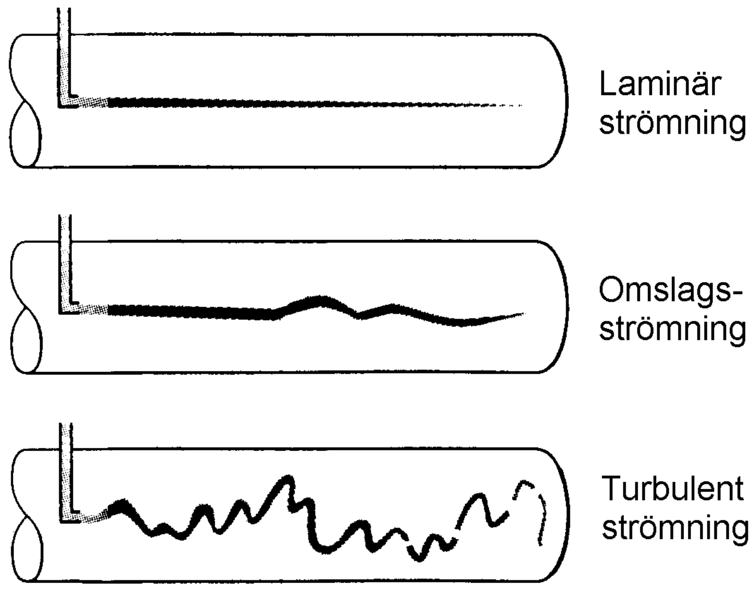
Re *sufficiently high* ⇒ **turbulent flow**

Disordered, seemingly chaotic motion; high mixing capability

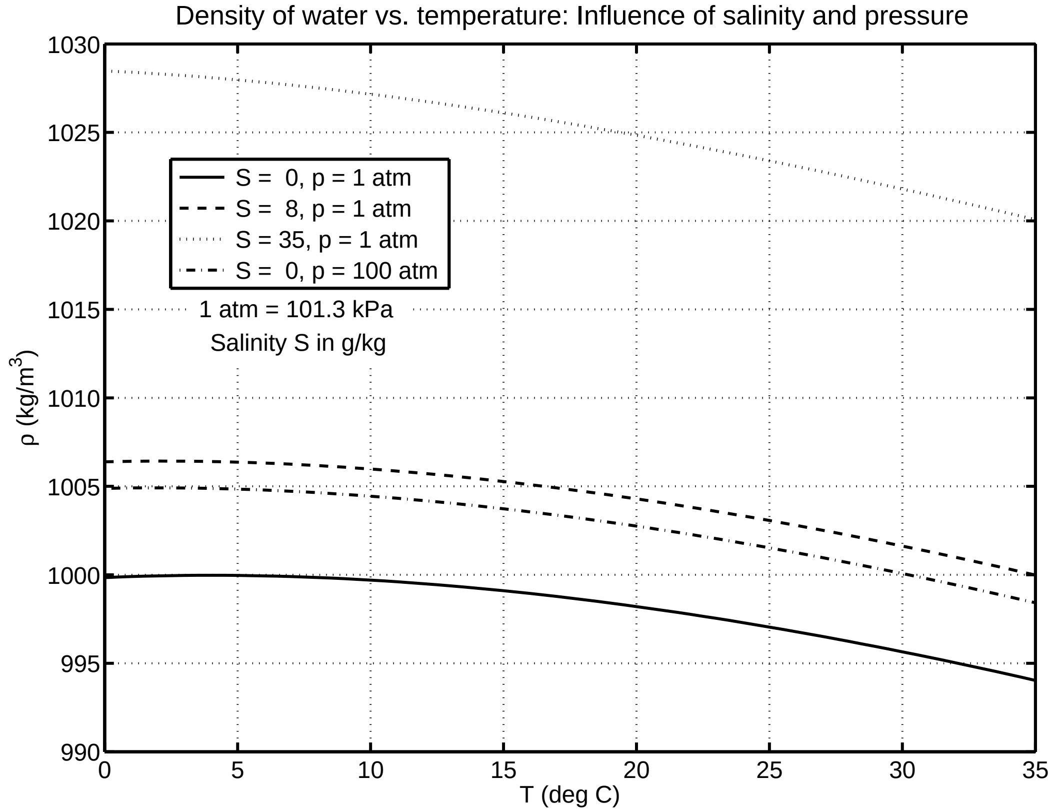
Ex. pipe flow (circular cross section): $\mathrm{Re} = \rho V d/\mu = V d/\nu$

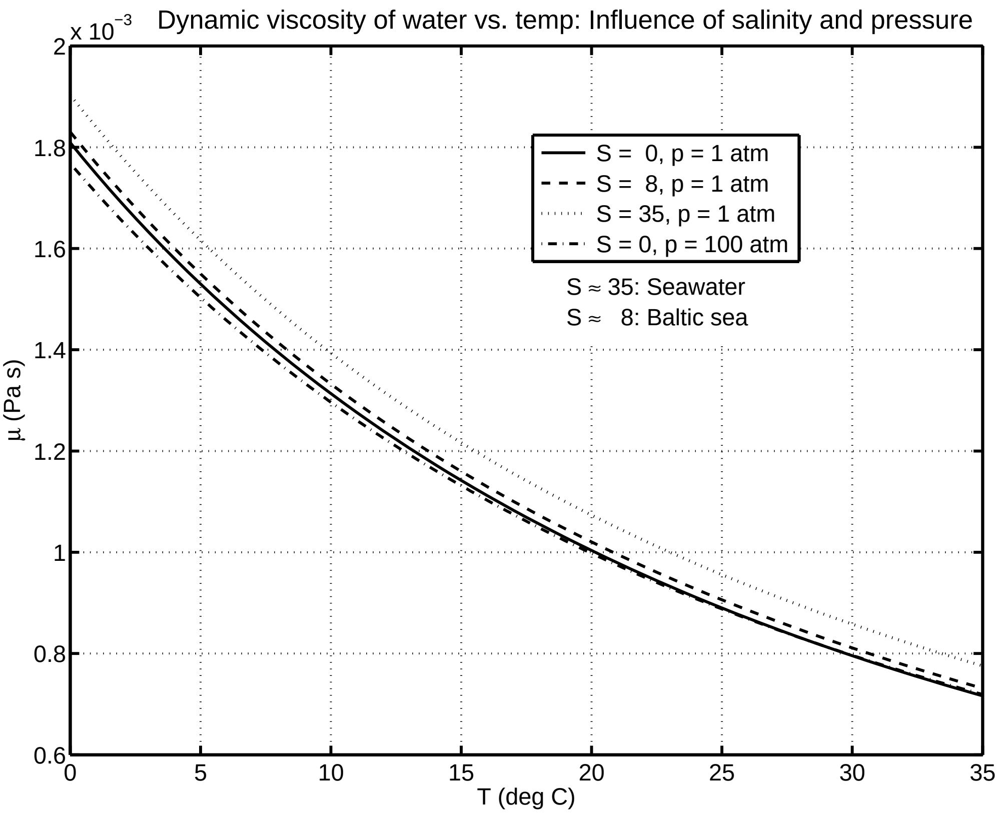
$\mathrm{Re} < 2100 \Rightarrow$ laminar flow

$\mathrm{Re} > 4000 \Rightarrow$ turbulent flow

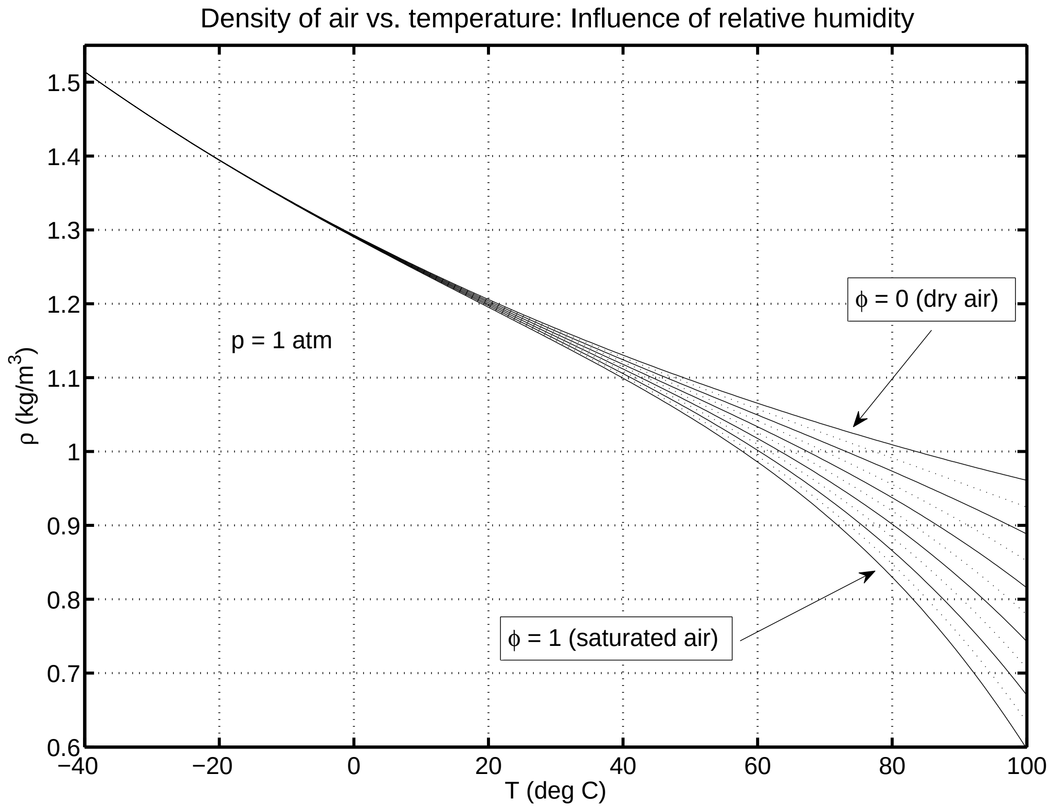


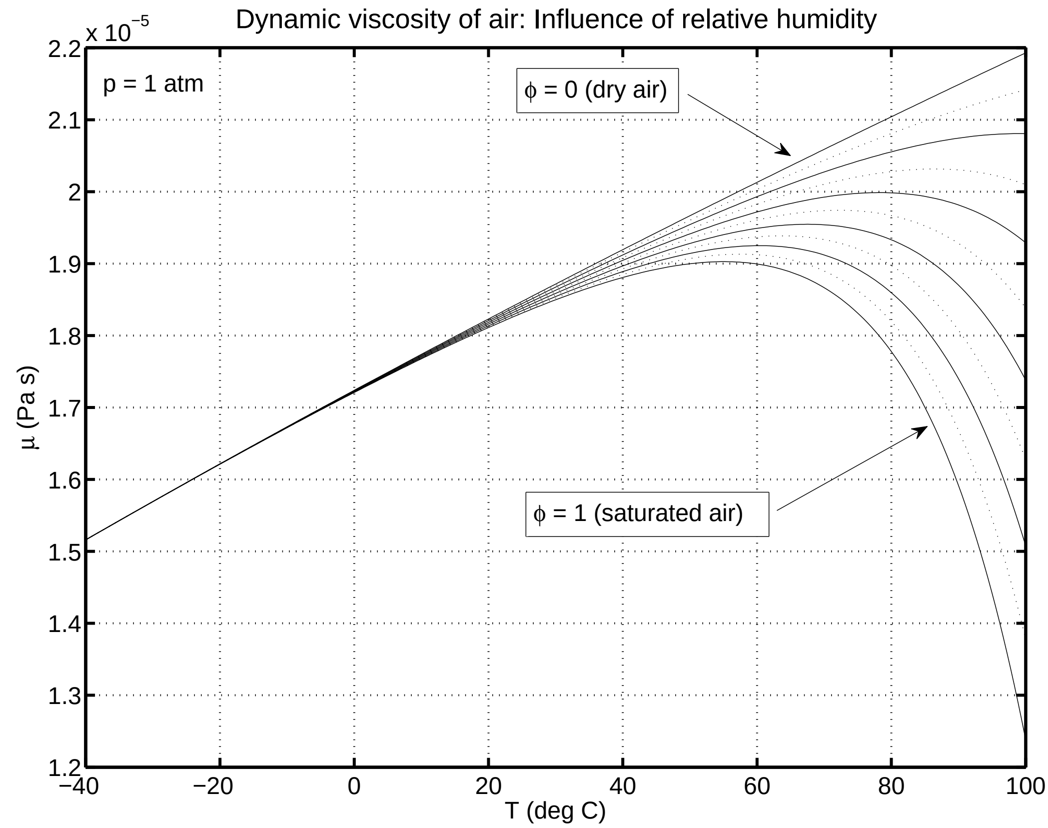
EFFECTS OF SALINITY AND PRESSURE (WATER)





MOISTURE AIR: DENSITY AND VISCOSITY

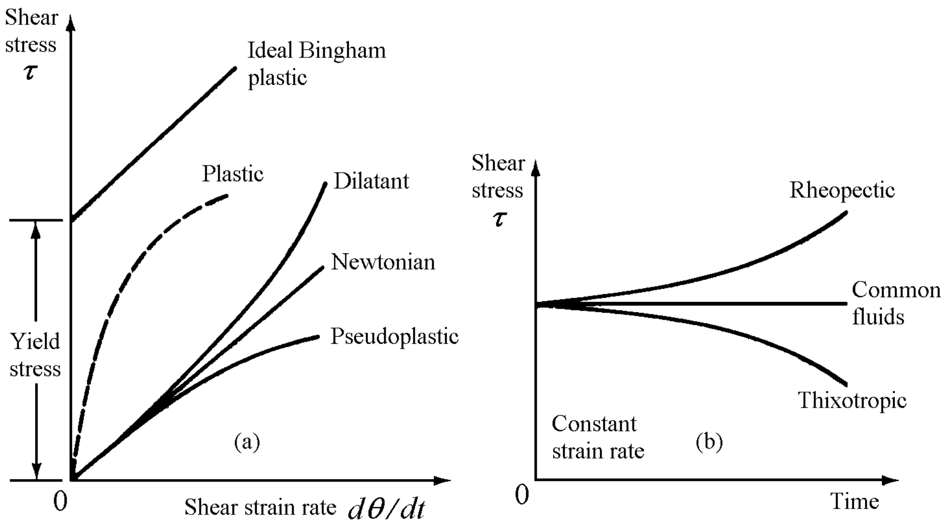




Newtonian: linear relation between viscous stresses and strain rates, e.g. shear stress, $\tau \propto d\theta/dt = du/dy$ for a simple shear flow; valid for (i) all gases at low to moderate pressures and (ii) most low-molecular pure liquids. There are four main types of *non-Newtonian fluids*:

1. Non-linear relation $\tau = f(d\theta/dt) = f(\dot{\theta})$, slope at any point being called the apparent viscosity; *dilatant (hardening) fluids*: wet sand, starch in water; *pseudoplastic (thinning) fluids*: polymer and colloidal solutions, paper pulp in water, syrup, glue.
2. Combination elasticity – viscous effects, *viscoelasticity*; asphalt, molten metals (near melting point), liquid crystals; Bingham plastic: like a solid up to its yield point, thereafter like a fluid, e.g. mud, toothpaste, mayonnaise.
3. Time-dependent deformation – memory effects, e.g. $\dot{\theta} = const. \Rightarrow$ τ time-dependent; *rheopectic*, $\tau \uparrow$, e.g. gypsum paste; *thixotropic*, $\tau \downarrow$, e.g. ketchup, honey, thixotropic paint.
4. Shear $\Rightarrow$ normal stress, e.g. the Barus' dough effect.

Fig. 1.9
Rheological behavior of various viscous materials: (a) stress versus strain rate; (b) effect of time on applied stress.



Non-Newtonian behavior is common in all melts and solutions with very high molecular weights (polymers); also in many suspensions (slurries) and multi-phase mixtures, e.g. blood plasma, mixtures of crude oil and natural gas.

Velocity vector: $\mathbf{V} = \mathbf{V}(\mathbf{x}, t) = (u, v, w)$

- At a given instant and everywhere on a *streamline* the velocity vector is a tangent vector.

$$d\mathbf{x} \times \mathbf{V} = \mathbf{0} \Rightarrow \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

A collection of streamlines that pass through a closed curve forms a *streamtube*.

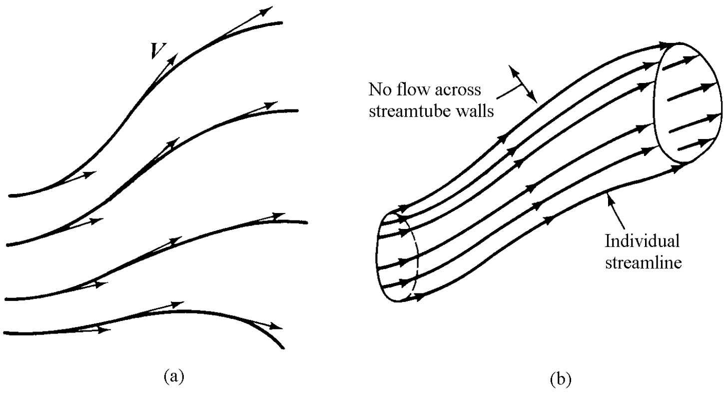
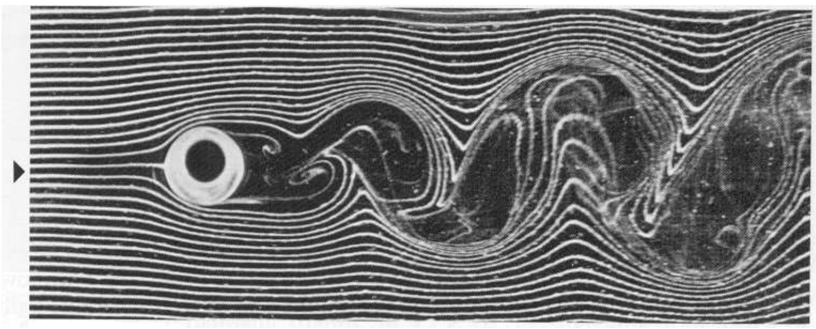


Fig. 1.16 The most common method of flow-pattern presentation: (a) streamlines are everywhere tangent to the local velocity vector; (b) a streamtube is formed by a closed collection of streamlines.

- A *streakline* is the locus of a continuous stream of fluid particles that have earlier passed through a prescribed point.



Streaklines. Flow around a cylinder, Re = 170 (Y. Nakayama 1988).

- A *pathline* is the actual path traversed by a fluid particle.
- A *timeline* is the locus of many fluid particles that at a certain previous time were along a (straight) line.

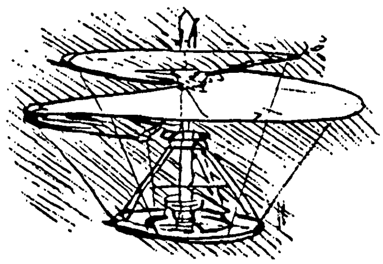
Stationary flow $\Rightarrow$

streamlines, streaklines and particle paths are equal

CHRONOLOGY OF FLUIDS HISTORY

Year	Name	Contribution
250BC	**Archimedes** of Syracuse, Ita.	Buoyancy
1500	**Leonardo** da Vinci, Ita.	Continuity equation, ...

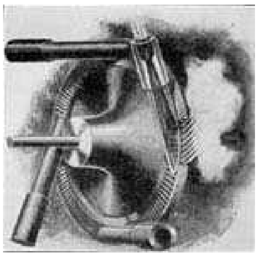
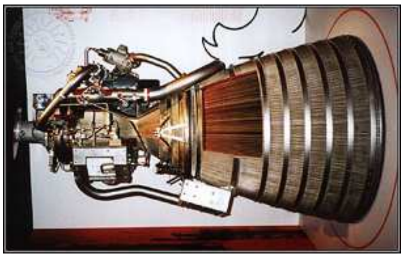
Figure 1.8. Leonardo: *Old man and Vortices;* probably a self-portrait (Windsor Castle, Royal Library, copyright reserved).



1612	**Galileo** Galilei, Ita.	Hydrostatics
1643	Evangelista **Torricelli**, Ita.	Theorem, manometer
1663	Blaise **Pascal**, Fra.	Pressure and capillary forces
1686	Edmé **Mariotte**, Fra.	Fluid forces (exp.)
1687	Isaac **Newton**, GB	Viscosity, $F \propto \rho V^2 A$
1732	Henri de **Pitot**, Fra.	Pitot tube
1738	Daniel **Bernoulli**, Hol.	*Hydrodynamica*
1752	Jean le Rond **d'Alembert**, Fra.	Paradox, streamline
1755	Leonhard **Euler**, Swi.	Continuum mechanics
1768	Antoine de **Chézy**, Fra.	Flow in channels (exp.)
1781	Joseph L. de **Lagrange**, Fra.	Mathematical tools
1797	Giovanni B. **Venturi**, Ita.	Flow meter
1800	Charles A. de **Coulomb**, Fra.	$F \propto \mu V$, $\propto \rho V^2$ (exp.)
1809	Sir George **Cayley**, GB	Aerodynamics (exp.), flight theory
1816	Pierre S. Marquis de **Laplace**, Fra.	Mathematical tools
1827	Claude L. M. -H. **Navier**, Fra.	Viscous flows
1836	John **Ericsson**, Swe.	Propeller design
1839	Gotthilf H. L. **Hagen**, Ger.	Laminar/turbulent pipe flow
1840	Jean L. M. **Poiseuille**, Fra.	Laminar pipe flow
1845	Sir George G. **Stokes**, GB	Navier-Stokes equations
1852	Ferdinand **Reech**, Fra.	Free-surface flows
1855	Julius **Weisbach**, Ger.	Hydraulics
1858	Hermann L. F. v. **Helmholtz**, Ger.	Mathematical tools
1860	Georg F. B. **Riemann**, Ger.	Gas dynamics
1867	William J. M. **Rankine**, GB	Wave and gas dynamics
1869	Lord **Kelvin**, GB	Mathematical tools
1872	William **Froude**, GB	Surface friction, free-surface flow
1877	Joseph V. **Boussinesq**, Fra.	Turbulent viscosity

CHRONOLOGY OF FLUIDS HISTORY ...

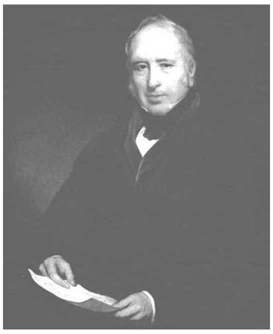
År	Namn	Bidrag
1878	Čeněk **Strouhal**, Cz.	Vortex tones
1880	Lord **Rayleigh**, GB	Stability, ...
1883	Osborne **Reynolds**, GB	Transition to turbulence
1885	Horatio **Phillips**, GB	First wind tunnel
1887	Ernst **Mach**, Aust.	Compressible flow
1887	Pierre H. **Hugoniot**, Fra.	Compressible flow
1889	Carl Gustav Patrik de **Laval**, Swe.	Laval nozzle

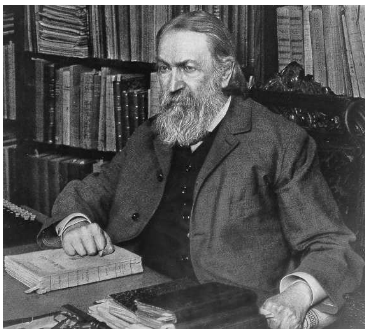
1894	Osborne **Reynolds**, GB	Turbulence modeling
1902	M. Wilhelm **Kutta**, Ger.	Aerodynamics (theory)
1903	Bröderna **Wright**, USA	Aerodynamics, aircraft (exp.)
1904	Ludwig **Prandtl**, Ger.	Boundary layer
1905	Vagn Walfrid **Ekman**, Swe.	Rotating flow, oceanography
1906	Nikolai E. **Zhukovskii**, Rus.	Lift per unit width = $\rho V \Gamma$
1907	Frederick W. **Lanchester**, GB	*Aerodynamics*, theory of flight
1908	Henri C. **Bénard**, Fra.	Vortex shedding
1908	P. R. Heinrich **Blasius**, Ger.	Boundary layer, pipe flow
1909	Dmitri P. **Riabouchinski**, Rus.	Hot-wire anemometry, CCA
1909	Martin H. C. **Knudsen**, Den.	Flow of rarified gases
1912	Gustave A. **Eiffel**, Fra.	Aerodynamics, "Drag crisis"(exp.)
1912	Theodore von **Kármán**, Hun.	von Kármán vortex street
1912	John T. **Morris**, GB	Hot-wire anemometry, CTA
1918	Ludwig **Prandtl**, Ger.	Three-dimensional wing theory
1920	Sir Geoffrey Ingram **Taylor**, GB	Rotating flow, gas dynamics
1925	Theodore von **Kármán**, Hun.	Turbulence theory, log-law
1928	Alexander **Thom**, GB	Numerical solution of N-S equations
1935	Sir Geoffrey Ingram **Taylor**, GB	Turbulence modeling
1941	Carl-Gustav Arvid **Rossby**, Swe.	Rotating flow, meteorology
1941	Andrei N. **Kolmogorov**, Rus.	Turbulence theory
1964	Y. **Yeh** & H. Z. **Cummins**, USA	Laser-Doppler Anemometry, LDA
1979	Jackson R. **Herring**, GB	Large-Eddy Simulation, LES
1981	Parviz **Moin** & John **Kim**, USA	Direct Numerical Simulation, DNS
1984	Ronald J. **Adrian**	Particle-Image Velocimetry, PIV


Daniel Bernoulli 1700–1782


Leonhard Euler 1707–1783

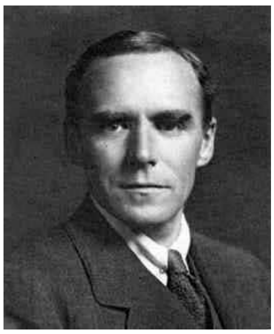

George Cayley 1773–1857


George G. Stokes 1819–1903

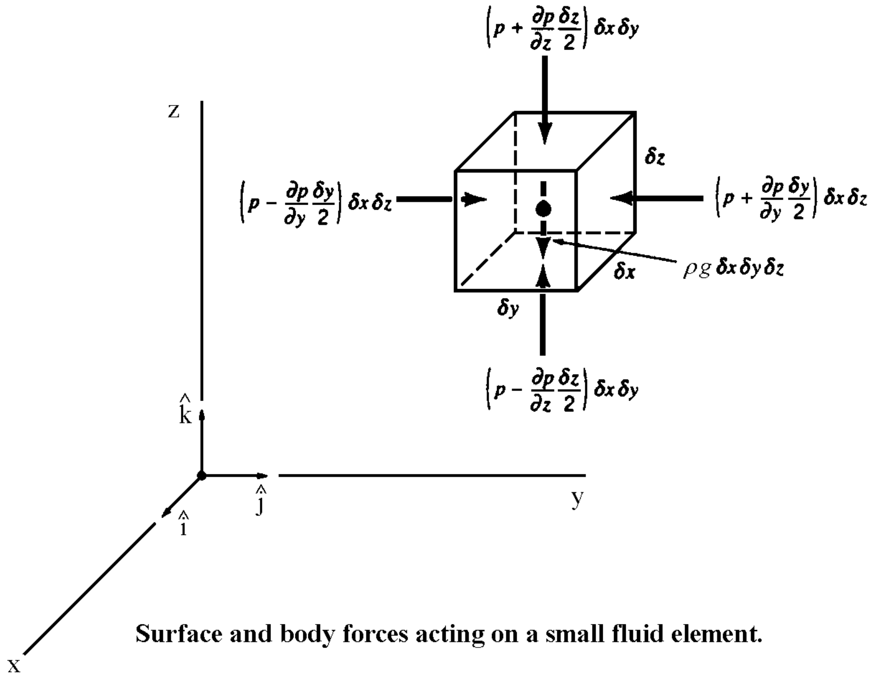

Ernst Mach 1838-1916


Osborne Reynolds 1842–1912


Ludwig Prandtl 1875–1953


G. I. Taylor 1886-1975

Consider an infinitesimal fluid element, formed as a parallelepiped, see figure; pressure at center: $p = p(x, y, z)$, gravity downwards (in negative z-direction). Per definition the pressure acts normal to the surfaces and inwards towards the center of the element.



Surface and body forces acting on a small fluid element.

On the upper surface, at level $z+\delta z/2$, where δz is at the continuum limit, the pressure equals $p+(\partial p/\partial z)(\delta z/2)$; on the lower surface at $z - \delta z/2$, pressure is $p + (\partial p/\partial z)(-\delta z/2)$. Net pressure force in z-direction: $(\partial p/\partial z)(\delta z\,\delta x\,\delta y) = (\partial p/\partial z)\delta\mathcal{V}$. Sine the fluid is at rest, this force must balance the gravity force, $-\rho\,g\,\delta\mathcal{V}$, i.e.

$$\frac{\partial p}{\partial z} + \rho\,g = 0$$

There is no gravity force in the other two directions, i.e.

$$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0 \Rightarrow p = p(z) \Rightarrow$$

$$\frac{dp}{dz} = -\rho\,g = -\gamma$$

$\gamma = \rho\,g = const. \Rightarrow$ pressure increases linearly with depth.