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Stellar collapse in field theories of gravitation

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Abstract

Collapse in general relativity encounters contradictions unless the Einstein-Hilbert theory is reformulated as a field theory on a space metric that is homogeneous and isotropic at large distance (Minkowski space-time). The classic 'black hole' solution of Oppenheimer and Snyder fails to satisfy a matching condition at the surface of a collapsing 'dust' ball. Correcting this error leads to a new solution in which the contraction process stops at the Schwarzschild radius and particles accumulate at the surface of the ball. Catastrophic collapse into a black hole is prevented by the increasing gravitational energy inside the ball, which results in gravity changing from attractive to repulsive. The result confirms Einstein's and Eddington's judgements about gravitational collapse; the process, throughout the ball, comes to a halt as the escape velocity at the surface approaches that of light.

Keywords: Einstein-Hilbert equation; gravitational field theory; gravitational collapse; Oppenheimer-Snyder; harmonic coordinates; black hole; event horizon

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1 Puzzle of continual collapse of a large gravitating mass

Theorists on stellar structure from the early 1930s realised that the gravitational forces in a large enough contracting mass would overcome atomic pressures. Einstein's general relativity (GR) or gravitation theory is relevant, because high gravitational red shift is equivalent to high gravitational 'curvature' in time (Schutz, 2003) so that the electrons become relativistic. Their limiting pressure suffices only to hold up "white dwarf" stars smaller than about 1.3 solar masses (the Chandrasekhar mass). At higher masses, neutron pressure comes in, but suffices only up to about 3 solar masses (neutron star limit).

At still larger masses, gravity would overcome all physical forces. Eddington (1935) argued that this is absurd:

> The star has to go on radiating and radiating and contracting and contracting until, I suppose, it gets down to a few km radius, when gravity becomes strong enough to hold in the radiation, and the star can at last find peace.

He was confident some physics would intervene:

> I think there should be a law of Nature to prevent a star behaving in this absurd way!

Both Einstein and Eddington recognised that the speed-of-light limit in strong gravity intervenes against unending collapse, but neither fully recognized exactly where the fault lay. Until Einstein, gravity was thought of as a force, like the electric force. Einstein described gravity instead as a distortion of geometry, based on his Principle of Equivalence (PE). The weak form of this principle describes the bending of light past the sun, for example, but the strong form equates gravity to distortion of space-time universally; this includes rotating systems, so there is no way to distinguish between gravity and a 'centrifugal force'. Though Eddington did enter some reservations in his text (Eddington,1924), he, like Einstein, considered both 'forces' and the distinction between kinetic and potential energy to be obsolete hangovers from the days when inertial frames were privileged.

Hilbert (1917) pointed out that in GR, where space-time is defined by the gravitational metric $g_{\mu\nu}$, it is not possible to construct an energy tensor. Other critics pointed out an ambiguity in GR, that subsidiary assumptions were needed to solve the Einstein-Hilbert equation. The consequence that gravitational energy cannot be localized has generated almost a century of unresolved controversy (Kennefick, 2007) about the reality of gravitational waves. Even Vladimir Fock, who was an ardent critic of GR but also a proponent of such waves, did not dissent from this position, that is non-localizability of energy.

However, Fock in his pioneering text (Fock, 1966) rejected PE, insisting that what Einstein had achieved was a new theory of gravity, with only a partial geometrization of the force field. Whereas the GR insistence on a coordinate-free description leads us to accept considerable ambiguity in the solutions of the Hilbert-Einstein equation, Fock showed that imposing certain asymptotic conditions removes that ambiguity and imposes harmonic coordinates as four additional conditions on the Riemannian metric. He justified this as ensuring that the field at a large distance from an island system satisfies a correspondence principle, that is it gives the galilean metric of Special Relativity at large spatial distance, with only outgoing waves superimposed. Fock showed that if gravitational forces are neglected (Minkowski space-time), all the equations of motion of particles and non-gravitational fields may be written in covariant notation with a flat metric (that is having a zero curvature tensor). For the harmonic choice of coordinates this reduces to the galilean metric everywhere. Note that Fock's position was that "special" relativity is as "general" as GR. The presence of gravitating bodies means that the field $g_{\mu\nu}$ is not flat, so that in harmonic form it is not, like Minkowski space, homogeneous and isotropic; the space of GR is less, and not more, symmetric than Minkowski space. For this reason Fock rejects altogether the concept of a general relativity, and refers therefore to "Einstein's Theory of Gravitation" rather than "Einstein's Theory of General Relativity".

Nathan Rosen (Rosen, 1940 and 1963), introduced the idea of keeping the Minkowski metric $\gamma_{\mu\nu}$ alongside $g_{\mu\nu}$, and this use of a bimetric theory was followed by Logunov and colleagues (Logunov and Mestvirishvili, 1989, Logonov, 2001) in developing the Relativistic Theory of Gravitation (RTG). The adoption of Fock's harmonic metric ensures a real energy-momentum tensor (not the pseudo-tensor of Einstein), with localised energy density. A further key result is the realisation that gravity becomes repulsive at ultra-high density of matter. This arises from the non-linearity of the Einstein-Hilbert equation, whereby the field energy has an equivalent mass.

A parallel analysis by Babak & Grishchuk (1999) defines a 'metrical' energy tensor, which they show bears the same relation to the Landau-Lifshitz pseudotensor as does Logunov's. As Grishchuk comments (2009) "this modification of GR has far-reaching implications, affecting the polarization states and propagation of gravitational waves, the event horizons of black holes, and the early-time and late-time cosmological evolution".

The existence of a real energy-momentum tensor establishes gravity to be a Faraday-Maxwell field. Gravity does make certain objects, for example light rays and the bodies of our planetary system, behave as though they were travelling in a curved space, but nevertheless gravity is first and foremost a force and only secondarily geometry.

The concepts of gravitational energy being localized, and in certain conditions exerting a repulsive force, have stimulated further work on solving the Einstein-Hilbert equations for a contracting star. In 1939 there were two attempts, by Einstein (1939) and by Oppenheimer and Snyder(OS) (1939) that came to opposite conclusions. Both considered the simplified spherically symmetric case with mass particles (cold 'dust') having zero pressure. OS took a mathematical approach, fitting interior and exterior solutions at the event horizon and finding that particles reach it in finite proper time (ie. time as measured travelling with the particles). Einstein argued that the nothing-faster-than-light principle stopped the particles crossing the event horizon, preventing the formation of black holes. Subsequent work has discounted Einstein's objection and assumed the OS solution to be correct. We re-examine the latter solution, in view of an anomalous discontinuity between the external and internal metrics which it exhibits.

In bimetric theories (Rosen, 1940, Logunov, 2001), coexisting with the field, or Riemann metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \quad , \tag{1}$$

there is a space, or Minkowski metric

$$d\sigma^2 = \gamma_{\mu\nu} dx^\mu dx^\nu \quad . \tag{2}$$

Unlike in General Relativity (GR), where the field is synonymous with the geometry, we recognize that the field, as in the electromagnetic case, is propagated through an underlying Minkowski space. Then there is a preferred coordinate system, namely that for which the space metric is galilean. It may be called the inertial system or frame, and its corresponding field metric $g_{\mu\nu}$ gives rise to a gravitational potential $\Phi^{\mu\nu} = g^{\mu\nu}\sqrt{-g}$ whose divergence is zero. The latter condition may be made covariant by requiring the covariant divergence of the gravitational field in the Minkowski metric to be zero. This is the coordinate system which Einstein (Einstein, 1918) used in order to derive his formula for the gravitational radiation emitted from a time-varying quadrupole source.

In this article we show how the comoving coordinate frame introduced by Tolman (1934), and developed by Oppenheimer and Snyder (OS) (1939) and Landau and Lifshitz (1975) to describe gravitational collapse, may be transformed to the inertial, or harmonic frame. It is then possible to track the trajectories of individual particles in the collapse of a dust ball, for which the equation of state is simply $p = 0$.

2 The Oppenheimer-Snyder metric

The Oppenheimer-Snyder (OS) field metric for a dust ball may be written

$$ds^2 = d\tau^2 - V^2 dR^2 - W^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \quad , \tag{3}$$

where

$$\frac{W}{2m} = \left(\sqrt{R^3} - \frac{3\tau}{4m}F(R)\right)^{2/3}, \quad \frac{V}{2m} = \frac{2m\sqrt{R} - \tau F'(R)}{\sqrt{2mW}} \quad . \tag{4}$$

In this comoving metric the free-fall radial geodesics are simply $R =$constant, and the coordinate τ is the particle's proper time; in particular the surface of the ball is specified by a fixed R, which will be put as 1. The function $F(R)$, or rather the product FF', gives the mass distribution of the dust particles. For $R > 1$ (the external region), we put

$$F(R) = 1 \quad (R > 1) \quad , \tag{5}$$

giving zero density there, and for $R < 1$ (the internal region), F is left arbitrary for the moment, but we note that, for the metric to be continuous it has to satisfy

$$F(1-) = 1, \quad F'(1-) = 0 \quad . \tag{6}$$

At this point OS made a fatal error by choosing an F which fails to satisfy the second of these, so our choice of F will constitute a corrected version of OS. The cumulative mass distribution is given by the function

$$M(R) = \int_0^R 4\pi T^{00}(R')\sqrt{-g(R')}dR' = mF^2(R) \quad . \tag{7}$$

The harmonic coordinates (Fock, 1966, Logunov and Mestvirishvili, 1989 , Weinberg, 1972) (t, r, θ, ϕ) for this system are given by the solutions of

$$\Box t = 0, \quad \Box r = -\frac{2r}{W^2} \quad , \tag{8}$$

where the spherical d'Alembertian operator is given by

$$\Box = \partial_\tau^2 - \frac{1}{V^2}\partial_R^2 + \left(\frac{\dot{V}}{V} + \frac{2\dot{W}}{W}\right)\partial_\tau + \frac{1}{V^2}\left(\frac{V'}{V} - \frac{2W'}{W}\right)\partial_R \quad , \tag{9}$$

and we use dot and prime to signify differentiation with respect to τ and R respectively. The solution will be chosen to satisfy the asymptotic conditions

$$t \sim \tau, \quad r \sim W \quad (\tau \to -\infty) \quad , \tag{10}$$

and in that case the evolution $R =$constant describes, in the asymptotic region, the Newtonian collapse of a dust ball of uniform density. This may be demonstrated from the relation between r and t, which is

$$r(R,t) \sim \left[\frac{9m|t|^2 F^2(R)}{2}\right]^{1/3} \quad (t \to -\infty) \quad , \tag{11}$$

which, combined with (7), gives

$$M(R) = m\left[\frac{r(R,t)}{r(1,t)}\right]^3 \quad , \tag{12}$$

and shows that the mass contained in a ball of radius r is proportional to the ball's volume. The t-dependence of r has, of course, been known since Michell discovered it in the eighteenth century.

In the exterior region the solutions of (8) are (Logunov and Mestvirishvili, 1989)

$$t = \tau - 2\sqrt{2mW} + 2m\ln\frac{\sqrt{W}+\sqrt{2m}}{\sqrt{W}-\sqrt{2m}}, \quad r = W - m \quad (R > 1) \quad , \tag{13}$$

and the OS metric becomes, in this region,

$$ds^2 = \frac{r-m}{r+m}dt^2 - \frac{r+m}{r-m}dr^2 - (r+m)^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \quad , \tag{14}$$

which is the Schwarzschild metric, except that the Schwarzschild "radius" r has been replaced by $r+m$. We shall see that the solution of (8) may be continued into the interior region all the way to $R=0$, and that this corresponds to $r=0$ for all t. We deduce that it is this r, rather than Schwarzschild's, which should be regarded as the true radius, a conclusion which will be reinforced by making a closer examination of the internal solutions of (8). It is clear that the above solution (13) gives, as t goes to plus infinity, that W approaches $2m$ and r approaches m, and that these limits are reached at a finite value of proper time

$$\tau_f(R) = \frac{4m}{3}\left(\sqrt{R^3} - 1\right) \quad (R > 1) \quad . \tag{15}$$

The fact that this is finite has led to the widespread, indeed almost universally held, conclusion that a falling particle goes on to cross the "event horizon" at $r = m$ for proper times greater than τ_f, and therefore to be swallowed by the black hole at $r = 0$. We shall show that this conclusion is incorrect.

All we have to do is demonstrate that the interior solutions of (8) give a limiting value $r_f(R)$ in $0 < R < 1$, with $r_f(0) = 0$ and $r_f(1) = m$, together with a corresponding $\tau_f(R)$, for which t goes to plus infinity. We have been able to make the demonstration numerically for $r_f(R)$ with a particular choice of the function $F(R)$. However, for general F, the values of $\tau_f(R)$ may be obtained simply by examining the characteristics of (8). These satisfy, for both of these partial differential equations, the same pair of ordinary differential equations

$$\frac{d\tau}{dR} = \pm V(\tau, R) = \pm\left(2m\sqrt{R} - \tau F'\right)\sqrt{\frac{2m}{W}} \quad . \tag{16}$$

It is a simple matter to verify that the above value of $\tau_f(R)$ satisfies this with the upper sign in $R > 1$, where $F = 1$ and $F' = 0$. In the interior region the upper-sign characteristic through the point

$$\tau_f(1) = 0 \tag{17}$$

is the first one to be met by a geodesic $R=$constant coming from $\tau = -\infty$. The situation is illustrated in Figure 1, where we have plotted this characteristic, in both exterior and interior regions, for the case $2m = 1$, using in the interior the function F of a previous article of ours (Marshall, 2007), namely

$$F(R) = R^{3/2}e^{3X/2}, \quad X = 1 - R \quad (R < 1) \quad . \tag{18}$$

The figure represents the whole of space, that is $0 < r < +\infty$, and the whole of time, that is $-\infty < t < +\infty$; there are no singularities and no trapped surfaces of the kind supposed by Hawking and Penrose (1970). Such strange objects are to be found in the region beyond $t = +\infty$, and belong in the realm of science fiction. The finite value of the proper time τ_f is an indication that a falling particle, as it approaches the boundary characteristic passing through the point $R = 1, \tau = 0$, suffers an infinite gravitational red shift. We shall see a natural explanation for this in the infinite surface density of the dust ball in the limit $t \to +\infty$.

The integration of (8) in the interior region may be best achieved by changing to the characteristic coordinates (R, S), defined by

$$\tau = \tau(R, S), \quad \tau(1, S) = -2mS, \quad D_R\tau = V = \left(2m\sqrt{R} - \tau F'\right)\sqrt{\frac{2m}{W}} \quad . \tag{19}$$

The family of curves given by $S=$constant are the upper characteristics of (8), and in particular the boundary characteristic is $S = 0$. In terms of these coordinates the d'Alembertian (9) is transformed by putting

$$D_R = \partial_R + V\partial_\tau \quad , \tag{20}$$

where D_R is the derivative with respect to R along the characteristic, giving

$$\Box = -\frac{1}{V^2}D_R^2 + \frac{2}{V}\partial_\tau D_R + \frac{1}{V^2}\left(\frac{V'}{V} - \frac{2V}{W}\right)D_R + \frac{2}{W}\left(1 + \dot{W}\right)\partial_\tau \quad . \tag{21}$$

Note that ∂_τ does not commute with D_R, but that if we write it as

$$\partial_\tau = \Psi^{-1}D_S, \quad \Psi = \partial_S\tau(R, S) \quad , \tag{22}$$

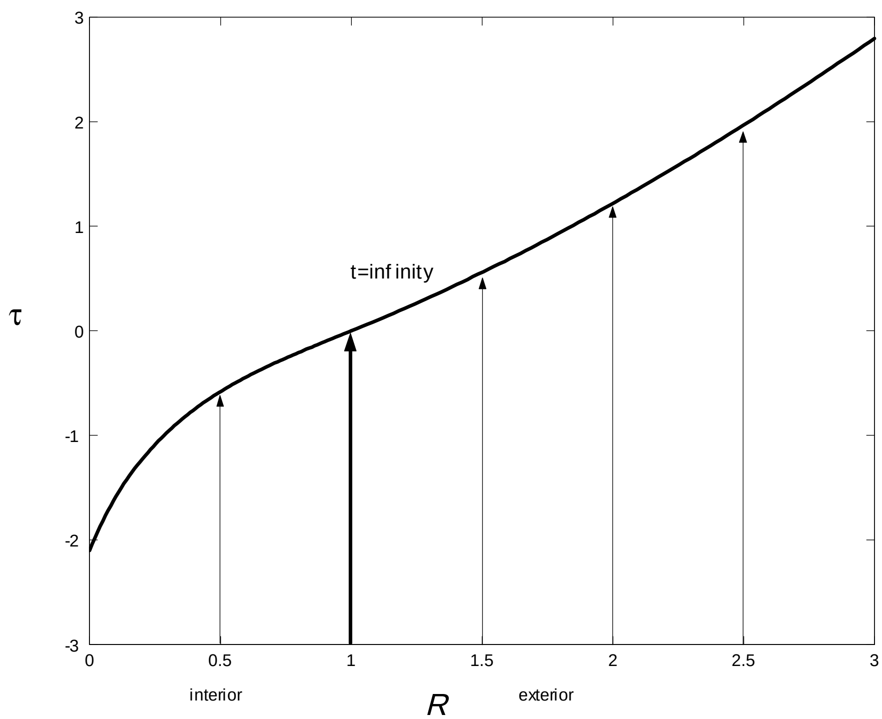

Figure 1: The limit of physical space-time with $2m = 1$. τ is the proper time of a dust particle and R is its comoving coordinate, so that $R = 1$ indicates a surface particle. A given dust particle, or in the exterior region a test particle, moves along the abcissa R =constant, arriving at the boundary curve after an infinite time t.

then D_R and D_S commute. In the numerical procedure, described in the next Section, there is no need to find the function Ψ explicitly, because we work with the differential operator ∂_τ rather than D_S. Although this integration is rather formidable, it may be guaranteed that no singularity occurs in the whole physical region, because the coefficients of the PDEs are nonzero and nonsingular, and the determinant of the second-order coefficients is negative throughout, thereby preserving their hyperbolic character.

Note that the characteristic curves we just introduced are also the null geodesics of the (corrected) OS metric. They are the paths taken by outward going light signals, and since the local light velocity is V, their form underlines the importance of the correction we made to the original OS metric, insisting on V being continuous at $R=1$. The time required for such a signal to go from an internal point (R_1, S) to an external point (R_2, S) may be written as

$$t\left(R_2,S\right)-t\left(R_1,S\right)=t\left(1,S\right)-t\left(R_1,S\right)+G\left(R_2,S\right) \quad , \tag{23}$$

where G is the travelling time in the exterior region, that is

$$G\left(R_2,S\right)=t\left(R_2,S\right)-t\left(1,S\right) \quad . \tag{24}$$

In view of the simple form (14) of the exterior metric, this latter integral may be simplified to give

$$G\left(R_2,S\right)=r\left(R_2,S\right)-r\left(1,S\right)+2m\ln\frac{r\left(R_2,S\right)-m}{r\left(1,S\right)-m} \quad . \tag{25}$$

Now, since $r(1,S)>m$ for all t, this result shows that any light signal emitted from inside the ball eventually reaches the exterior region, and this should lay to rest all preexisting ideas regarding trapped surfaces. But note that we say "eventually"; owing to the infinite red shift suffered at the boundary $R=1$, this travel time becomes infinite as $r\left(1,S\right)$ approaches m, that is in the closing stages of the compression process.

3 Numerical integration

It is convenient to define the operator

$$P=-V^2\Box=D_R^2-2\xi W\partial_\tau D_R-2\xi^2W\left(1+\dot{W}\right)\partial_\tau+\left(\xi-\frac{\xi'}{\xi}\right)D_R \quad , \tag{26}$$

where $\xi=V/W$, that is, for the choice we made in (18) for F,

$$\xi=R^{-1}\left(1+\frac{3\tau}{4m}Xe^{3X/2}\right)\left(1-\frac{3\tau}{4m}e^{3X/2}\right)^{-1} \quad , \tag{27}$$

and the harmonic coordinates then satisfy

$$Pt=0, \quad \left(P-2\xi^2\right)r=0 \quad . \tag{28}$$

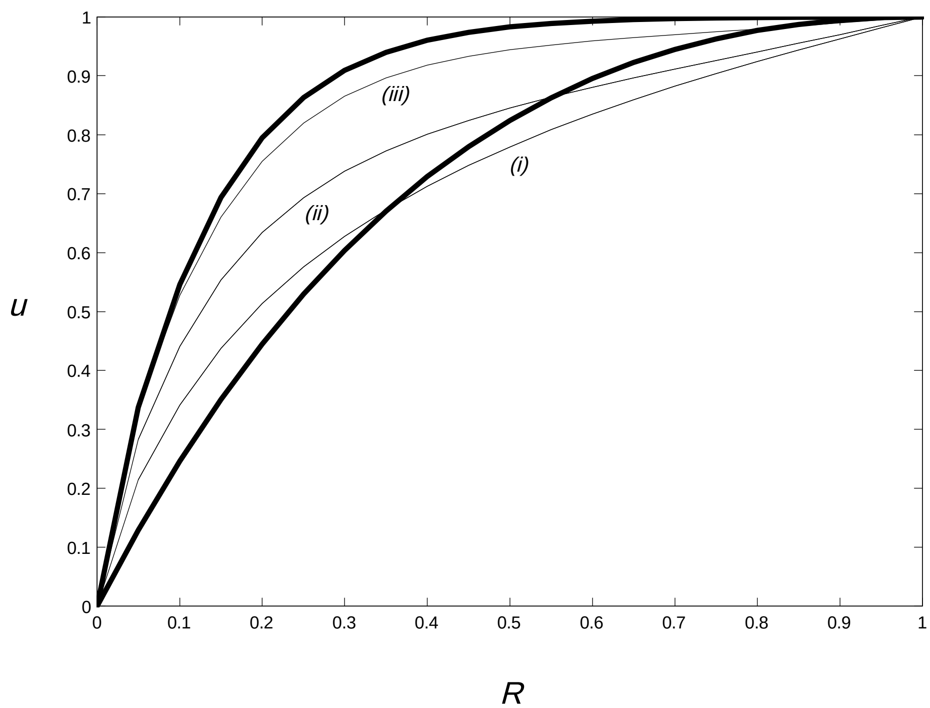

Figure 2: Evolution of the particle distribution $u\left(R\right)$. u is the position of a given particle relative to the surface, indexed by its comoving coordinate R. The lower bold curve gives $u(R)$ in the limit $r_0 \to \infty$, that is $t \to -\infty$, while the upper bold curve gives $u\left(R\right)$ for $r_0 = m$, that is $t \to +\infty$. The lighter curves give $u\left(R\right)$ for the values *(i)* $r_0 = 2m$ *(ii)* $r_0 = 1.4m$ *(iii)* $r_0 = 1.1m$.

These PDEs must be integrated in $0 < R < 1, S > 0$ with surface boundary conditions at $R = 1$

$$t\left(1,S\right) = -2mS - 2\sqrt{2mW_0} + \ln\frac{\sqrt{W_0}+\sqrt{2m}}{\sqrt{W_0}-\sqrt{2m}}, \quad r\left(1,S\right) = W_0 - m \quad , \tag{29}$$

where

$$W_0\left(S\right) = W\left(1,S\right) = 2m\left(1+\frac{3S}{2}\right)^{2/3} \quad , \tag{30}$$

and also with the asymptotic condition (10) for $S \to \infty$ and with finite values at $R = 0$.

Because $t\left(1,S\right)$ becomes infinite at $S = 0$, the integration must be over a range $S \geq S_1 > 0$, and we may then convert each PDE into a set of coupled ODEs in R, starting from $t\left(1,S\right)$ and $r\left(1,S\right)$, for a discrete set of N values of S between S_1 and an upper limit S_2 for which the asymptotic values may be used. Because S is itself defined by the first-order ODE (19), we retain τ as the dependent variable, so there are a total of $3N$ coupled ODEs for t, Dt and τ. Fuller details, including the asymptotic matching procedure, are given in the Appendix section of our previous article (Marshall, 2009).

From the solutions $r\left(R,S\right)$ and $t\left(R,S\right)$ we interpolated to obtain $r\left(R,t\right)$. Putting $r_0\left(t\right) = r\left(1,t\right)$, the relative position of a dust particle whose position in the ball is indexed by (R,θ,ϕ), with $0<R<1$, may then be described by the coordinate

$$u\left(R,r_0\right)=\frac{r\left[R,t\left(r_0\right)\right]}{r_0}\quad\left(0<R<1\right),\qquad . \tag{31}$$

We have plotted, in Figure 2, $u\left(R,r_0\right)$ against R for various values of r_0. In the early stage of collapse the slopes at both $R=0$ and 1 increase as r_0 decreases, indicating that particles near the centre move towards the surface and particles near the surface move towards the centre. However, towards the end of the process the curve $u(R,r_0)$ almost immediately crosses the curve $u\left(R,\infty\right)$ (the lower bold curve in Figure 2) near $R=1$, and in the limit $r_0\to m$ approaches the upper bold curve. The latter has both zero slope and zero curvature at $R=1$, indicating that the particle distribution in the final state has an infinite density at the surface. Note that the asymptotic series for $r\left(R,S\right)$ and $t\left(R,S\right)$, given in (Marshall,2009), are sufficient to provide a stable solution down to a dustball radius r_0 of around $1.1m$, and indeed, because we know the exact values of $r'\left(R,S\right)$ for $\left(R=1,S>0\right)$, the solution for $r\left(R,S\right)$ is known for the whole of the physical space-time region, that is all the way to $r_0=m$.

4 Conclusion

Fock's reformulation of GR transforms it into a proper field theory with real energy-momentum tensor. Gravitational waves then have a firm basis as waves on this field carrying real energy. Earlier doubts on the reality of such waves have been dispelled by the discovery of the Hulse-Taylor binary pulsar (and others), which show it spiralling down at a rate that agrees closely with Einstein's quadrupole formula. As noted above, in deriving the quadrupole formula for emission of gravitational waves, Einstein actually adopted the harmonic (that is asymptotically galilean) metric.

The present re-examination of gravitational collapse reveals not only that the event horizon of a collapsing dust ball goes continuously from the surface to the centre of the ball, but also that, in the harmonic frame, a freely falling test particle takes an infinite time to reach the horizon. This thereby confirms the judgments of both Eddington (1935) (see the quotes in our opening section) and Einstein (1939) on black holes: their existence would violate the Principle of Locality – the basis of Special Relativity (SR) – which does not allow a material particle to cross the superluminal speed barrier.

• It is natural and appropriate to reach this conclusion by privileging the inertial frame of SR

• Such a point of view requires restricting the Equivalence Principle to its original (1913) weak, form.

A notable feature of Figure 2 is that the density of dust particles, which started off uniform in the Newtonian region of t, becomes infinite at the surface of the ball as $t\to+\infty$; this also may be considered as confirming Eddington's intuition, referred to in section 1, that something intervenes to prevent the "absurdity" of the ball collapsing to a point. We see that the finite radius of the end state is a consequence of a purely gravitational field, without the need for any other forces to counter the gravitational attraction. It is more appropriate to call the collapse process one of gravitational compression; the combination of an overall attraction of the surface particles with a repulsion of the particles beneath the surface produces an infinite density at the surface;

- the collapse can go no further than the Schwarzschild radius
- the collapse to this state takes an infinite time
- the density of particles in the limit becomes infinite at the surface of the cloud
- gravity changes from being attractive to repulsive for certain high-density conditions. It is the accumulation of gravitational energy inside the ball which prevents it from collapsing to a point.

The "continued gravitational contraction" of Oppenheimer and Snyder is incorrect. The trapped surface they identified is associated with their erroneous fit of inner and outer solutions. Hawking and Penrose (1970) do no more than construct an elegant topological argument on this faulty premise of a trapped surface. Both Weinberg and Landau and Lifshitz repeat the error of OS; this is unexpected in the case of the latter authors, because they acknowledge the importance of correct matching. The new 'dustball' solution to the Einstein-Hilbert equations demonstrates that there does exist a solution to the contracting star without a black hole in an ideal case. We anticipate that a less trivial equation of state than the $p = 0$ of the dust medium will result in a larger asymptotic radius, near which the surface density remains finite for non-zero temperature.

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