Could time be logarithmic?

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Abstract

This paper explores the consequences of the hypothesis that time might be logarithmic in absolute time, the age of the universe. Newton's laws, Maxwell's equations and certain relativistic Einstein's relations are considered. Two versions of logarithmic time are explored: first where the physical laws are written in fixed space, and second where space itself is expanding. Both imply that what we thought were physical constants like the gravitational constant and the speed of light are time-dependent in linear time, and only constant when derived in log-time coordinates. New definitions of mass, velocity and acceleration are also needed. Implications for dark matter and dark energy could be profound.

1 Introduction

The description of a universe evolving in '*epochs*' of logarithmic time is familiar to every cosmologist (see Table 1 taken from Wikipedia [10]). There appears to be no evidence that it has ever been previously considered that logarithmic time should be applied to physical laws. This paper simply explores the implications and consequences if our physical laws should have been written using log-time as well.

The idea of time varying logarithmically has been present in turbulence theory for some time, but only recently recognized [3]. There it is the growth of length scales with time in the decaying homogeneous turbulence which dictates that turbulence evolves in logarithmic time increments. In effect 'time' slows down as the turbulence decays. Originally it was thought that perhaps such 'solutions' applied to the universe as well, and indeed they might. If so, what we think we perceive as the universe expanding might simply be the scales growing – as in the turbulence solutions. But this paper explores an entirely different idea – that time itself might be logarithmic. And that perhaps the

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Log-time	Seconds after the Big Bang	Period
-45 to -40	10^{-45} to 10^{-40}	Plank Epoch
-40 to -35	10^{-40} to 10^{-35}	
-35 to -30	10^{-35} to 10^{-30}	Epoch of the Grand
-30 to -25	10^{-30} to 10^{-25}	Unification
-25 to -20	10^{-25} 10 10^{-20}	
-20 to -15	10^{-20} to 10^{-15}	
-15 to -10	10^{-15} to 10^{-10}	Electroweak Epoch
-10 to -5	10^{-10} 10 10^{-5}	
-5 to 0	10^{-5} to 10^{-0}	Hadron Epoch
0 to +5	10^{0} to 10^{5}	Lepton Epoch
+5 to + 10	10^{5} to 10^{10}	Epoch of Nucleosynthesis
+10 to +15	10^{10} to 10^{15}	Epoch of Galaxies
+15 to +20	10^{15} to 10^{20}	

Table 1: Each row is defined in seconds after the Big Bang epochs of logarithmic time in cosmology with earliest at the top. The present time is approximately 4.3×10^{17} seconds after the Big Bang.[10]

laws of physics should have been written using logarithmic time. Note that we are not talking about simply a coordinate transformation of existing equations and laws here, but instead completely new equations and laws. But equations and laws close enough to those we have believed for over a century now that we would have accepted them as being accurate descriptions of nature.

The most important difference in a log-time universe from a linear time one would be that we must modify our definition of mass. And of course velocity and acceleration as well. This will be seen to have no measurable consequences over the span of our human existence. But it has great consequences for how we interpret the results of applying our physical laws to astronomical observations of events that happened long ago. In particular, when viewed using linear time, mass will appear to us to be missing, even when it is not. The gravitational constant will appear to be time-dependent. And the speed of light will be slowing down. Also the universe might appear to be expanding, even if it is not.

This paper is organized in three parts. First it defines logarithmic time and its relation to absolute time (as measured from a beginning). Then it examines the consequences for two of our physical laws if they should have been written as log-time but in fixed space variables. And finally it considers how things might be different if we had to write our laws in both log-time and spatially changing coordinates. An attempt has been made to make this as understandable as possible to those with even a high school and early college level physics background.

2 The hypothesis of logarithmic time

It is hypothesized that a *cosmos time*, say τ, might describe the cosmos and all physical laws governing it. It is further postulated that this *cosmic time* varies as the logarithm of t/t_o; i.e.,

$$\tau = \ln t/t_o \tag{1}$$

where t is the time (or *absolute time*) measured in linear increments since the beginning at $t = 0$. t_o is any convenient time scale and sets the units of t (e.g., seconds or years). The exact beginning chosen is arbitrary, but for our purposes the Big Bang is probably acceptable. No quantum mechanical effects will be considered (although they could be) so the equations considered herein apply only to any post-inflationary period, even though time itself could vary logarithmically even to the beginning of time.

3 Would we have noticed any difference?

Physics (and Mechanics in particular) are mostly concerned with time differences. But any cosmic (or logarithmic) time difference between two cosmic times, say τ_p and $\tau_p + \delta\tau$, can expressed in linear time increments by the difference of their Taylor series expansions; i.e.,

$$\begin{aligned}
\delta\tau &= \ln[(t_p + \delta t)/t_o] - \ln[t_p/t_o] &(2)\\
&= \ln[(t_p/t_o)(1 + \delta t/t_p)] - \ln(t_p/t_o) &(3)\\
&= \ln(t_p/t_o) + \ln(1 + \delta t/t_p) - \ln(t_p/t_o) &(4)\\
&\approx (\delta t/t_p) + (\delta t/t_p)^2 + \cdots &(5)
\end{aligned}$$

where t_p and $t_p + \delta t_p$ are the corresponding absolute times.

It is estimated that there have been approximately 13.7 billion years ($t_p \approx 4.3 \times 10^{17}$ s) since the Big Bang. Mankind has only been on the earth for approximatly 250,000 years. So even if we had been keeping careful track since then the differences we would have noticed between the hypothesized cosmic time and linear time would have been $\delta t/t_p \approx 2.5 \times 10^5/13.7 \times 10^{12} \approx 3.4 \times 10^{-8}$. And the differences we would have needed to observe to discover a discrepancy are the square of this, or of order 10^{-15}. But we have been doing mechanics for only the past 500 years, so even had we started measuring carefully at Galileo, $\delta t/t_p \approx 500/13.7 \times 10^{12} = 3.6 \times 10^{-11}$. So the leading error term would have been of order 10^{-21}, and clearly beyond our ability to distinguish from experimental data alone.

'*Physical laws*' are descriptions of experiments, and generally cannot be derived from more fundamental considerations. So the point of the above is that time could have been logarithmic all along, but we might have never suspected nor noticed. And even if we had, new ideas which challenge those long-established have seldom been welcomed with open minds [7].

4 'Velocity' and displacement in logarithmic time

Velocity depends on time derivatives – linear ones according our history. But suppose it should be computed in logarithmic time increments. Tben we should have been taking logarithmic time derivatives of spatial position change. Note that throughout the remainder of this paper quantities denoted by $*$ will identify logarithmic derivatives, or constants and quantities associated with equations written using logarithmic time.

For example if a body is moving though space and its coordinates can be given by $\vec{x} = \vec{x}_p(t) = \vec{X}_p(\tau)$, then the proper **cosmic velocity**, say $\vec{V^*}$, would be given by:

$$\vec{V^*} = \frac{d\vec{X_p}}{d\tau} = t\frac{d\vec{x_p}}{dt} = t\ \vec{v} \tag{6}$$

where again t is absolute time and $\vec{v}$ is the velocity we would have computed with normal linear time increments. But in our post-Newton era we would have never known the difference, even if it mattered for physics, because t is effectively constant (to one part in 10^{11} or less) during the span of our human existence.

It follows immediately that any displacement field between two times, say τ_1 and τ_2, is given by the logarithmic time integral of $\vec{V^*}$; i.e.,

$$\vec{X_p}(\tau_2) - \vec{X_p}(\tau_1) = \int_{\tau_1}^{\tau_2} \vec{V^*}(\tau)d\tau = \int_{t_1}^{t_2} [tv_g(t)]\frac{dt}{t} = \int_{t_1}^{t_2} v_g(t)dt, \tag{7}$$

which is the linear time result.

We note for future use the relation, $c^* = t\ c$, between a logarithmic speed of light, c_* and the usual linear speed of light, c. Clearly both cannot be constant. Any difference of course would be virtually impossible to determine – at least without looking far back past the span of our human existence. But if c^* is the constant one (as will be suggested in Section 10 below), a glance at the earliest times in Table 1 shows that $c = c^*/t$ will produce the extraordinarily large magnitudes for the speed of light near the Big Bang suggested as necessary by Albrect and Magueijo [1] from other considerations.

5 Accelerations in logarithmic time

A logarithmic acceleration would be given by:

$$\begin{aligned} \vec{A^*} &= \frac{d\vec{V^*}}{d\tau} = \frac{d^2\vec{X}_p}{d\ln(t/t_o)^2} && (8) \\ &= t^2\frac{d^2\vec{x}_p}{dt^2} + t\frac{dx_p}{dt} && (9) \\ &= t^2\left\{\vec{a}_p + \frac{1}{t}\vec{v}_p\right\} && (10) \end{aligned}$$

If we assume that $|\vec{v}_p|$ is bounded by the speed of light, say $c \approx 3 \times 10^8$ m/s, and in our human era of physics $t > 10^{17}$s, then contribution of the last term is of order 10^{-9} m/s^2 or less.

6 A logarithmic time replacement for Newton's Law

If time were indeed logarithmic, then our replacement for Newton's Law probably should be:

$$\vec{f} = m^* \frac{dV_p^*}{d\tau} = m^* \frac{d^2 X_p}{d\tau^2} \tag{11}$$

where m^* is the real *cosmic mass*, the relation of which to what we have believed to be mass, say m, is derived below.

Expanding equation 11 yields:

$$\begin{aligned} \vec{f} &= m^* \frac{dV_p^*}{d\tau} = m^* \frac{d^2 X_p}{d\tau^2} && (12) \\ &= m^* t^2 \left\{ \frac{d^2 \vec{x}_p}{dt^2} + \frac{1}{t} \frac{dx_p}{dt} \right\} && (13) \\ &\approx [m^* t^2]\ \vec{a}_p && (14) \end{aligned}$$

The last term on the right-hand-side of equation 13 would have most probably have not been noticed by any experimentalist, nor measurable even if they were seeking to find it. On the other hand, this term lends itself easily to a perturbation analysis, and might well be used to explain some long-noted orbital anomaly. Or to disprove the idea of logarithmic time entirely.

To summarize this section, if time is logarithmic, what we thought previously was the mass, m, is really the true cosmic mass, m^*, times the age of the universe squared; i.e.,

$$m \approx m^* t^2 \tag{15}$$

Clearly the farther back we travel in time for our observations, the more important the departures from the classical Newton's law become. Moreover any attempt to determine mass by balancing the equations using observations of events a long time ago would have appeared to have mass missing. And the farther back in time we look, the worse it would seem to be. We will explore this further in the discussions below of Kepler's law and the virial theorem.

An interesting consequence of our definitions of cosmic mass and velocity is that kinetic energy remains the same in both log and linear time. Just the distribution among mass and speed differs. I.e,

$$m|\vec{v}|^2/2 = m^* |\vec{V}*|^2/2. \tag{16}$$

And since the speed of light will need to be measured in cosmic time increments as well; $mc^2 = m^* c_*^2$ as well, since $c_* = t\ c$.

7 Gravity and mass

There is nothing fundamental about gravity which changes if we change our equations to reflect a dependence on logarithmic time. We would need, however, to change the definition of the gravitational constant to reflect the differences in the definition of mass. It is not the gravitational force which changes, but definitions of the masses we associate with a given force; i.e., m^* instead of m.

Also there is no reason to assume our previous experiments to determine G are incorrect, only that the definitions of accelerated mass we have associated with a given gravitational attraction need to change. We should be able to associate a new gravitational constant, say G^*, with the masses m^* by incorporating the appropriate factors of t when the measurements were made, say t_p.

Newton's gravitational law is commonly written as:

$$F = G\frac{m_1 m_2}{r^2} \tag{17}$$

where $G = 6.67408 \times 10^{-11}\ m^3/kg\ s$ to one part in 4.7×10^{-5}.

If time were logarithmic, then $m = m^* t^2$. So we should rewrite Newton's gravitational law as:

$$F = G^* \frac{m_1^* m_2^*}{r^2} \tag{18}$$

where G^* the new log-time gravitational constant.

Clearly our old linear time gravitational constant, G, is not so constant after all since

$$G = G^*\ t^4, \tag{19}$$

where we have absorbed the time dependence from the m's into G. Interestingly, this produces values of $(1/G)dG/dt$ of the same order of magnitude as that proposed many years ago by Brans and Dicke [2] (see also Haymes [6]) in trying to resolve special and general relativity.

Since our measurements have been made in the present time, say t_p, we need to include only the factor of t_p^4 into our definition of G^* where t_p is the absolute time the measurement of G was made. Thus our best estimate for G^* is:

$$G^* = G\ t_p^4 \tag{20}$$

The estimated age of the universe in linear seconds is approximately $t_p \approx 4.320432 \times 10^{17}$ seconds, so:

$$G^* = Gt_p^4 \approx 3.48425^{70} \times 6.6705 \times 10^{-11} = 2.325 \times 10^{62}\ m^3 s^3/kg \tag{21}$$

Note that the units of cosmic mass, m^*, are kg/s^2. Clearly given the size of the numbers, there must be a better choice of units. Also, since G enters the Planck scale definitions, introduction of G^* will change some of these as well.

So in summary, there appears to be nothing fundamentally different about gravity. BUT, as we shall see below, the different definitions of mass, m^*, and the gravitational constant, G^*, have enormous consequences for the application of our physical laws at different cosmic times.

8 Rotating dynamics and Kepler's third law

The laws for rotating systems are just a special case of the equations for linear momentum obtained by defining angular velocities and taking moments of Newton's Law. The only essential difference for logarithmic time is that the angular logarithmic velocity is defined as $\vec{V}^* = \vec{\Omega}^* \times \vec{r}$ instead of $\vec{v} = \vec{\Omega} \times \vec{r}$, where $|\vec{\Omega}| = d\theta/dt$ where θ is the angular position and $|\vec{\Omega}^*| = d\theta/d\tau$ is its log-time equivalent. The angular acceleration is similarly defined by $\vec{\Omega}^* \times \vec{\Omega}^* \times \vec{r}$ instead of $\vec{\Omega} \times \vec{\Omega} \times \vec{r}$.

So for a simple mass rotating about an axis of rotation the appropriate form of our Newton's Law replacement for logarithmic time is:

$$m^*(\vec{r} \times \vec{\Omega}^* \times \vec{\Omega}^* \times \vec{r}) = \vec{r} \times \vec{F} \tag{22}$$

For a simple gravitational system of two identical masses with rotation at distance r perpendicular to the plane of rotation this reduces to just:

$$2m^* r^2 |\Omega^*|^2 = r\frac{G^* {m^*}^2}{(2r)^2} \tag{23}$$

or

$$r^3|\Omega^*|^2 = \frac{1}{16} G^* m^* \tag{24}$$

Since the logarithmic period of an orbit is just $2\pi/|\Omega^*|$, this is just Kepler's third law, but for time defined logarithmically.

We can see the important consequences of logarithmic versus linear time if we transform this equation back to linear time quantities. First we note $|\Omega^*| = d\theta/d\ln t/t_o = t\, d|\Omega|/dt$ while $m* = m/t^2$. But since we have measured G at the present time, t_p, we must substitute using t_p instead of t; i.e., $G^* = G\, t_p^{-4}$. It follows immediately that the proper linear time equivalent of our log-time law is:

$$r^3|\Omega^*|^2 = \frac{1}{16} Gm \left[\frac{t_p}{t}\right]^4 \tag{25}$$

This equation (or more general versions of it) are commonly used by astronomers to determine mass of celestial objects when they are in rotation. If time is logarithmic, then the mass they estimate should in reality be given by:

$$m^* = \frac{16 r^3 |\Omega^*|^2}{G^*}. \tag{26}$$

Or in linear time quantities using G determined at t_p:

$$m = \frac{16 r^3 |\Omega|^2}{G} \left[\frac{t}{t_p}\right]^4 \tag{27}$$

where G is the previously thought to be 'universal constant'. Clearly any estimate of mass at times in the distant past using gravity and rotation-rate alone will substantially over-estimate the amount present by the time ratio to the fourth power.

Astronomers also estimate mass another way – by measuring luminosity and comparing the result to the luminosity and the mass of the sun. [8]. But the mass of the sun has itself been estimated using the rotation of the earth and other planets about it. [6, 8]. So in effect one of the t^2's in the substitution should be replaced by t_p^2. So in this case this method of mass estimation is only off by a factor of $(t/t_p)^2$.

Both of these provide a possible explanation for the missing matter in galaxies, since all estimates use some form of the linear time rotational forms of Newton's equations. And it also provides an explanation as to why the various estimates do not agree. In fact, if time is logarithmic, both are wrong and both over-estimate the actual mass present. A correct application of these log-time equations could quantitatively account for the differences in the various means of estimation. And elimnate the need for the missing 'dark matter' as well. If there is less mass, there is less gravitational attraction, so less 'dark energy' would be needed to keep the system from collapsing or to maintain the expansion rate.

9 Energy and the virial theorem

It should be clear from the above that in a system where time is logarithmic, we must be willing to reconsider the basic ideas of what is invariant and what is not. Of particular importance for cosmology is the Hamiltonian defined as $H = \Sigma p_i r_i$ where p_i is the momentum of the i-th member of the collection of masses, r_i is its position, and the summation is over all i.? For simplicity here, we use only the scalar form of the equations.

Following the proof outline of Thayer Watkins in notes at San Jose University [9], we define

$$H^* = \Sigma P_i^* r_i \tag{28}$$

where as before the subscript i denotes the i-th member of the system, m_i^* is its cosmic mass, r_i is its position relative to it's center of mass, $P_i^* = m_i^* V_i^*$ is the newly defined momentum and $V_i^* = dr_i/d\tau$. For now we only consider the forms when the coordinate system is *not* expanding.

First take the logarithmic derivative to obtain:

$$\frac{dH^*}{d\tau} = \Sigma r_i \frac{dP_i^*}{d\tau} + \Sigma P_i^* \frac{dr_i}{d\tau} \tag{29}$$

But $F_i = dP_i^*/d\tau$. So

$$\frac{dH^*}{d\tau} = \Sigma r_i F_i + \Sigma P_i^* \frac{dr_i}{d\tau} \tag{30}$$

And since $P_i^* = m_i^*(dr_i/d\tau)$ the last term reduces to

$$\Sigma P_i^*(dr_i/d\tau) = \Sigma m_i^*(dr_i/d\tau)^2 = \Sigma m_i^* V_i^2 \quad (31)$$

This is just twice the kinetic energy of the system , say $2K$; so,

$$\frac{dH^*}{d\tau} = \Sigma F_i r_i + 2K \quad (32)$$

(Interestingly, as noted above, since $m \approx m^* t^2$, K is the kinetic energy in both logarithmic and linear time variables).

Second we assume the forces acting to be conservative so $F_i = -d\Phi/dr_i$ where Φ is the gravitational potential field. Substituting yields:

$$\frac{dH^*}{d\tau} = -\Sigma r_i \frac{d\Phi}{dr_i} + 2K \quad (33)$$

If the system is cyclical, like two galaxies in rotation, then averaging over a period (in τ) yields $\langle dH/d\tau \rangle = 0$. So the average kinetic energy is just half the potential energy; i.e.,

$$\langle K \rangle = \frac{1}{2}\langle \Sigma r_i \frac{d\Phi}{dr_i} \rangle \quad (34)$$

This looks exactly like the result for linear time. Only the gravitational potential is different – both because of the definition of mass and G^*.

Third we need to compute the average potential energy of n galaxies. The average gravitational potential energy for two identical galaxies separated by distance R is $G^* m^{*2}/R$. If there are n galaxies, there are $n(n-1)/2$ pairs of galaxies. So the average kinetic energy of n identical galaxies is $nm^*\langle V^{*2} \rangle/2$. And this must be equal to half the average potential energy, so:

$$n\frac{1}{2}m^*\langle V^{*2} \rangle = \frac{1}{2}(n)(n-1)\frac{1}{2}\left[G^* \frac{m^{*2}}{R}\right] \quad (35)$$

Finally, solving for m^* yields:

$$m^* = \frac{2\langle V^{*2} \rangle R}{G^*(n-1)} \quad (36)$$

Thus the total mass of the n-galaxies in the cluster is:

$$nm^* = \left[\frac{2n}{n-1}\right]\frac{\langle V^{*2} \rangle R}{G^*} \quad (37)$$

At first glance this appears to be the previous result for linear time. But it is not, since the definition of mass, m^*, velocity, V, and gravitational constant, G^*, are different.

We can put this result in terms of our previous definitions using $m^* = mt^{-2}$, $V^* = tv$ and $G^* = Gt_p^4$. The result is:

$$nmt^{-2} = \left[\frac{2n}{n-1}\right] \frac{t^2 v^2 R}{G t_p^4} \tag{38}$$

or

$$m = \left[\frac{2}{n-1}\right] \left[\frac{v^2 R}{G}\right] \left[\frac{t}{t_p}\right]^4 \tag{39}$$

So if time is logarithmic, then the mass at time t will be over-estimated by a factor of $(t_p/t)^4$ using the linear time analysis. This is exactly what have might been guessed from the preceding section.

10 Maxwell's equations

All of astronomy depends on the propagation of radiation of some form. And radiation is governed by Maxwell's equations. So it makes sense to re-write them as well in log-time variables and examine the consequences.

Maxwell's equations (in Gaussian units) are given by [11]:

$$\nabla \cdot \vec{E} = 0 \tag{40}$$

$$\nabla \cdot \vec{B} = 0 \tag{41}$$

$$\nabla \times \vec{B} = -\frac{1}{c}\frac{\partial \vec{E}}{\partial t} \tag{42}$$

$$\nabla \times \vec{E} = \frac{1}{c}\frac{\partial \vec{B}}{\partial t} \tag{43}$$

But $(1/c)\partial/\partial t$ is exactly $(1/c_*)\partial/\partial\tau = (1/c^*)\partial/\partial \ln t/t_o$ since $c^* = t\ c$. So Maxwell's equations in log-time variables are exactly the same in either set of time variables; i.e.,

$$\nabla \cdot \vec{E} = 0 \tag{44}$$

$$\nabla \cdot \vec{B} = 0 \tag{45}$$

$$\nabla \times \vec{B} = -\frac{1}{c^*}\frac{\partial \vec{E}}{\partial \tau} \left(= -\frac{1}{c^*}\frac{\partial \vec{E}}{\partial \ln t/t_o} \right) \tag{46}$$

$$\nabla \times \vec{E} = \frac{1}{c^*}\frac{\partial \vec{B}}{\partial \tau} \left(= \frac{1}{c^*}\frac{\partial \vec{B}}{\partial \ln t/t_o} \right) \tag{47}$$

There is, however, one important difference when one considers the wave equation forms which can be derived by defining a vector magnetic potential, $\vec{B} = \nabla \times \vec{A}$, and a scalar potential for the electric field, $\vec{E} = \nabla\phi$. The resulting equations are:

$$\nabla^2 \vec{A} = \frac{1}{c^{*2}} \frac{\partial^2 \vec{A}}{\partial \tau^2} \tag{48}$$

$$\nabla^2 \phi = \frac{1}{c^{*2}} \frac{\partial^2 \phi}{\partial \tau^2} \tag{49}$$

In linear variables these become;

$$\nabla^2 \vec{A} = \frac{1}{c^2} \left\{ \frac{\partial^2 \vec{A}}{\partial t^2} + \frac{1}{t} \frac{\partial \vec{A}}{\partial t} \right\} \tag{50}$$

$$\nabla^2 \phi = \frac{1}{c^2} \left\{ \frac{\partial^2 \phi}{\partial t^2} + \frac{1}{t} \frac{\partial \phi}{\partial t} \right\} \tag{51}$$

where we have substituted the linear (and presumably *time-dependent*) phase speed, c, for the logarithmic one, c^*, using $c^* = c\ t$. Like the corresponding term in the reformulation of Newton's Law (equation 13), these extra terms, $td\vec{A}/dt$ and $td\phi/dt$, might actually be useful for verifying the logarithmic time idea (or invalidating it) experimentally by looking for anomalous attenuation of very high frequency radiation. They surely would seem to present an ideal opportunity for perturbation analysis.

11 Special relativity

We have seen in the preceding section that Maxwell's equations have an identical form in both linear time and log time variables. So aside from the fact that most of the relativistic equations of physics need to be rewritten in terms of cosmic time derivatives, nothing appears to change very much. But we have to be careful to make sure we distinguish between absolute time and time differences.

One reason for this is that the Minkowski metric is unchanged by the transformation to log-time. This is easy to see from the fact that $c^* = t\ c$ and $d\tau = d \ln t/t_o = dt/t$. So

$$dx^2 + dy^2 + dz^2 - c^2 dt^2 = dx^2 + dy^2 + dz^2 - {c^*}^2 d\tau^2 \tag{52}$$

The same is true for the Lorentz transformation since $v^2/c^2 = {V^*}^2/{c^*}^2$. The only difference is that c^* is presumed constant instead of c. And Einstein's law of physics that the speed of light is constant is similarly modified so that it is c^* that is constant.

Finally we note that the important relativistic relationships (see Feynman vol I, Chap 16[4]),

$$E^2 - P^2 c^2 = m_o c^2 \tag{53}$$

$$Pc = Ev/c \tag{54}$$

are also preserved in log-time coordinates. I.e.

$$E^{*2} - P^{*2}c^{*2} = {m^*}_o c^{*2} \quad (55)$$

$$P^*c^* = E^*V^*/c^* \quad (56)$$

Note that there may still be some question about how these apply to the stretched coordinates considered below. In particular, is it reasonable to apply to this very special coordinate system the hypothesis that laws of nature should be independent of coordinate system?

12 Propagation of radiation at constant phase

Let's consider what happens if an oscillator radiates into space with logarithmic time, $\tau = \ln(t - t_{osc})/t_o$ instead of the usual linear time. Its location is at fixed $\vec{x}_{osc}$ and it begins at time t_{osc}. The phase at any time and location would be given by:

$$\phi = \vec{\kappa} \cdot (\vec{x} - \vec{x}_{osc}) - \omega_*(\tau - \tau_{osc}). \quad (57)$$

$\vec{k} = \nabla_{\vec{x}}\phi$ is the wavenumber and $\omega^* = -d\phi/d\tau$ are the wavenumber vector and frequency in our logarithmic time space.

The important question for us on earth is: what do we see in our linear time, linear space coordinates? Or more to the point: can logarithmic time account for the 'Red Shift'? Interpretations of the Red Shift form the basis for much of modern cosmology.

The wavenumber vector is of course the same since we have assumed space fixed. But the 'frequency' and 'phase velocity' are given respectively as $\omega = -d\phi/dt$, and $c = \omega/|\vec{k}|$. It follows immediately that the frequency we see is given by:

$$\omega = \frac{d\phi}{dt} = \frac{\omega^*}{t} \quad (58)$$

At first glance, Equation 58 appears to imply a red-shift, since the farther away and earlier it began, the greater the frequency shift. *In fact it seems to imply a red-shift even if there no expansion at all.* But things are not what they first appear, because the speed of light is time-dependent as well.[1] The phase velocity seen in our earthly linear-time system would be:

$$c = \frac{\omega^*}{k^*}\left[\frac{1}{t}\right] = c^*/t \quad (59)$$

Or $c^* = c\ t$. If $\omega^*/k^* = c^* = constant$ then c measured by us would be time dependent. Note that expanding $1/t$ about any time interval in today's epoch means any discrepancies noted would be of order $\delta t/t_p$, and as noted above well below our ability to measure it. It should also be noted that since we now define the standard meter to be exactly the distance traveled by the speed of light in

[1] This point was missed in earlier versions of this work [5], but is corrected herein.

a given time, then that definition makes it impossible to see any effect at all. Clearly the log-time hypothesis if verified will require some changes to even our standards for measurement.

So the wavenumber of radiation (in propagation through fixed space) remains constant. Thus any observed wavenumber shift must come from other sources; e.g., source moving away or space itself expanding. The first possibility is like all previous analyses of linear time, so is not of interest here. The possibility that space itself is expanding is considered below.

13 Accounting for an expanding universe

The computation of velocities and accelerations above do not account for any effects that might arise if the coordinate system we need to describe space might be expanding. Or that the universe itself might be expanding. Either way, we can examine this possibility by defining a scale length, say $\delta(t)$, that is time-dependent. Note that we have used t instead of τ, but either is acceptable. Also, special relativity could be included as well.

Now it makes sense to define our physical laws using both logarithmic time and our expanding coordinate system. So we define a position within it to be given by $\vec{\eta} = \vec{x}/\delta(t)$ and a displacement field to be defined by $\vec{\eta}_p = \vec{x}_p(\vec{\eta}, \tau)/\delta(t)$.

The logarithmic velocity in this field would be given by:

$$\vec{V_p}^* = \frac{d\vec{\eta}_e}{d\tau} \tag{60}$$

$$= \left[\frac{t}{\delta}\right] \left.\frac{\partial \vec{x}_p}{\partial t}\right|_t - \vec{\eta}_p \left[\frac{t}{\delta}\frac{d\delta}{dt}\right] \tag{61}$$

Or putting it in linear- time, linear-space variables, the velocity $\vec{v}_p = \partial \vec{x}_p/\partial t|_t$ would be given by:

$$\vec{v}_p = \left[\frac{\delta}{t}\right] \vec{V_p}^* + \left[\frac{d\delta}{dt}\right] \vec{\eta} \tag{62}$$

The farther away in η-space we view things, the faster they will appear to be moving, even if there is no relative velocity at the same place (i.e., $\vec{V^*}_p = 0$). In fact, it does not matter where we put the origin for η, the result will be the same everywhere. Locally (i.e., near $\eta = 0$), the two velocities, $\vec{v}_p$ and $\vec{V_p}^*$ are simply proportional to each other by the factor δ/t. Obviously the special case for $\delta \propto t$ is of great interest.

Now the acceleration in our four-dimensional expanding system can be similarly computed by defining it to be:

$$\vec{A_p}^* = \frac{d\vec{V_p}^*}{d\tau} \tag{63}$$

$$= \left[\frac{t^2}{\delta}\right]\frac{d^2\vec{x}_p}{dt^2} + \left[\frac{t}{\delta}\right]\frac{d\vec{x_p}}{dt} - \left[\frac{t^2}{\delta^2}\frac{d^2\delta}{dt^2}\right]\frac{d\vec{x_p}}{dt} - \eta_p\left[\frac{d^2\ln\delta}{d(\ln t/to)^2}\right] + \eta_p\left[\frac{d\ln\delta}{d\ln t/to}\right]^2$$

The first two terms correspond to the terms we saw above for a fixed coordinate system, while the last two are a result of the expanding coordinates. As for the velocity, the acceleration due to coordinate expansion are negligible near $\eta = 0$ but increase with distance from the origin. The middle term is zero if $\delta \propto t$. All of the last three terms are zero if $\delta = constant$ (i.e., no expansion of coordinate system), and the result is to within the factor δ the same as equation 10.

Let's consider what happens if an oscillator operates in expanding space, $\vec{\eta} = (\vec{x} - \vec{x}_{osc})/\delta(t)$, and with logarithmic time, $\tau = \ln(t - t_{osc})/t_o$. Its location is at fixed $\vec{\eta}_{osc}$ and it begins at time t_{osc}. The phase at any time and location would be given by:

$$\phi = \vec{\kappa}_* \cdot (\vec{\eta} - \vec{\eta}_{osc}) - \omega_*(\tau - \tau_{osc}). \tag{64}$$

$\vec{k}^* = \nabla_{\vec{\eta}}\phi$ and $\omega^* = -d\phi/d\tau$ are the dimensionless wavenumber vector and frequency in our spatially scaled and logarithmic time space.

The important question for us on earth is: what do we see in our linear time, linear space coordinates? The answer can again be found from our earthly definitions of 'frequency', 'wavenumber' and 'phase velocity' which given respectively as the $\omega = -d\phi/dt$, $\vec{k} = \nabla_{\vec{x}}\phi$, and $c = \omega/|\vec{k}|$.

It follows immediately that:

$$\omega = \frac{d\phi}{dt} = \frac{\omega^*}{t} - \vec{k}^* \cdot [\vec{\eta} - \vec{\eta}_{osc}] \left[\frac{1}{\delta}\frac{d\delta}{dt}\right] \tag{65}$$

$$\vec{k} = \nabla_{\vec{x}}\phi = \frac{\vec{k}^*}{\delta(t)} \tag{66}$$

Equation 66 clearly implies a wavenumber red-shift for all non-negative expansion rates. And for frequency, equation 65 implies that the farther away, the greater the shift.

The phase velocity seen in our earthy system would be:

$$c = \frac{\omega^*}{k^*}\left[\frac{\delta(t)}{t}\right] - |\vec{\eta} - \vec{\eta}_{osc}| \left[\frac{d\delta}{dt}\right] \tag{67}$$

If $d\delta/dt = 0$, the speed of light we see would be proportional to just the inverse age of the universe in linear time, our previous result for fixed spatial coordinates. Our proposed new physical law that $c^* = \omega^*/k^* = constant$ demands that (as before) c measured by us be both time and space dependent.

The linear expansion rate case is of particular interest since $\delta = [d\delta/dt]t$, so equation 59 reduces to:

$$c = \left[\frac{d\delta}{dt}\right]\{c^* - |\vec{\eta} - \vec{\eta}_{osc}|\} \tag{68}$$

If we measure only at $\eta = 0$, at least directly, c and c^* are directly proportional to each other. Only by looking far away could we see differences.

14 Summary and conclusions

In view of the above it is clear that we could have equally written Newton's law using logarithmic time without ever being able to tell the difference, at least as long as we only applied it to times within our human experience. Any differences would have shown up only for very large times – on the order of billions of years.

The most important difference for mechanics is that the definitions of mass and velocity change. In particular our traditional definition of mass becomes dependent on absolute time squared. This has important implications for the gravitational law and our applications of it. In particular what we might have previously believed to be the gravitational constant in fact varies as the fourth power of absolute time. Not recognizing this can lead to gross over-estimates of celestial masses, and incorrect inferences that mass is missing.

It is further suggested if time is indeed logarithmic, then the remaining laws of physics must be treated the same way. Maxwell's equations in particular have exactly the same form when expressed in logarithmic time. And the same is true for the important relativistic energy equations and the Minkowski invariant.

Finally, two different scenarios have been considered, the first where space is not expanding, and a second where it is. The first shows no redshift unless the source is moving away from the observer. The second with a constant expansion rate leads to a wavenumber redshift which depends on the expansion rate, and a frequency shift which is linear in distance from the observer. Both are consistent with a constant logarithmic speed of light.

Assuming there is nothing obviously wrong with the analyses presented above, the possibility of that time might be logarithmic should be a boon for cosmologists and astronomers. It is for the former to flesh out the mathematical consequences on other aspects of our knowledge. And the latter alone have the data and wherewithal to test whether it describes their data. The late great solid mechanics experimentalist, James C. Bell (of the Johns Hopkins University), often remarked "Experimentalists test and sort theories." At very least the experimental astrophysics community has another theory to add to the mix.

Acknowledgments

An earlier version of this work was presented at the 2016 APS/DFD meeting in Portland OR. [5]. It was stimulated by listening to a Canadian radio program *As it Happens* on 'Dark Matter' while driving to the 2015 APS/DFD annual meeting in Boston, MA. I remarked to my wife: "How could so much mass be missing? Surely there must be a problem with the equations." But what? The next day my two co-workers from Princeton, Marcus Hultmark and Clay Byers, were discussing our forthcoming paper on how time evolved logarithmically in decaying turbulence. [3]. And in a flash it occurred to me: why not in nature as well? The author is particularly grateful to his many colleagues and students who puzzled with him over the past year about this. Only time will tell (pun intended) whether it does evolve logarithmically. But the exercise of asking the

question has been wonderfully stimulating.

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