The Special Theory of Double Relativity

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ABSTRACT

Observations show a substantial amount of anomalies with respect to the laws of physics when distances become cosmological large, thus necessitating the introduction of dark matter and dark energy. The objective of this paper is to introduce new theoretical concepts that eliminate the need for these "dark entities", explaining cosmological observed anomalies at large length-scales while leaving well-established and accepted theories intact for cosmological small distances. This paper introduces a noncommutative geometry that is defined by an extra universal length-scale constant. Besides a constant speed of light c there is a second constant of length-scales b, equal to the distance between the observer and the cosmic microwave background radiation, the horizon of the universe. Extending the laws of physics with a universal length-scale constant does not violate Einstein's Theory of Relativity. Namely, Einstein's theories of relativity describes the laws of physics by means of a causal structure of space-time, it is inherently background independent. So, Einstein's theory fixes most features of the space-time universe, i.e. just about the only feature that remains to be fixed are relative length-scales. This paper strives to give a clear and persuasive discussion of extending Einstein's Special Theory of Relativity for cosmological large distances. This paper quantitatively demonstrates that the new extended formulation can account for the experimental data at these cosmological large scales. It also demonstrates that the proposed extension does account for all the experimental data at cosmological small scales, as it does not significantly alter the accepted theories at these length-scales. The size and age of the universe, and constants in Physics like e and π, emerge out of equations that use a limited number of basic geometry constants only.

Key words: *special relativity, non-commutative geometry, cosmological constant, universe expansion, universe acceleration, horizon problem, flatness problem, cosmic microwave background.*

1 INTRODUCTION

The laws of physics describe the observations well for cosmological small distances. Only at cosmological large distances, anomalies have been observed, e.g. distant stars rotating very fast around a black-hole, distant outer galaxies rotating too fast around its centre. This paper claims that extending Einstein's postulates with a second universal constant, can explain these anomalies at large length-scales; this while leaving Einstein's well-established Special Theory of Relativity in tact for cosmological small length-scales. Einstein's postulates (= traveling at constant speeds, excluding gravity) are extended as follows:

Einstein's Postulate-1: The laws of physics must be the same for any two objects, no matter how fast they are moving relative to one another. Speeds are always relative and one cannot detect the difference between moving at constant speed and standing still. Any observer may claim to be standing still.

Einstein's Postulate-2: The speed of light c is constant. One can never obtain a speed larger that c. Note: For the hypothetical case that an object would travel faster than the value of c, then that object can never decelerate down to the value of c.

Extension of postulate-1: The laws of physics must be the same for any two objects, no matter how much distance there is between them. Distances are always relative and one cannot detect the central location of the universe. Any observer may claim to be in the unique central location of the universe.

Extension of postulate-2: The distance to the cosmic microwave background radiation b is a constant. One can never obtain a distance larger than b. Note: For the hypothetical case that an object would be further away than the value of b, then that object's distance can never be reduced to the value of b.

2 THE SPECIAL THEORY OF DOUBLE RELATIVITY IN MATHEMATICAL FORMAT

2.1 Einstein's Special Theory of Relativity

The laws of physics are co-variant with respect to the Lorentz transformations. This can be observed when relative speeds become large (in reference to the speed of light).

$$V = \frac{V_1 + V_2}{1 + \left(\frac{V_1 * V_2}{c^2} \right)} \qquad \textit{(Equation-1)}$$

With $|V_1| \le c$ $\quad V_1 \ge 0$, when object-1 is moving towards the observer

With $|V_2| \le c$ $\quad V_2 \ge 0$, when object-2 is moving towards the observer, and space direction of V_2 being the opposite of V_1

Note: The values of V_1 and V_2 can be positive or negative.

V = relative speed between object-1 and object-2 [m/s]
V_1 = relative speed between object-1 and the observer [m/s]
V_2 = relative speed between object-2 and the observer [m/s]
c = speed of light in vacuum [m/s]

2.2 Extending Einstein's special relativity theory with a length-scale constant

This paper claims that the laws of physics are also co-variant with respect to the Lorentz transformations when relative distances become cosmological large (in reference to the size of the observable universe).

$$D = \frac{D_p + D_f}{1 + \left(\frac{D_p * D_f}{b^2} \right)} \qquad \textit{(Equation-2)}$$

With $|D_p| \le b$ $\quad D_p \ge 0$, when object-p is in the past of the observer

With $|D_f| \le b$ $\quad D_f \ge 0$, when object-f is in the future of the observer, i.e. time direction of D_f is the opposite of D_p

Note: the values of D_p and D_f can be positive or negative.

D = relative distance between object-p and object-f [billion light years]
D_p = relative distance between object-p and the observer [billion light years]
D_f = relative distance between object-f and the observer [billion light years]
b = distance to the cosmic microwave background radiation [billion light years]

Note: an object is related to an event.

2.3 Describing the special double relativity equation with both objects in the past

$$d_y = \frac{D_y}{b} \qquad \textit{(Equation-3)}$$

With $|d_y| \leq 1$ $\quad d_y \geq 0$, when object-y is in the past of the observer

$D_y = D_p$ = relative distance between object-y and the observer [billion light years]
d_y = relative distance between object-y and the observer [-]
b = distance to the cosmic microwave background radiation [billion light years]

$$d_x = -\frac{D_x}{b} \qquad \textit{(Equation-4)}$$

With $|d_x| \leq 1$ $\quad d_x \geq 0$, when object-x is in the past of the observer

$D_x = - D_f$ = relative distance between object-x and the observer [billion light years]
d_x = relative distance between object-x and the observer [-]
b = distance to the cosmic microwave background radiation [billion light years]

$$d = \frac{D}{b} \qquad \textit{(Equation-5)}$$

D = relative distance between object-y and object-x [billion light years]
d = relative distance between object-y and object-x [-]
b = distance to the cosmic microwave background radiation [billion light years]

By combining *equation-2*, *equation-3*, *equation-4* and *equation-5*, one can define d as:

$$d = \frac{d_y - d_x}{1-(d_y * d_x)} \qquad \textit{(Equation-6)}$$

Note: the values of d_y and d_x can be positive or negative.

d = relative distance between object-y and object-x [-]
d_y = relative distance between object-y and the observer [-]
d_x = relative distance between object-x and the observer [-]

2.4 The ratio equations

$$R = \frac{d}{(d_y - d_x)}$$ *(Equation-7)*

Combining *equation-6* and *equation-7*:

$$R = \frac{1}{1-(d_y * d_x)}$$; With $d_y \neq d_x$ *(See graph-1)* *(Equation-8)*

R = ratio between the values of d and (d_y - d_x) [-]
d = relative distance between object-y and object-x [-]
d_y = relative distance between object-y and the observer [-]
d_x = relative distance between object-x and the observer [-]

2.5 The delta equations

$$\Delta = (d_y - d_x) - d$$ *(Equation-9)*

Combining *equation-6* and *equation-9*:

$$\Delta = \frac{(d_y - d_x)}{1-\left(\frac{1}{(d_y * d_x)}\right)}$$ *(See graph-2)* *(Equation-10)*

Δ = delta between the values of (d_y - d_x) and d [-]
d = relative distance between object-y and object-x [-]
d_y = relative distance between object-y and the observer [-]
d_x = relative distance between object-x and the observer [-]

2.6 The "sum-over-histories" ratio equation

By means of *equation-8*, one can average out the R_y-values ($0 < d_y \leq 1$) into a "sum-over-histories" ratio $\overline{R}_N$; this for any specific value of d_x. One example is to average out the R_y-values for y = 1, 7/8, 3/4, 5/8, 1/2, 3/8, 1/4, 1/8; see the line $\left(\overline{R}_N\right)_{N=8}$ on *graph-1*:

$$\overline{R}_N = \sum R_y \Big/ N \qquad ; \text{With } y > 0 \qquad \textit{(See graph-1) (Equation-11)}$$

$\left(\overline{R}_N\right)_{x->1} -> \infty$ is related to the idea of a period of universal inflation just after Big Bang.

$\overline{R}_N$ = "sum-over-histories" ratio; this over N R_y-values [-]
R_y = ratio for value d_y= y; this at a specific value of d_x [-]
N = number of R_y-values that are averaged out [integer]
y, x = short descriptions for the relative distances d_y and d_x respectively [-]

By means of *equation-8*, one then can define the "sum-over-histories" value $\overline{d}_y$ as:

$$\overline{d}_y = \left(1 - \frac{1}{\overline{R}_N}\right) * \frac{1}{d_x} \qquad \textit{(Equation-12)}$$

$\overline{d}_y$ = "sum-over-histories" d_y value; this for a certain value of d_x [-]
$\overline{R}_N$ = "sum-over-histories" ratio; this over N R_y-values [-]
d_x = relative distance between object-x and the observer [-]

The averaged out value of $\overline{d}_y$ is here related to the idea of the "sum-over-histories" in Quantum Mechanics. Classic physics (re: Isaac Newton) says a particle follows a unique trajectory from A to B. But Richard Feynman's version of Quantum Mechanics says that one must calculate the effect of <u>all</u> possible paths from A to B and add them together. The above described "sum-over-histories" is also closely related to one of the basic axioms of Quantum Mechanics, i.e. the principle of superposition. So, the concept behind the ratio $\overline{R}_N$ is to use an averaged out value for R_y, so to be able to calculate a $\overline{d}_y$ value; this being the "sum-over-histories" value for all the possible d_y's.

This paper claims that for cosmological large relative distances, so-called superposition calculations are indeed necessary due to the non-commutative aspect of geometry at these cosmological large length-scales. Namely, there is always an un-preciseness, an uncertainty, regarding the relative position of d_y; this for each and every given value of d_x.

Note that for cosmological small relative distances, this non-commutative component becomes in-significant and thus classic physics remain valid for these observations.

2.7 The universal superposition ratio equation

The universal superposition ratio equation $\overline{R}_u = f(d_x)$ is defined as the line that approx. connects the various "sum-over-histories" $\overline{R}_N$ points for $d_x > 0$ and $d_y > 0$; this while:

- The boundaries condition at the position of the observer $\left(\overline{R}_u\right)_{x->0} = \left(\overline{R}_N\right)_{x->0} = 1$.
- The maximum future boundaries condition $\left(\overline{R}_u\right)_{x->-1} \approx \left(\overline{R}_N\right)_{x->-1} \approx 0.7$. This value is close to the observed curvature of the universe, i.e. the cosmological constant.
- The maximum past boundaries condition $\left(\overline{R}_u\right)_{x->1} \approx 3$, thus satisfying the observed expansion of the universe, instead of the exponential doubling of $\left(\overline{R}_N\right)_{x->1} = \infty$.

$$\boxed{\overline{R}_u = \left[(d_x+1)^3\right] - \left[\frac{3}{\sqrt{2}} * (d_x+1)^2\right] + \left[\frac{2}{\sqrt{2}} * (d_x+1)\right] + \frac{1}{\sqrt{2}}}$$ *(See graph-1)* *(Equation-13)*

$\overline{R}_u$ = universal superposition ratio [-]
d_x = relative distance between object-x and the observer [-]

The boundary values can be calculated as follows:

$$\left(\overline{R}_u\right)_{x=1} = 8 - \frac{7}{\sqrt{2}}$$ *(See graph-1)* (= approximately 3.050)

$$\left(\overline{R}_u\right)_{x->0} = 1$$ *(See graph-1)* (= 1)

$$\left(\overline{R}_u\right)_{x=-1} = \frac{1}{\sqrt{2}}$$ *(See graph-1)* (= approximately 0.707)

x = short description for d_x

By means of *equation-6* and *equation-8*, one can define the universal superposition values for $\overline{d}_u$ and $\overline{y}_u$ respectively:

$$\overline{d}_u = \frac{\overline{y}_u - d_x}{1 - \left(\overline{y}_u * d_x\right)}$$ *(Equation-14)*

$$\overline{y}_u = \left(1 - \frac{1}{\overline{R}_u}\right) * \frac{1}{d_x}$$ *(Equation-15)*

$\overline{d}_u$ = universal superposition relative distance between object-y and object-x [-]
$\overline{y}_u$ = universal superposition d_y value; this for a certain value of d_x [-]
d_y = relative distance between object-y and the observer [-]
d_x = relative distance between object-x and the observer [-]
$\overline{R}_u$ = universal superposition ratio [-]

2.8 The universal superposition delta equation

By means of *equation-9*, one can define the universal superposition value $\overline{\Delta}_u$ as:

$$\overline{\Delta}_u = \left(\overline{y}_u - d_x\right) - \overline{d}_u \qquad \textit{(Equation-16)}$$

$\overline{\Delta}_u$ = universal superposition delta [-]
$\overline{d}_u$ = universal superposition relative distance between object-y and object-x [-]
$\overline{y}_u$ = universal superposition d_y value; this for a certain value of d_x [-]
d_y = relative distance between object-y and the observer [-]
d_x = relative distance between object-x and the observer [-]

Combining *equation-16* with *equation-14* and *equation-15*:

$$\boxed{\overline{\Delta}_u = \left(1 - \overline{R}_u\right) * \left[\left(\left(1 - \frac{1}{\overline{R}_u}\right) * \frac{1}{d_x}\right) - d_x\right]} \qquad \textit{(See graph-2)} \quad \textit{(Equation-17)}$$

$\overline{\Delta}_u$ = universal superposition delta [-]
$\overline{R}_u$ = universal superposition ratio [-]
d_x = relative distance between object-x and the observer [-]

The boundary values can be calculated as follows:

$$\left(\overline{\Delta}_u\right)_{x=1} = 1 - \left[\frac{1}{\left(\overline{R}_u\right)_{x=1}}\right] \Rightarrow \left(\overline{\Delta}_u\right)_{x=1} = 1 - \left[\frac{1}{8 - \frac{7}{\sqrt{2}}}\right] \qquad \textit{(See graph-2)} \quad \text{(= approx. 0.672)}$$

$$\left(\overline{\Delta}_u\right)_{x \to 0} = 0 \qquad \textit{(See graph-2)} \quad \text{(= 0)}$$

$$\left(\overline{\Delta}_u\right)_{x=-1} = \left[\frac{1}{\left(\overline{R}_u\right)_{x=-1}}\right] - 1 \Rightarrow \left(\overline{\Delta}_u\right)_{x=-1} = \sqrt{2} - 1 \qquad \textit{(See graph-2)} \quad \text{(= approx. 0.414)}$$

The "zero Delta" range can be calculated as follows:

$\left(\overline{\Delta}_u\right)_{x=0 \to -1/2} \approx 0$ (=the value of $\overline{\Delta}_u \approx 0$ over the whole range of d_x = 0 -> -0.5)

The value of $\overline{\Delta}_u$ changes from decreasing to increasing at d_x=1/2. And the second order differential value of $\overline{\Delta}_u$ changes "+/- sign" at d_x=3/8.

$$\left(\frac{d\left(\overline{\Delta}_u\right)}{d\left(d_x\right)}\right)_{x=1/2} = 0 \qquad \left(\frac{d^2\left(\overline{\Delta}_u\right)}{d\left(d_x^2\right)}\right)_{x=3/8} = 0$$

x = short description for d_x

2.9 Basic universal geometry constants

One can derive the basic universal constants, related to cosmology, out of *equation-13*:

$k_u = 2$ => related to opposite directions

$l_u = 1/\sqrt{2}$ => related to the curvature of the universe

$m_u = 3$ => related to the expansion of space itself; this for the critical density case

The basic universal constants can be defined by means of the following descriptions:

- k_u = universal horizon constant [-]. The maximum span that an observer can measure is two times the distance to the cosmic microwave background radiation. Namely, one always can observe the universe in opposite directions.
- l_u = universal cosmological constant [-]. What characterizes the cosmological constant is a scale, which is the length-scale over which it curves the universe. This scale is indeed extremely large & approx. (1/√2) * 13.7 ≈ 9.7 [billion years].
- m_u = universal expansion constant [-]. Scale-factor of space for the critical density case is calculated to be 3. It is the distance to the horizon divided by the age of the universe; this for a universe that has got exactly the critical density.

There are in fact more basic universal constants, these related to algebraic properties:

$p_u = 1$ => related to the Boolean logic TRUE statement and integer value ONE

$q_u = 0$ => related to the Boolean logic FALSE statement and nothing value ZERO

$z_u = e^{i*\Pi}$ => related to complex numbers and direction => $z_u = i^k = i^2 = -1$

p_u = universal integer constant [-]

q_u = universal nothing constant [-]

z_u = universal direction constant [-]

i = short description for the universal complex number i_u [-]

e , π = the natural logarithm constant e and geometric circle constant pi [-]

The universal superposition ratio *equation-13* described by basic universal constants:

$$\boxed{\overline{R}_u = [(d_x+1)^{k+1}] - [m*l*(d_x+1)^k] + [k*l*(d_x+1)^{k-1}] + l} \qquad \textit{(Equation-18)}$$

$\overline{R}_u$ = universal superposition ratio [-]

d_x = relative distance between object-x and the observer [-]

k, l, m = short descriptions for the universal constants k_u, l_u, m_u [-]

3 EXPLAINING THE COSMOLOGICAL OBSERVATIONS

This chapter explains some of the cosmological observations. E.g., it explains why:

- The universe changed from deceleration to acceleration approx. 7 [billion years] ago.
- The acceleration of the universe became ever-faster approx. 5 [billion years] ago.
- The expansion factor of space itself is observed to be around 3.
- The curvature of the universe is observed to be approx. 10 [billion light years].
- The universe is observed to be spatial flat, i.e. Euclidean geometry (= flatness problem).
- The universe cosmic microwave background radiation is smooth (= horizon problem).
- The boundaries condition at the horizon of the universe is non-symmetric.
- Outer galaxies are observed to be rotating too fast around the centre of a galaxy cluster.

3.1 The universe changed from deceleration to acceleration, to ever-faster acceleration

The age of the universe is estimated to be approximately 13.7 [billion years] and observations show that the expansion of the universe changed from deceleration to acceleration around 7 [billion years] ago.

The universal superposition delta equation $\overline{\Delta}_u$ (*equation-17*) shows that:

$$\left(\frac{d\left(\overline{\Delta}_u\right)}{d\left(d_x\right)}\right)_{x=1/2} = 0 \qquad \textit{(See graph-2)}$$

The value of $\overline{\Delta}_u$ changes from decreasing to increasing at $d_x = ½$. So, calculations show that the deceleration changed to acceleration around 0.5 * 13.7 $\approx$ 6.9 [billion years] ago.

Also, current observations show that the acceleration of the universe became ever-faster around 5 [billion years] ago.

The universal superposition delta equation $\overline{\Delta}_u$ (*equation-17*) shows that:

$$\left(\frac{d^2\left(\overline{\Delta}_u\right)}{d\left(d_x^2\right)}\right)_{x=3/8} = 0 \qquad \textit{(See graph-2)}$$

The second order differential value $\frac{d^2\left(\overline{\Delta}_u\right)}{d\left(d_x^2\right)}$ changes "+/- sign" at $d_x = 3/8$. So, calculations show that acceleration became ever-faster around 0.375 * 13.7 $\approx$ 5.1 [billion years] ago.

3.2 The universal expansion constant

For the universe critical density case, the expansion factor of space is exactly 3. The observable universe is close to the critical density case.

The boundaries value for the universal superposition ratio equation $\overline{R}_u$ (*equation-13*):

$\left(\overline{R}_u\right)_{x=1} = 8 - \frac{7}{\sqrt{2}}$ *(See graph-1)* (= approximately 3.050)

$\left(\overline{R}_u\right)_{x=1}$ is indeed close to the value for the universal expansion constant $m_u = 3$.

3.3 The universal cosmological constant

Observations show that the age of the universe is estimated to be around 13.7 [billion years] and the universe curvature has been estimated to be approx. 10 [billion years]. This is a curvature ratio of $10/13.7 \approx 0.73$ [curvature as ratio of the age of the universe].

The boundaries value for the universal superposition ratio equation $\overline{R}_u$ (*equation-13*):

$\left(\overline{R}_u\right)_{x=-1} = \frac{1}{\sqrt{2}}$ *(See graph-1)* (= approximately 0.707)

$\left(\overline{R}_u\right)_{x=-1}$ is equivalent to the cosmological constant $l_u = {}^1\!/\!_{\sqrt{2}}$, close to 0.73 indeed.

3.4 The flatness problem of the universe

The universe is observed to be approximately spatial flat, i.e. Euclidean geometry applies.

The universal superposition delta equation $\overline{\Delta}_u$ (*equation-17*):

$\left(\overline{\Delta}_u\right)_{x=0->-1/2} \approx 0$ *(See graph-2)*

$\overline{\Delta}_u \approx 0$ over the cosmological large range of $d_x = 0$ to -0.5. This means that the universal superposition relative distance $\overline{d}_u$ between object-y and object-x is indeed close to the commutative geometric value of $\overline{y}_u$ - d_x (*eq-16*). Euclidean geometry applies when $\overline{\Delta}_u = 0$.

Measurements can only be carried out as fast as the constant speed of light c allows it. This ultimately means that at cosmological large scales, the observer always measures object-x as it was in the distant past. This means that the "actual" d_x for object-x, at the very present moment of the observer, has always got a negative value. So, is always positioned in the future of the observer. But as long as this value d_x is in between -½ and 0, the universal superposition relative delta $\overline{\Delta}_u$ between object-y and object-x is indeed close to zero, and thus any non-Euclidean geometry influence is insignificant.

3.5 The horizon problem of the universe

The universe cosmic microwave background radiation has been observed to be incredible smooth. Even when comparing areas of the universe that should not have been in contact.

The non-commutative nature of The Special Theory of Double Relativity explains that the whole universe is "in contact". Namely, the extension of postulate-2 states the value of the maximum relative distance to be b. And b is a universal constant, similar to the speed of light constant c. The relative distance between two objects can thus never be further apart than the observable distance to the cosmic microwave background radiation, i.e. b.

Also, the boundaries condition of the universal superposition delta equation $\overline{\Delta}_u$ *(equation-17)* shows values at the horizon ($d_x = 1$ and $d_x = -1$ respectively) that are in between 0 and 1 *(see graph-2)*. So, there are no infinities at the universe horizon.

In a nutshell, the Special Theory of Double Relativity shows that the observable universe has no infinities at both the maximum past as well as maximum future, distances $\leq$ b. This means that any two objects in the observable universe can have been in contact.

3.6 Non-symmetric boundaries condition at the horizon of the universe

One would expect that at the largest scales of the observable universe symmetry should prevail, i.e. any one direction should be similar. But experimental observations of fluctuations in the cosmic microwave background show that the random motion (=*heat*) in these large-scale modes is not symmetric; there is a preferred direction (*Kate Land and Joao Magueijo*). The WMAP images show also evidence of a non-symmetric boundaries condition, e.g. the red tilt in the amplitudes of energy density fluctuations in the cosmic microwave background radiation temperature. This red tilt is a deviation from perfect scale-variance, having slightly smaller amplitude as the wave-length decreases.

The Special Theory of Double Relativity shows that the observable universe is "closed", with a geometry that is tilted towards the direction of the observer (re: direction of time).

Note: As the theory of relativity is inherently background independent, one also could describe the observed acceleration of the expansion of the universe as being a relative ever faster shrinking of geometry, say compared to a static universe boundaries condition of length-scale constant b. The theory of relativity allows this without jeopardizing any physical law. So, the above mentioned tilting, i.e. geometry starting to relatively shrink compared to the static boundaries of the universe, in the direction towards the observer, may indeed explain this non-symmetric aspect.

3.7 Outer galaxies rotate too fast around its centre

On cosmological large scales, relative distances cannot be calculated by $d = d_y - d_x$. Instead, one has to use *equation-6*. The superposition equations in paragraph 2.7 may also play a role here, but for simplicity reasons are left out of the under-mentioned discussion.

$$d = \frac{d_y - d_x}{1 - (d_y * d_x)} \qquad \textit{(See paragraph 2.3)} \quad \textit{(Equation-6)}$$

d = relative distance between object-y and object-x [-]
d_y = relative distance between object-y and the observer [-]
d_x = relative distance between object-x and the observer [-]

Equation-6 can be modified into *equation-19* when the relative distances from the observer to both objects are very large; this compared to the relative distance between the objects. So, the Δd value is relatively small compared to d_y and d_x.

$$d_\Delta \approx \frac{\Delta d}{1 - d_x^2} \qquad \text{; With } \Delta d = d_y - d_x \text{ and } \Delta d << d_x \qquad \textit{(Equation-19)}$$

d_Δ = relative distance between object y & x, both objects far away [-]
Δd = relative observed distance between object y & x, by observer [-]
d_y = relative distance between object-y and the observer [-]
d_x = relative distance between the object-x and the observer [-]

For example: $d_x = 0.3$ (= 0.3 times the distance to the horizon):

$$d_\Delta \approx \frac{\Delta d}{1 - 0.3^2} \qquad => \qquad d_\Delta \approx 1.1 * \Delta d$$

So, when measuring distances on cosmological large scales, the observer measures distances between the centre of a galaxy cluster, and its outer galaxies, to be smaller than d_Δ. Indeed, special double relativity has got a non-commutative aspect at its core.

According to classic Newtonian Physics, the orbital speed of the outer galaxies around the centre of a galaxy cluster should be proportional to the reciprocal value of the square root of its distance to this centre. But distances are observed to be smaller than the actual relative distance d_Δ, and thus the orbital speed is perceived to be too fast. Please note that the postulates, as described in the introduction (*chapter-1*), remain valid.

Note: One can make an analogy between the phenomena of bending of light around large cosmological objects (re: Einstein's General Theory of Relativity) & the above mentioned phenomena of the discrepancy between the observed & actual relative orbiting distances.

4 EXPLAINING THAT THERE ARE NO COSMOLOGICAL ANOMALIES

This paper explains that there are no cosmological anomalies:

- There is no need to introduce so-called dark matter.
- There is no need to introduce so-called dark energy.
- There are no singularities in the maximum past.
- There are no infinities in the maximum future.

The non-commutative nature of The Special Theory of Double Relativity explains these anomalies. The extension of postulate-2 states the value of the maximum relative distance to be b. And b is a universal constant, similar to the speed of light constant c.

4.1 There is no need to introduce so-called dark matter

On cosmological large scales, relative distances cannot be calculated by $d = d_y - d_x$. Instead, one has to use *equation-6*. The superposition equations in paragraph 2.7 may also play a role here, but for simplicity reasons are left out of the under-mentioned discussion.

$$d = \frac{d_y - d_x}{1 - \left(d_y * d_x\right)}$$ *(See paragraph 2.3)* *(Equation-6)*

d = relative distance between object-y and object-x [-]
d_y = relative distance between object-y and the observer [-]
d_x = relative distance between object-x and the observer [-]

Instead of introducing dark matter, the anomalies related to missing matter emerges out of the non-commutative geometry, as relative distances are indeed not equal to $d = d_y - d_x$. The equations in paragraph 2.7 may also play a role regarding the matter discrepancies.

4.2 There is no need to introduce so-called dark energy

The universal superposition ratio equation of $\overline{R}_u$ (*equation-13*) shows that:

$$\left(\overline{R}_u\right)_{x=1} = 8 - \frac{7}{\sqrt{2}}$$ *(See graph-1)* (= approximately 3.050)

$$\left(\overline{R}_u\right)_{x=-1} = \frac{1}{\sqrt{2}}$$ (=cosmological constant) *(See graph-1)* (= approximately 0.707)

The expansion factor of space itself is observed to be approximately 3. The curvature of the universe is observed to be approximately $\left(1/\sqrt{2}\right) * 13.7 \approx 9.7$ [billion years]. Indeed, both values already emerge out of the boundaries condition of the universal superposition ratio equation $\overline{R}_u$, and thus there is no need to introduce so-called dark energy.

4.3 There are no singularities in the maximum past

The boundaries condition of the universal superposition delta equation $\overline{\Delta}_u$ *(equation-17)* shows values at the horizon ($d_x = 1$ and $d_x = -1$) that are always larger than zero *(see graph-2).* So, there are no singularities at the boundaries of the observable universe.

For example, the universal superposition delta equation $\overline{\Delta}_u$ (*equation-17*) shows:

$$\left(\overline{\Delta}_u\right)_{x=1} = 1 - \left[\frac{1}{8 - \frac{7}{\sqrt{2}}}\right] \qquad \textit{(See graph-2)} \ \ (= \text{approximately } 0.672)$$

The universal superposition delta *equation-16* $\overline{\Delta}_u = \left(\overline{y}_u - d_x\right) - \overline{d}_u$ shows that in the maximum past the <u>observed</u> relative distances subtraction $\left(\overline{y}_u - d_x\right)$, and the <u>actual</u> universal superposition relative distance $\overline{d}_u$ between both objects, has a positive superposition delta-value of around 0.672. So, the <u>actual</u> relative distance between both objects in the maximum past has thus always got a finite value, which indeed is larger than the perceived singularity at the so-called Big Bang. The <u>true</u> size of the universe in the maximum past has thus always got a finite value.

4.4 There are no infinities in the maximum future

The boundaries condition of the universal superposition delta equation $\overline{\Delta}_u$ *(equation-17)* shows values at the horizon ($d_x = 1$ and $d_x = -1$) that are always smaller than the value of one *(see graph-2).* So, there are no infinities at the boundaries of the observable universe.

For example, the universal superposition delta equation $\overline{\Delta}_u$ (*equation-17*) shows:

$$\left(\overline{\Delta}_u\right)_{x=-1} = \sqrt{2} - 1 \qquad \textit{(See graph-2)} \ \ (= \text{approximately } 0.414)$$

The universal superposition delta *equation-16* $\overline{\Delta}_u = \left(\overline{y}_u - d_x\right) - \overline{d}_u$ shows that in the maximum future the <u>observed</u> relative distances subtraction $\left(\overline{y}_u - d_x\right)$, and the <u>actual</u> universal superposition relative distance $\overline{d}_u$ between both objects, has a positive superposition delta-value of around 0.414. So, the <u>actual</u> relative distance between both objects in the maximum future has thus always got a finite value, which indeed is much smaller than the perceived infinity of the far future. Also, the <u>true</u> size of the universe in the maximum future has always got a finite value, because the extension of postulate-2 in *chapter-1* defines the maximum relative distance in the universe to remain being the finite value of b, the distance to the cosmic microwave background radiation.

5 EXTENSION OF SPECIAL RELATIVITY TO GENERAL RELATIVITY

This paper makes some predictions related to gravity, in reference to general relativity:
- Why so-called black-holes are perceived to be so massive.
- Why the entropy of a black-hole is related to the horizon area, and not its volume.

5.1 Describing gravity (acceleration) by means of the universal horizon constant

The Special Theory of Double Relativity excludes gravity and acceleration. But, conceptually, one can extrapolate how it may also describe an extension to Einstein's General Theory of Relativity.

The basic universal horizon constant is described in paragraph 2.9:

$k_u = 2$

k_u = universal horizon constant [-]. The maximum span that an observer can measure is two times the distance to the cosmic microwave background radiation. Namely, one always can observe the universe in opposite directions.

Including gravity (acceleration), this paper claims that the value of k_u changes to:

$$k_u = 2 * F_g \quad ; \; F_g \leq 1 \qquad \textit{(Equation-20)}$$

k_u = universal horizon constant [-]
F_g = gravity (or acceleration) non-symmetric correction-factor [-]

The Special Theory of Double Relativity has defined the universal horizon constant k_u to be 2, F_g = 1. Extrapolating Einstein's General Theory of Relativity, this may indeed be an ideal situation and only valid when gravity and acceleration are excluded. This paper claims that the value of the universal horizon constant k_u is influenced by gravity and / or by acceleration, and that the value of the factor F_g can become smaller than 1. But F_g only becomes significantly smaller than 1 when gravity and / or acceleration obtain a very large value indeed.

Gravity and acceleration are here described as a non-symmetric factor when observing the universe in opposite directions, i.e. the relative distance to the cosmic microwave background radiation in one direction is not equal anymore to the distance in the other direction. In other words, gravity and acceleration are here described as a one directional "move closer" horizon positioning; this compared to the distance of the universe horizon in the other direction. A strong gravity (acceleration) force leads to a closer positioning to the horizon of the observable universe in one specific direction, and thus tilting away from the central location of the universe.

5.2 The position of the horizon of a black-hole in the observable universe

If *equation-20* is correct, then the universal horizon constant k_u for a black-hole may have been reduced to a value smaller than 2. The relative distance between the black-hole's horizon and the cosmic microwave background radiation may even be close to zero, thus making the shape of the universe for a black-hole in that boundaries direction two dimensional. So, the position of a black-hole's horizon is here described as being located at the universe horizon itself. Metaphorical speaking, a black-hole becomes a sort of umbilical cord for its observable surrounding space. One can speculate that the idea of an umbilical cord, explains why each galaxy seems to have a black-hole at its centre.

On cosmological large scales, relative distances cannot be calculated by $d = d_y - d_x$. Instead, one has to use *equation-6*. The superposition equations in paragraph 2.7 may also play a role here, but for simplicity reasons are left out of the under-mentioned discussion.

$$d = \frac{d_y - d_x}{1 - (d_y * d_x)} \qquad \textit{(See paragraph 2.3)} \quad \textit{(Equation-6)}$$

d = relative distance between object-y and object-x [-]
d_y = relative distance between object-y and the observer [-]
d_x = relative distance between object-x and the observer [-]

If one would observe a black-hole (or object with astronomical large acceleration) in such a geometric direction that it correlated well with a "k_u corrected" that leads to a universal value of $d_y \approx 1$, then *equation-6* can be modified into *equation-21*:

$$d_b \approx \frac{1 - d_x}{1 - (1 * d_x)} \qquad d_b \approx 1 \qquad \textit{(Equation-21)}$$

d_b = relative distance between the black-hole-y and the object-x [-]
d_x = relative distance between the object-x and the observer [-]

The observer measures the distance between the black-hole and object-x to be much smaller than d_b. Special double relativity has got a non-commutative aspect at its core.

According to classic Newtonian Physics, the orbital speed of an object around a black-hole should be proportional to the reciprocal value of the square root of its distance to the black-hole. But this distance is observed to be much smaller than the <u>actual</u> relative distance $d_b \approx 1$, and thus the orbital speed is perceived to be far too fast. Please note that the postulates, as described in the introduction (*chapter-1*), remain valid.

Note: The black-hole's link to the universe horizon may also explain why the entropy of a black-hole is anticipated to be a function of area instead of volume. Namely, the black-hole sits in one specific geometric direction on the boundaries of the universe and thus is directly related to the area of the universe horizon. In one specific geometric direction, volume means little to a black-hole, i.e. the geometry of a black-hole is holographic.

6 THE CONNECTION BETWEEN QUANTUM MECHANICS AND RELATIVITY

This paper highlights some of the deeper philosophical aspects of the laws of physics:

- The connection between the Theories of Quantum Mechanics and Relativity.
- The need for an observer.

6.1 Heisenberg's uncertainty principle

Heisenberg's uncertainty principle states that one can never measure a quantum particle's position and momentum (= speed) both precise. There always is a degree of uncertainty.

One can extend Heisenberg's uncertainty principle for cosmological large scales. Namely, special double relativity has got two universal constants, i.e. one a very large speed and the other a cosmological large distance. This leads to non-commutative geometry. So, when observing two objects on cosmological large scales, one never can measure the relative distance and relative speed both precise. There always is a degree of uncertainty.

6.2 Pauli's exclusion principle

Pauli's exclusion principle states that two quantum particles can never be positioned at the same location at the same time.

This may also be embedded in The Special Theory of Double Relativity. Namely, an observer always perceives to be positioned central to the horizon of the universe. And because any other observer far away also perceives to be in a similar exact central location, there is thus always a relative distance between both so-called central locations.

6.3 The Copenhagen Interpretation

The Copenhagen Interpretation states that the observer plays a central role. The "probability wave" collapses in a way that is dependent on what the observer measures.

Paragraph 2.6 describes the connection between Quantum Mechanics' basic axiom of the superposition principle and the non-commutative aspect of special double relativity.

Similar to Quantum Mechanics, the observer has a key role in The Special Theory of Double Relativity as well. For example, the location of the observer plays a central role.

6.4 Planck's length-scale

The Universe is observed to be approximately spatial flat, Euclidean geometry applies. However, the cosmological constant shows that the universe is minuscule curved. But although the universe curvature angle α is small indeed, its distance-influence $d * \sin\alpha$ is normally still much larger than the Planck length-scale. Only when measuring distances on quantum scale, $d * \sin\alpha$ is of similar order of magnitude as the Planck's length-scale.

7 CONSTANTS IN PHYSICS EMERGE OUT OF BASIC GEOMETRY CONSTANTS

This paper claims that constants in physics emerge out of a set of basic universal geometry constants. The basic universal constants are described in paragraph 2.9.

$k_u = 2$ => related to opposite directions

$l_u = 1/\sqrt{2}$ => related to the curvature of the universe

$m_u = 3$ => related to the expansion of space itself; this for the critical density case

7.1 The size and age of the observable universe

The multi-dimensional expansion scale-factors are defined as follows:

$$s_u = m_u^{\ k} \quad => \quad s_u = m_u^{\ 2} \quad ; \ s_u = 3^2 = 9 \qquad \textit{(Equation-22)}$$

$$t_u = s_u^{\ k} \quad => \quad t_u = s_u^{\ 2} \quad ; \ t_u = \left(3^2\right)^2 = 81 \qquad \textit{(Equation-23)}$$

s_u = 2-dimensional expansion scale-factor, for the critical density case [-]
t_u = 3-dimensional expansion scale-factor, for the critical density case [-]
m_u = universal expansion constant [-]
k = short description for the universal horizon constant k_u [-]

This paper claims that the size of the observable universe is defined by:

- Addition *(and in the hypothetical case subtraction)* of the differential of volume-expansion AND the integral of surface-expansion; this by means of $d(s)$.
- The combined value to be multiplied by the curvature of the universe.

Equation-24 is speculative. Philosophical wise, it is deeply puzzling.

$$\boxed{B_u = l_u * \left[\frac{d(t)}{d(s)} \pm \int s * d(s)\right]} \quad => \quad B_u = \frac{1}{\sqrt{2}} * \left[\left(2 * s_u\right) + \left(\frac{s_u^2}{2}\right)\right] \approx 41.37 \ \textit{(and -15.91)} \qquad \textit{(Eq-24)}$$

$$A_u = \left(\frac{1}{m_u}\right) * B_u \quad => \quad A_u = \frac{B_u}{3} \approx 13.79 \qquad \textit{(and -5.30)} \qquad \textit{(Eq-25)}$$

B_u = distance to the universe horizon, for the critical density case [billion light years]
A_u = age of the observable universe [billion years]
l_u = universal cosmological constant [-]
m_u = universal expansion constant [-]
s, t = short description for the multi-dimensional expansion scale-factors s_u , t_u [-]

If *equation-24* and *25* are correct, then one can conclude that the size and age of the observable universe are both constant, irrespective where & when observations are made. The 'critical density' distance to the universe horizon will always be 41.37 [billion light years], the real-time age of the universe always 13.79 [billion years], both irrespective of the actual duration of a year! This all if the basic universal geometry constants stay as is.

WMAP estimates the age of the universe to be 13.73 [billion years]; this with an accuracy of +/- 0.12 [billion years]. Indeed, the calculated value of $A_u \approx 13.79$ [billion years] is within the observed accuracy limits. If *equation-25* is correct, then the value of c emerges out of a mathematical equation that uses basic universal geometry constants only.

7.2 The value of e

The universal superposition ratio equation $\overline{R}_u$ (*equation-13*) boundary value for $d_x = 1$:

$$\left(\overline{R}_u\right)_{x=1} = 8 - \frac{7}{\sqrt{2}}$$ *(See graph-1)* (= approximately 3.050)

The boundaries ratio value for the maximum past $\left(\overline{R}_u\right)_{x=1}$ is close to the value of the universal expansion constant m_u =3. *Equation-26* corrects for the discrepancy:

$$F_{\exp} = k_u - \frac{m_u}{\left(\overline{R}_u\right)_{x=1}} \quad => \quad F_{\exp} = 2 - \left[\frac{3}{8 - \frac{7}{\sqrt{2}}}\right] \quad => F_{\exp} \approx 1.016474876 \quad \textit{(Equation-26)}$$

$$\boxed{e_u = k_u + (l_u * F_{\exp})} \quad => \quad e_u = 2 + \left(\frac{F_{\exp}}{\sqrt{2}}\right) \quad => \; e_u \approx 2.7188 \qquad \textit{(Equation-27)}$$

Note that $e \approx 2.7183$.......

e_u = approximated value for the natural logarithm constant e [-]
e = exact value for the natural logarithm constant e [-]
$F_{\exp}$ = universal logarithm correction-factor [-]
$\overline{R}_u$ = universal superposition ratio [-]
k_u = universal horizon constant, $k_u = 2$ [-]
l_u = universal cosmological constant, $l_u = 1/\sqrt{2}$ [-]
m_u = universal expansion constant, $m_u = 3$ [-]
d_x = relative distance between object-x and the observer [-]
x = short description for d_x

If *equation-27* is correct, then the approximated value of e emerges out of an equation that uses basic universal geometry constants.

7.3 The value of Pi

The universal superposition delta equation $\overline{\Delta}_u$ (*equation-17*) boundary values $d_x = 1$ & -1:

$$\left(\overline{\Delta}_u\right)_{x=1} = 1 - \left[\frac{1}{\left(\overline{R}_u\right)_{x=1}}\right] \quad => \quad \left(\overline{\Delta}_u\right)_{x=1} = 1 - \left[\frac{1}{8 - \frac{7}{\sqrt{2}}}\right]$$

(See graph-2) (= approx. 0.672)

$$\left(\overline{\Delta}_u\right)_{x=-1} = \left[\frac{1}{\left(\overline{R}_u\right)_{x=-1}}\right] - 1 \quad => \quad \left(\overline{\Delta}_u\right)_{x=-1} = \sqrt{2} - 1$$

(See graph-2) (= approx. 0.414)

Adding up the boundaries delta values for the maximum future and past $\left(\overline{\Delta}_u\right)_{x=-1} + \left(\overline{\Delta}_u\right)_{x=1}$ gives a value that is close to 1. *Equation-28* corrects for the discrepancy:

$$F_{phi} = \left(\overline{\Delta}_u\right)_{x=-1} + \left(\overline{\Delta}_u\right)_{x=1} = \left[\frac{1}{\left(\overline{R}_u\right)_{x=-1}}\right] - \left[\frac{1}{\left(\overline{R}_u\right)_{x=1}}\right] \quad => \quad F_{pi} \approx 1.086371854 \quad \textit{(Equation-28)}$$

$$\boxed{\pi_u = k_u * \frac{(1 + l_u)}{F_{pi}}} \quad => \quad \pi_u = \frac{\left(2 + \sqrt{2}\right)}{F_{pi}} \quad => \quad \pi_u \approx 3.1428 \qquad \textit{(Equation-29)}$$

Note that $\pi \approx 3.1416.......$

π_u = approximated value for the geometric circle constant pi [-]
π = exact value for the geometric circle constant pi [-]
F_{pi} = universal circle correction-factor [-]
$\overline{\Delta}_u$ = universal superposition delta [-]
k_u = universal horizon constant, $k_u = 2$ [-]
l_u = universal cosmological constant, $l_u = 1/\sqrt{2}$ [-]
d_x = relative distance between object-x and the observer [-]
x = short description for d_x

If *equation-29* is correct, then the approximated value of pi emerges out of an equation that uses basic universal geometry constants.

As the approximated value π_u is only slightly larger than π, it also hints towards an observable space that has minute differences only compared to Euclidean geometry.

Appendix-1 SPECULATIONS THAT BASIC CONSTANTS CAN BE SUBSTITUTED BY NATURAL CONSTANTS

Combining *equation-26* with *equation-28* leads to a relationship between $F_{\exp}$ and F_{pi} :

$$F_{\exp} = k_u + \left[m_u * \left(F_{pi} - \frac{1}{l_u} \right) \right] \qquad \textit{(Equation-30)}$$

$F_{\exp}$ = universal logarithm correction-factor, $F_{\exp} \approx 1.016474876$ [-]
F_{pi} = universal circle correction-factor, $F_{pi} \approx 1.086371854$ [-]
k_u = universal horizon constant, $k_u = 2$ [-]
l_u = universal cosmological constant, $l_u = 1/\sqrt{2} \approx 0.707106781$ [-]
m_u = universal expansion constant, $m_u = 3$ [-]

The definitions of e and pi by means of basic natural constants

The algebraic axioms for natural constants calculations are different from commutative algebra. But some addition and multiplication calculations between the natural constants are believed to have enough symmetry to allow commutative arithmetic's.

This paper claims that the basic universal constants can be substituted by basic natural constants, which values are calculated by a combination e and π, and are defined by *Eq-27 and 29*; this with the natural correction-factors F_e and F_Π being equal to the integer 1.

$$\boxed{e = k_n + l_n} \quad \text{; with } k_{n1} \approx 1.298059561 \text{ and } l_{n1} \approx 1.420222268 \ (\approx \sqrt{2}) \qquad \textit{(Eq-31)}$$

$$\boxed{\pi = k_n * (1 + l_n)} \quad \text{; with } k_{n2} \approx 2.420222268 \text{ and } l_{n2} \approx 0.298059561 \ (\approx \frac{1}{2\sqrt{e}}) \qquad \textit{(Eq-32)}$$

$$k_n = \left(\frac{e+1}{2} \right) \pm \sqrt{\left(\left(\frac{e-1}{2} \right)^2 - (\pi - e) \right)} \text{ and } l_n = \left(\frac{e-1}{2} \right) \mp \sqrt{\left(\left(\frac{e-1}{2} \right)^2 - (\pi - e) \right)}$$

$$1 + e = k_{n1} + k_{n2} \quad \text{; with } k_{n1} - l_{n2} = 1 \qquad \textit{(Equation-33)}$$

$$\pi = k_{n1} * k_{n2} \quad \text{; with } k_{n2} - l_{n1} = 1 \qquad \textit{(Equation-34)}$$

e = exact value for the natural logarithm constant e [-]
π = exact value for the geometric circle constant pi [-]
k_n = natural horizon constant [-]
l_n = natural cosmological constant [-]
k_{n1}, k_{n2} = 1st and 2nd solution values for k_n [-]
l_{n1}, l_{n2} = 1st and 2nd solution values for l_n [-]

Defining the natural expansion and natural integer constants

Combining *equation-30* with the natural constants boundaries condition of F_e and F_Π being both 1, and using the values for k_n and l_n as calculated by *equation-31* and *32*:

$$\boxed{m_n = \frac{k_n - 1}{\frac{1}{l_n} - 1}} \quad \Rightarrow \quad m_{n1} \approx -1.007349819 \quad \text{and} \quad m_{n2} \approx 0.603058041 \qquad \textit{(Equation-35)}$$

And the natural integer constant p_n is defined by:

$$1/p_{n1} = -1 * m_{n1} \quad \text{and} \quad 1/p_{n2} = \sqrt{e} * m_{n2} \qquad \text{Note: } \left(\overline{R}_u\right)_{x=1} \approx \left[1 + \frac{(p_{n2} - 1)}{l_{n2}}\right] * m_u \quad \textit{(Eq-36)}$$

$$\Rightarrow \quad p_{n1} \approx 0.992703807 \quad \text{and} \quad p_{n2} \approx 1.005758349$$

Symmetry equations are defined by adjusting geometry, so that natural integer → 1

$$\left(\overline{R}_n\right)_{sym,x=1} = m_u \qquad \textit{(Equation-37)}$$

$$\frac{1}{(m_n)_{m,1}^2} + \frac{1}{(m_n)_{m,2}^2} = k_{n1} + k_{n2} \qquad \textit{(Equation-38)}$$

$$\sqrt{(m_n)_{m-k}} + \sqrt{(m_n)_{m+k}} = \pm 2 * \varphi_2 \qquad \textit{(Equation-39)}$$

$$(m_n)_{m-k} + (m_n)_{m+k} = 2 * m_u \qquad \textit{(Equation-40)}$$

Deriving out of *Equation-33* and *38*: $(m_n)_{m,1} = -1$ and $(m_n)_{m,2} = 1/\sqrt{e}$

Deriving out of *Equation-39* and *40* : $\sqrt{(m_n)_{m-k}} = \pm 1$ and $\sqrt{(m_n)_{m+k}} = \mp\sqrt{5}$

$(m_n)_m$ = natural expansion constant for symm. natural geometry, the observed space [-]
$(m_n)_{m-k}$ = natural expansion constant for symmetric natural geometry & static space [-]
$(m_n)_{m+k}$ = natural expansion constant for symmetric natural geometry, expanding by 5 [-]
k_n = natural horizon constant [-]
l_n = natural cosmological constant [-]
m_n = natural expansion constant [-]
p_n = natural integer constant [-]
m_u = universal expansion constant, $m_u = 3$ [-]
$\left(\overline{R}_n\right)_{sym,x=1}$ = natural symmetric $(p_n \to 1)$ superposition ratio, at the horizon of the past [-]
$\left(\overline{R}_u\right)_{x=1}$ = universal superposition ratio, at the horizon of the past [-]
$(m_n)_{m,1}$, $(m_n)_{m,2}$ = 1st and 2nd solution values for $(m_n)_m$ [-]
m_{n1}, m_{n2} = 1st and 2nd solution values for m_n [-]
e = exact value for the natural logarithm constant e [-]
φ = golden ratio constant, $\varphi_1 = (1+\sqrt{5})/2$ and $\varphi_2 = (1-\sqrt{5})/2$ [-]

The basic natural constants for symmetric natural geometry, expanding by 5

Substituting the observed expanding geometry ($m_u = 3$) with a natural expansion of 5, and defining symmetry between the natural horizon & cosmological constants as follows:

$(m_n)_{m+k} = m_u + k_u$ => $(m_n)_{m+k} = 5$ *(Equation-41)*

$(k_n)_{m+k} - (l_n)_{m+k} = 1$ *(Equation-42)*

By means of *equation-35* , *41* and *42* one can calculate the natural constants:

$$(k_n)_{m+k} = -3\varphi$$

$$(l_n)_{m+k} = -3\varphi - 1$$

$(k_n)_{m+k}$ = natural horizon constant for symmetric natural geometry, expanding by 5 [-]
$(l_n)_{m+k}$ = natural cosmological constant for symm. natural geometry, expanding by 5 [-]
$(m_n)_{m+k}$ = natural expansion constant for symmetric natural geometry, expanding by 5 [-]
k_u = universal horizon constant, $k_u = 2$ [-]
m_u = universal expansion constant, $m_u = 3$ [-]
φ = golden ratio constant, $\varphi = \left(1 \pm \sqrt{5}\right)/2$ [-]

The definition of the natural square root of pi

The natural square root of pi can be extrapolated out of *equation-34* ($\pi = k_{n1} * k_{n2}$):

$\rho^{\rho} = \pi = k_{n1} * k_{n2}$ => $\rho \approx 1.854105968$ *(Equation-43)*

Please note that: $\rho \approx (k_n)_{m+k,\varphi 2}$ with $(k_n)_{m+k,\varphi 2} \approx 1.854101966$

ρ = natural square root of π [-]
π = exact value for the geometric circle constant pi [-]
$(k_n)_{m+k,\varphi 2}$ = solution value for $(k_n)_{m+k}$; this with $\varphi_2 = \left(1 - \sqrt{5}\right)/2$ [-]
k_{n1}, k_{n2} = 1st and 2nd solution values for the natural horizon constant k_n [-]

The basic natural constants for symmetric and static natural geometry

Substituting the observed expanding geometry ($m_u = 3$) with a natural static geometry, and defining symmetry between the natural horizon & cosmological constants as follows:

$(m_n)_{m-k} = m_u - k_u$ => $(m_n)_{m-k} = 1$ *(Equation-44)*

$(k_n)_{m-k} - (l_n)_{m-k} = 1$ *(Equation-45)*

By means of *equation-35* , *44* and *45* one can calculate the natural constants:

$$(k_n)_{m-k} = \varphi$$

$$(l_n)_{m-k} = \frac{1}{\varphi}$$

$(k_n)_{m-k}$ = natural horizon constant for symmetric natural geometry & static space [-]
$(l_n)_{m-k}$ = natural cosmological constant symmetric natural geometry & static space [-]
$(m_n)_{m-k}$ = natural expansion constant for symmetric natural geometry & static space [-]
k_u = universal horizon constant, $k_u = 2$ [-]
m_u = universal expansion constant, $m_u = 3$ [-]
φ = golden ratio constant, $\varphi = \left(1 \pm \sqrt{5}\right)/2$ [-]

Substituting the universal constants in *equation-18* by these natural symmetric constants:

$$\boxed{(\overline{R}_n)_{m-k} = \left[(d_x+1)^{\varphi+1}\right] - \left[\frac{1}{\varphi}(d_x+1)^{\varphi}\right] + \left[(d_x+1)^{\varphi-1}\right] + \frac{1}{\varphi}} \quad \textit{(Equation-46)}$$

With the boundaries conditions:

$$\left[(\overline{R}_n)_{m-k}\right]_{x=-1} = \frac{1}{\varphi}$$
$$\left[(\overline{R}_n)_{m-k}\right]_{x=0} = 2$$
$$\left[(\overline{R}_n)_{m-k}\right]_{x=1} = \left(\frac{5}{2} - \frac{1}{\varphi}\right) * 2^{\varphi} + \frac{1}{\varphi}$$

$(\overline{R}_n)_{m-k}$ = natural superposition ratio for symmetric natural geometry & static space [-]
d_x = relative distance between object-x and the observer [-]
x = short description for d_x [-]
φ = golden ratio constant, $\varphi = \left(1 \pm \sqrt{5}\right)/2$ [-]

The size and age of a universe with symmetric and static natural geometry

The values for the multi-dimensional expansion scale-factors (see *equation-22* and *23*):

$$(s_n)_{m-k} = (m_n)_{m-k}^k \quad => \quad (s_n)_{m-k} = 1^\varphi \quad => \quad (s_n)_{m-k} = 1 \qquad \textit{(Equation-47)}$$

$$(t_n)_{m-k} = (s_n)_{m-k}^k \quad => \quad (t_n)_{m-k} = 1^\varphi \quad => \quad (t_n)_{m-k} = 1 \qquad \textit{(Equation-48)}$$

The size and age of a universe with symmetric and static natural geometry (*eq-24* and *25)*:

$$(B_n)_{m-k} = (l_n)_{m-k} * \left[\frac{d(t)}{d(s)} \pm \int s * d(s)\right] \; => (B_n)_{m-k} = \frac{1+\varphi}{2} \quad (\textit{and } \frac{3-\varphi}{2}) \qquad \textit{(Equation-49)}$$

$$(A_n)_{m-k} = \left(\frac{1}{(m_n)_{m-k}}\right) * (B_n)_{m-k} \quad => \; (A_n)_{m-k} = (B_n)_{m-k} \qquad \textit{(Equation-50)}$$

The relationships between the universal and natural constants for this symm & static case:

$(x_\varphi)_{addition} + (x_\varphi)_{subtracion} = k_u$ and $(x_{\varphi 1})_{addition} + (x_{\varphi 2})_{subtracion} = \left(\frac{m_u}{k_u}\right) - (l_n)_{m-k,\varphi 2}$

and $(x_{\varphi 2})_{addition} + (x_{\varphi 1})_{subtracion} = \left(\frac{m_u}{k_u}\right) - (l_n)_{m-k,\varphi 1}$

$(x_\varphi)_{addition} - (x_\varphi)_{subtracion} = (l_n)_{m-k}$ and $(x_{\varphi 1})_{addition} - (x_{\varphi 2})_{subtracion} = \left(\frac{m_u}{k_u}\right) - k_u$

and $(x_{\varphi 2})_{addition} - (x_{\varphi 1})_{subtracion} = \left(\frac{m_u}{k_u}\right) - k_u$

$(B_n)_{m-k}$ = distance to horizon for symmetric & static natural geometry [b light years]
$(A_n)_{m-k}$ = age of a universe with symm. & static natural geometry [b y]; [b y] = $(1+s)^S$ [y]
$(k_n)_{m-k}$ = natural horizon constant for symmetric natural geometry & static space [-]
$(l_n)_{m-k}$ = natural cosmological constant symmetric natural geometry & static space [-]
$(m_n)_{m-k}$ = natural expansion constant for symmetric natural geometry & static space [-]
$(s_n)_{m-k}$ = 2-dimensional expansion scale-factor for symm. and static natural geometry [-]
$(t_n)_{m-k}$ = 3-dimensional expansion scale-factor for symm. and static natural geometry [-]
k, s, t = short description for $(k_n)_{m-k}$, $(s_n)_{m-k}$ and $(t_n)_{m-k}$ respectively [-]
$(x_\varphi)_{addition}$ = age of universe by means of addition equation, $(x_\varphi)_{addition} = (1+\varphi)/2$ [b y]
$(x_\varphi)_{subtraction}$ = age of universe by means of subtraction eq., $(x_\varphi)_{subtraction} = (3-\varphi)/2$ [b y]
k_u = universal horizon constant, $k_u = 2$ [-]
m_u = universal expansion constant, $m_u = 3$ [-]
φ = golden ratio constant, $\varphi_1 = (1+\sqrt{5})/2$ and $\varphi_2 = (1-\sqrt{5})/2$ [-]

The golden ratio constant with natural mathematics

Equation-49 and *50* show that the distance to the horizon, and the age of the universe, have two sets of solution values. For a universe with symmetric and static geometry:

$$(x_\varphi)_{addition} = \frac{1+\varphi}{2} \quad \text{; can also be expressed by} \quad (x_\varphi)_{addition} = \frac{\varphi^2}{2}$$

$$(x_\varphi)_{subtraction} = \frac{3-\varphi}{2} \quad \text{; can also be expressed by} \quad (x_\varphi)_{subtraction} = \frac{1+(1/\varphi^2)}{2}$$

One of the axioms of natural mathematics is defined in such a way that the arithmetic's lead to one set of x_φ solutions only:

$$(x_{\varphi n})_{addition} = (x_{\varphi n})_{subtraction} \qquad \textit{(Equation-51)}$$

This leads to the following set of solutions for φ_n :

$$\frac{1+\varphi_{n1}}{2} = \frac{3-\varphi_{n1}}{2} \quad => \quad \varphi_{n1} = 1$$

$$\frac{{\varphi_{n2}}^2}{2} = \frac{1+(1/{\varphi_{n2}}^2)}{2} \quad => \quad \varphi_{n2} = \sqrt{\varphi}$$

For a universe with symmetric and static geometry, and when using the above mentioned natural mathematics arithmetic's, one can express the natural constants as follows:

$$[(k_n)_{m-k}]_{natural\_mathematics} = {\varphi_{n2}}^2$$

$$[(l_n)_{m-k}]_{natural\_mathematics} = \frac{\varphi_{n1}}{{\varphi_{n2}}^2}$$

Note: l = m divided by k.

$$[(m_n)_{m-k}]_{natural\_mathematics} = \varphi_{n1}$$

$(k_n)_{m-k}$ = natural horizon constant for symmetric natural geometry & static space [-]
$(l_n)_{m-k}$ = natural cosmological constant symmetric natural geometry & static space [-]
$(m_n)_{m-k}$ = natural expansion constant for symmetric natural geometry & static space [-]
$(x_\varphi)_{addition}$ = age of universe by means of addition equation, $(x_\varphi)_{addition} = (1+\varphi)/2$ [b y]
$(x_\varphi)_{subtraction}$ = age of universe by means of subtraction eq., $(x_\varphi)_{subtraction} = (3-\varphi)/2$ [b y]
$(x_{\varphi n})_{addition}$ = age of universe by means of addition eq., with natural mathematics [b y]
$(x_{\varphi n})_{subtraction}$ = age of universe by means of subtraction eq., with natural mathematics [b y]
φ_n = golden ratio constant with natural mathematics, $\varphi_{n1} = 1$, $\varphi_{n2} = \sqrt{\varphi}$ [-]
φ = golden ratio constant with classic mathematics, $\varphi_1 = (1+\sqrt{5})/2$, $\varphi_2 = (1-\sqrt{5})/2$ [-]

Appendix-2 **DEFINITION OF THE GOLDEN UNIVERSAL SUPERPOSITION RATIO EQUATION**

For the special case of $d_y = \frac{d_x}{1+d_x}$ and $\Delta = 1$, *equation-10* is transformed into:

$$\left(\Delta_\varphi\right)_{\Delta=1} = \frac{\left(d_y - d_x\right)}{1-\left(\frac{1}{\left(d_y * d_x\right)}\right)} = 1 \qquad => \qquad \left(\Delta_\varphi\right)_{\Delta=1} = 1 \qquad \textit{(Equation-52)}$$

Please note the following symmetry for $\left(\Delta_\varphi\right)_{\Delta=1}$: $d_y - d_x = \varphi$ and $d_y * d_x = -\varphi$

By means of $d_y * d_x = -\varphi$ and *equation-8*:

$$\left(R_\varphi\right)_{\Delta=1} = \frac{1}{1-\left(d_y * d_x\right)} = \frac{1}{1+\varphi} \qquad => \qquad \left(R_\varphi\right)_{\Delta=1} = \frac{2}{3 \pm \sqrt{5}} \qquad \textit{(Equation-53)}$$

By means of $d_y = \frac{d_x}{1+d_x}$, and either $d_y - d_x = \varphi$ or $d_y * d_x = -\varphi$, one can calculate:

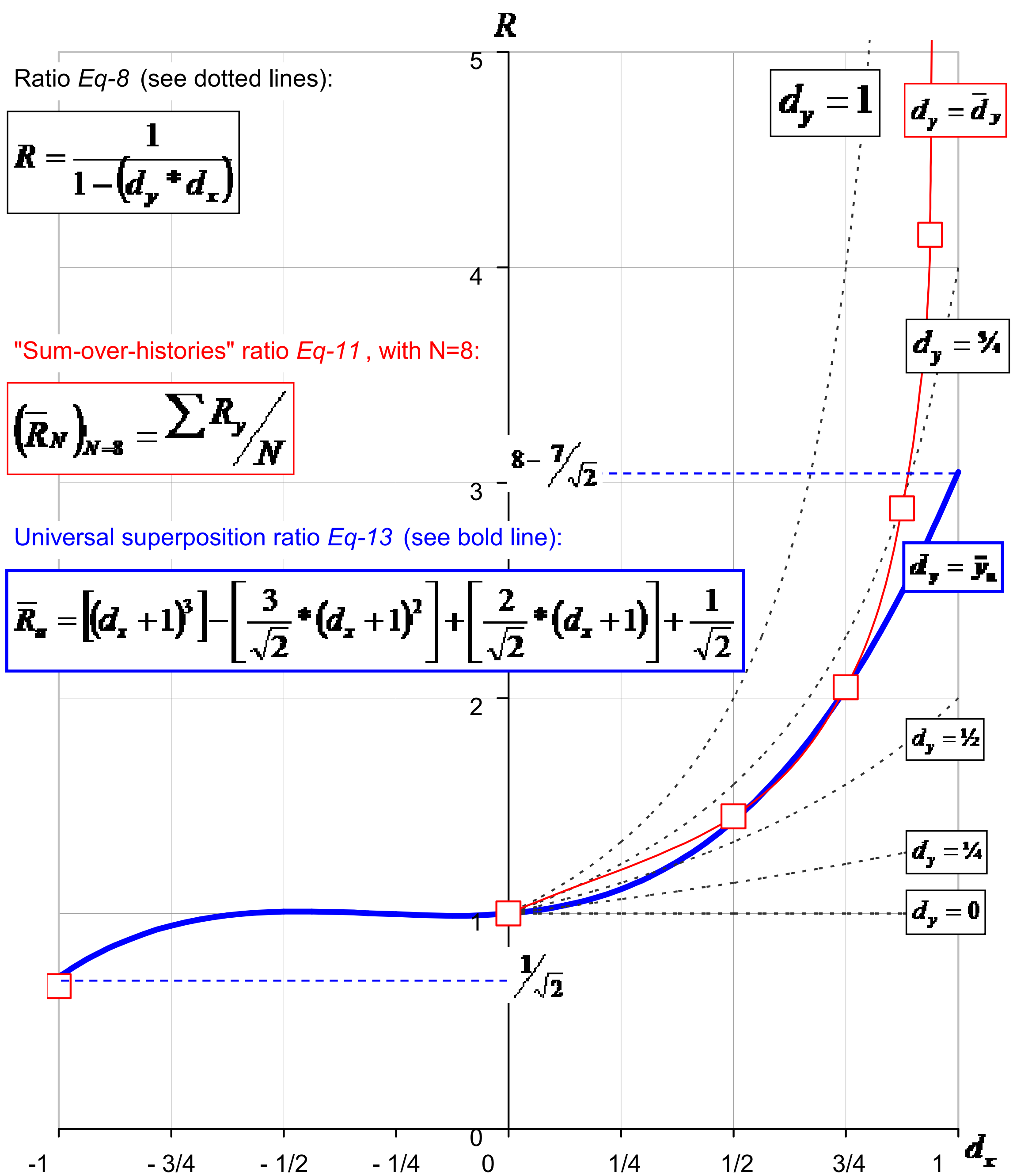
$$\left(d_x\right)_{\varphi_2} = \frac{-\varphi_2 \pm \sqrt{1-3\varphi_2}}{2} \quad => \quad \left(d_x\right)_{\varphi_2,x_1} \approx -0.535687387 \text{ and } \left(d_x\right)_{\varphi_2,x2} \approx 1.153721376$$

$$\left(d_y\right)_{\varphi_2} = \frac{\varphi_2 \pm \sqrt{1-3\varphi_2}}{2} \quad => \quad \left(d_y\right)_{\varphi_2,y_1} \approx -1.153721376 \text{ and } \left(d_y\right)_{\varphi_2,y_2} \approx 0.535687387$$

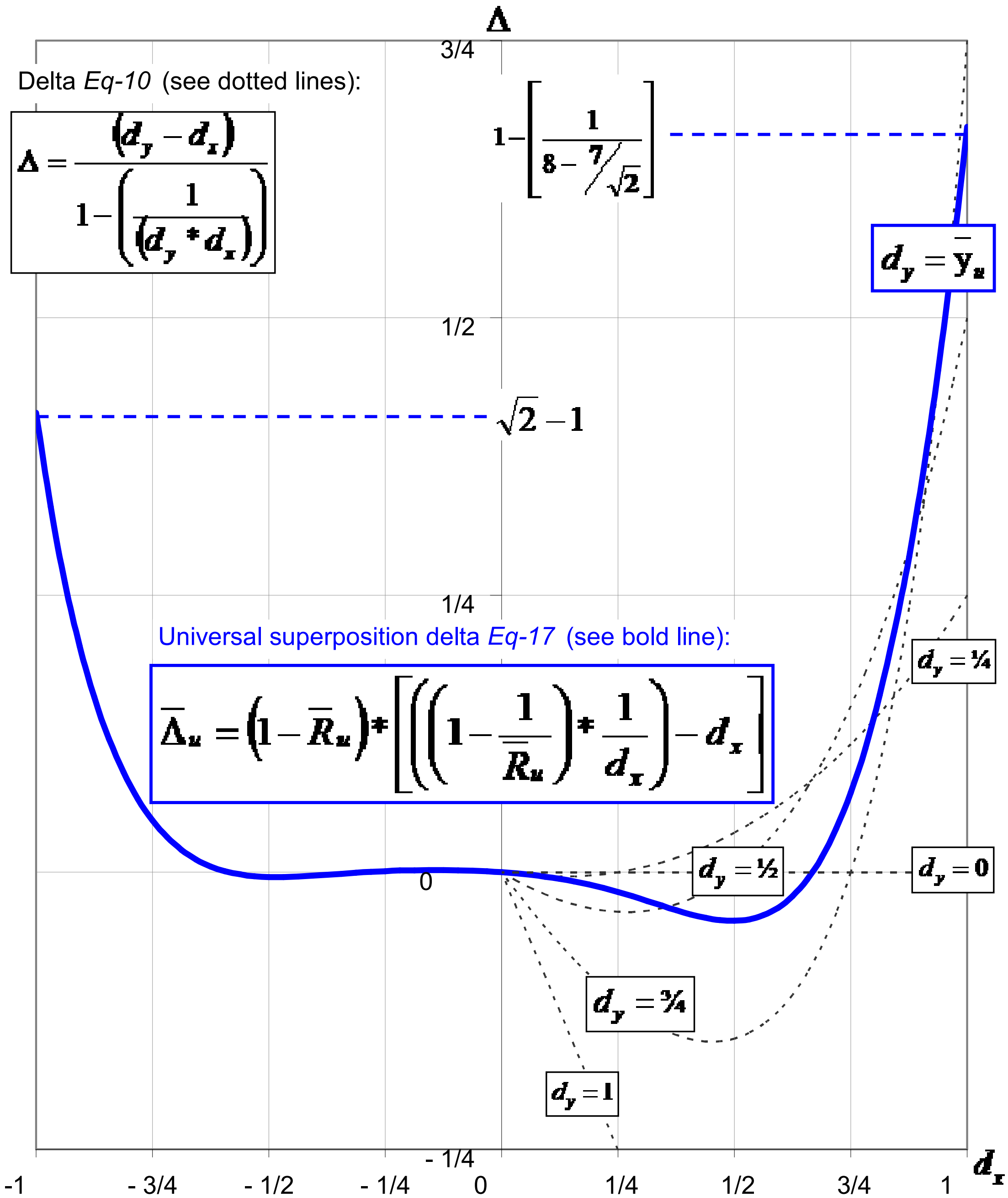
One can conclude that all these pairs of solutions are outside the observable universe.

$\left(\Delta_\varphi\right)_{\Delta=1}$ = golden universal superposition delta equation [-]
$\left(R_\varphi\right)_{\Delta=1}$ = golden universal superposition ratio equation [-]
Δ = delta between the values of (d_y - d_x) and d [-]
R = ratio between the values of d and (d_y - d_x) [-]
d = relative distance between object-y and object-x [-]
d_y = relative distance between object-y and the observer [-]
d_x = relative distance between object-x and the observer [-]
$\left(d_x\right)_{\varphi_2}$ = solution values for d_x; this while $d_y = d_x/\left(1+d_x\right)$ and $\Delta = 1$ [-]
$\left(d_y\right)_{\varphi_2}$ = solution values for d_y; this while $d_y = d_x/\left(1+d_x\right)$ and $\Delta = 1$ [-]
$\left(d_x\right)_{\varphi_2,x_1}$, $\left(d_x\right)_{\varphi_2,x_2}$ = 1st and 2nd solution values for $\left(d_x\right)_{\varphi_2}$ [-]
$\left(d_y\right)_{\varphi_2,y_1}$, $\left(d_y\right)_{\varphi_2,y_2}$ = 1st and 2nd solution values for $\left(d_y\right)_{\varphi_2}$ [-]
φ = golden ratio constant, $\varphi_1 = \left(1+\sqrt{5}\right)/2$ and $\varphi_2 = \left(1-\sqrt{5}\right)/2$ [-]

Graph-1


R
Ratio Eq-8 (see dotted lines):
$R = \frac{1}{1-(d_y * d_x)}$
"Sum-over-histories" ratio Eq-11, with N=8:
$(\bar{R}_N)_{N=8} = \sum R_y / N$
Universal superposition ratio Eq-13 (see bold line):
$\bar{R}_x = [(d_x+1)^3] - \left[\frac{3}{\sqrt{2}} * (d_x+1)^2\right] + \left[\frac{2}{\sqrt{2}} * (d_x+1)\right] + \frac{1}{\sqrt{2}}$
$d_y = 1$
$d_y = \bar{d}_y$
$d_y = ¾$
$d_y = \bar{y}_x$
$d_y = ½$
$d_y = ¼$
$d_y = 0$
$8 - 7/\sqrt{2}$
$1/\sqrt{2}$
5
4
3
2
1
0
-1
- 3/4
- 1/2
- 1/4
0
1/4
1/2
3/4
1
d_x

Graph-2


Δ
3/4
1/2
1/4
0
- 1/4
Delta Eq-10 (see dotted lines):
$\Delta = \frac{(d_y - d_x)}{1 - \left(\frac{1}{(d_y * d_x)}\right)}$
$1 - \left[\frac{1}{8 - \frac{7}{\sqrt{2}}}\right]$
$d_y = \bar{y}_u$
$\sqrt{2} - 1$
Universal superposition delta Eq-17 (see bold line):
$\bar{\Delta}_u = (1 - \bar{R}_u) * \left[\left(\left(1 - \frac{1}{\bar{R}_u}\right) * \frac{1}{d_x}\right) - d_x\right]$
$d_y = ¼$
$d_y = ½$
$d_y = 0$
$d_y = ¾$
$d_y = 1$
-1
- 3/4
- 1/2
- 1/4
0
1/4
1/2
3/4
1
d_x

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