On intense vortex structures in isotropic turbulence

A. Leonard
California Institute of Technology
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Johns Hopkins web-based data base
(http/turbulence.pha.jhu.edu)
DNS forced homogeneous isotropic turbulence
1024^3 $Re_\lambda = 433$

Intense vortex structures

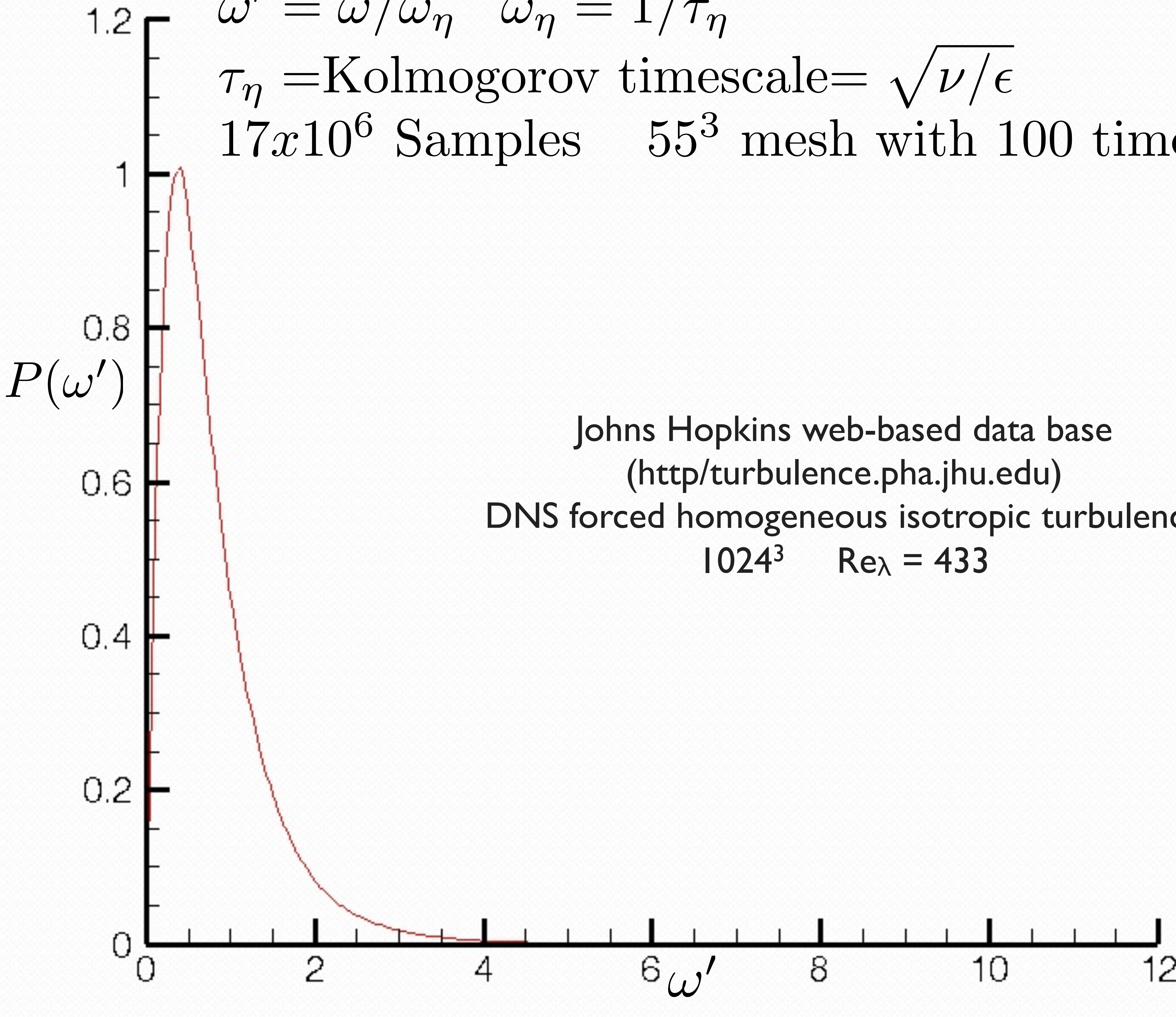
- PDF of vorticity amplitudes - asymptotics
- Geometry of intense structures, vorticity distribution
- Evolution in time

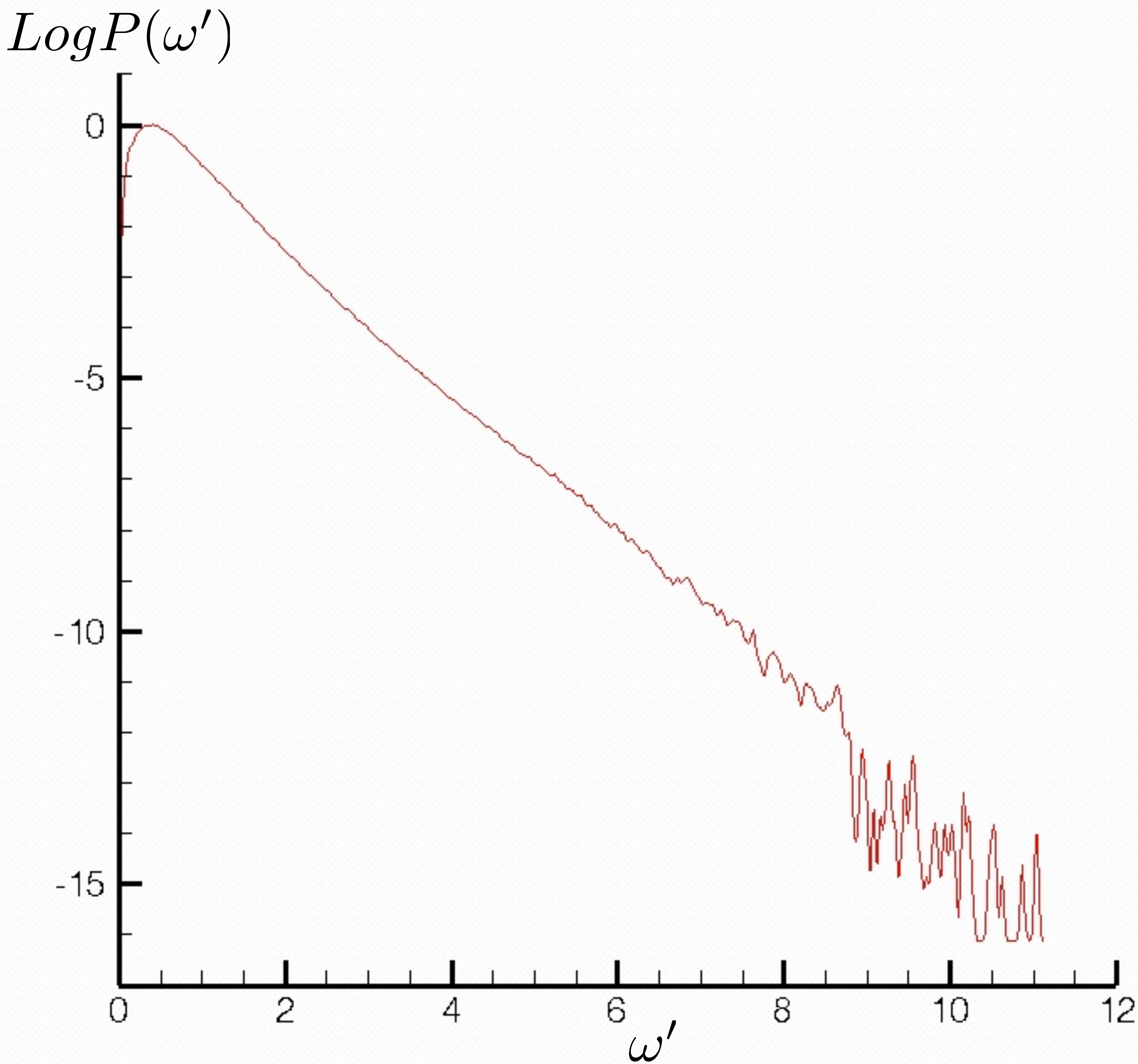
PDF of Normalized Vorticity Amplitudes

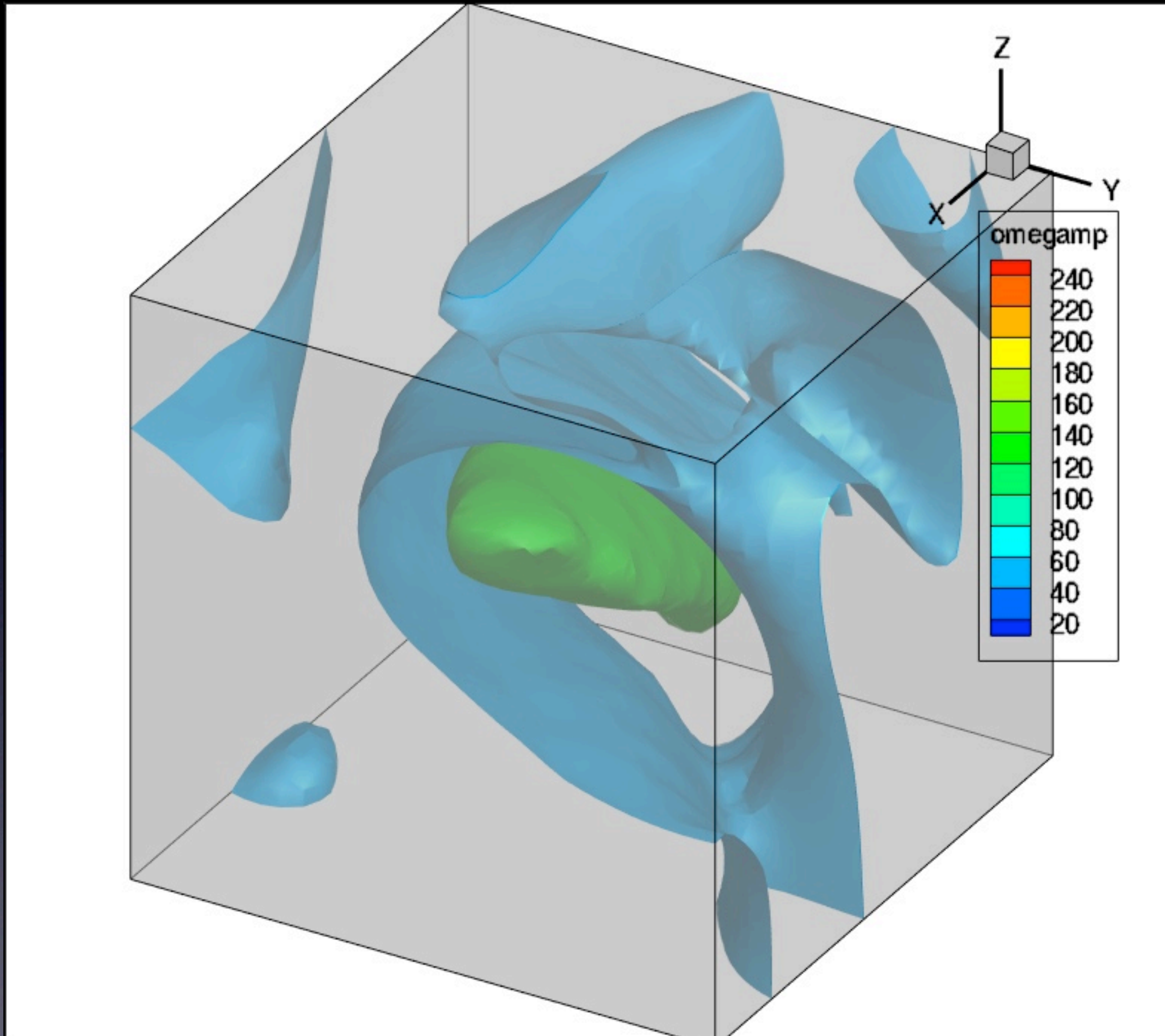
$\omega' = \omega/\omega_\eta \quad \omega_\eta = 1/\tau_\eta$

τ_η =Kolmogorov timescale= $\sqrt{\nu/\epsilon}$

$17x10^6$ Samples $\quad 55^3$ mesh with 100 timesteps

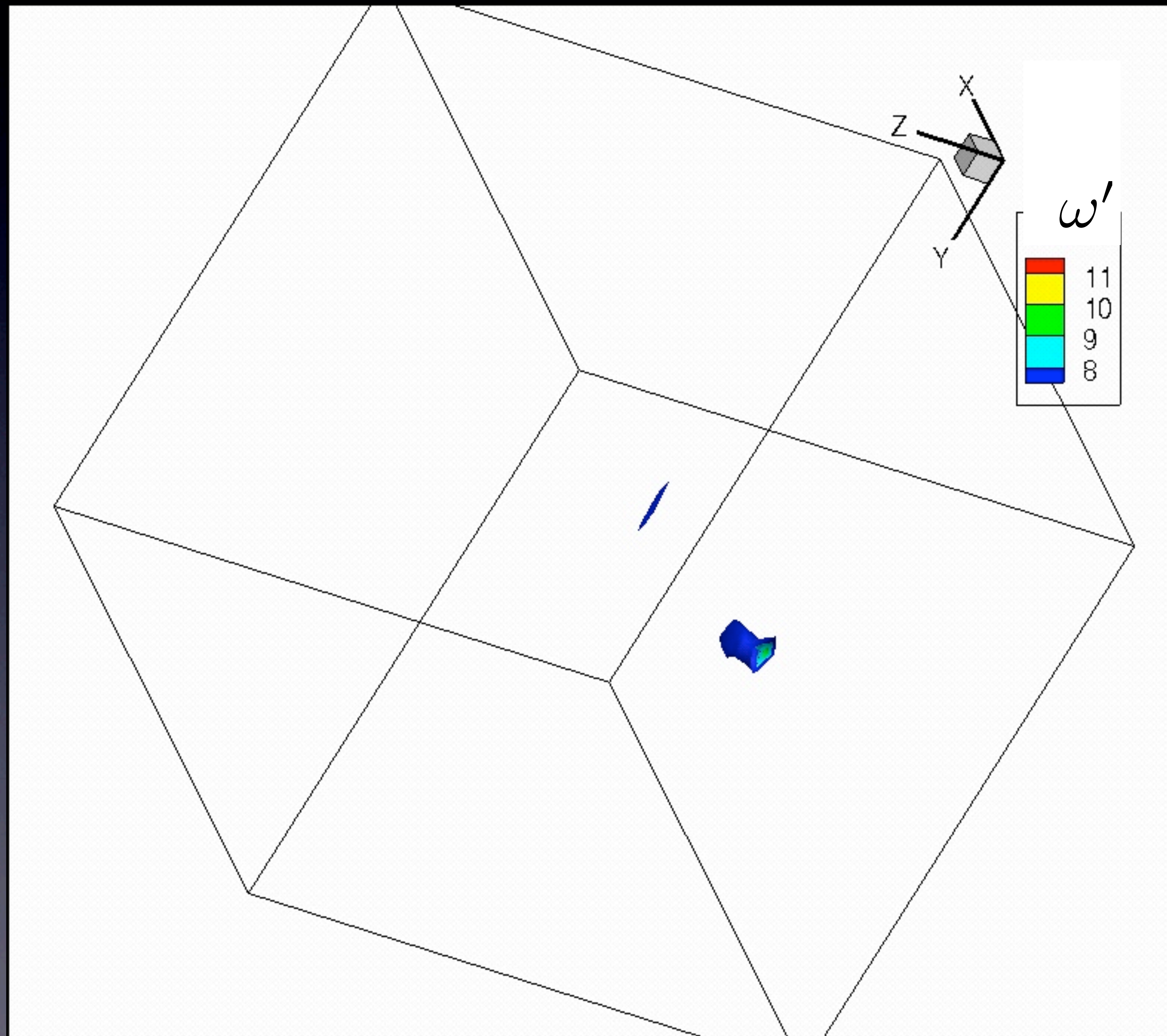



$LogP(\omega')$
0
-5
-10
-15
0
2
4
6
8
10
12
ω'

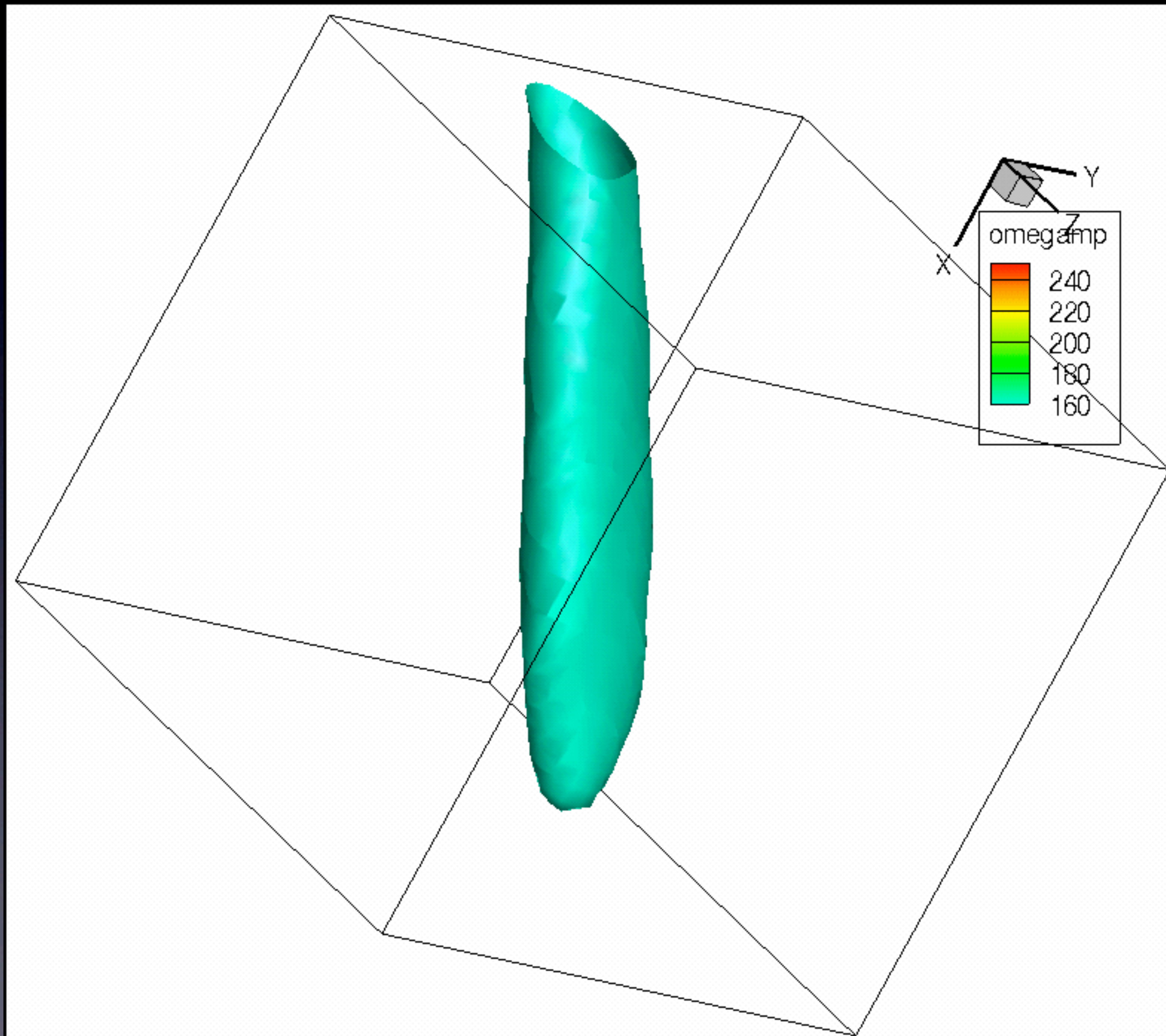

Z
Y
X
omegamp
240
220
200
180
160
140
120
100
80
60
40
20

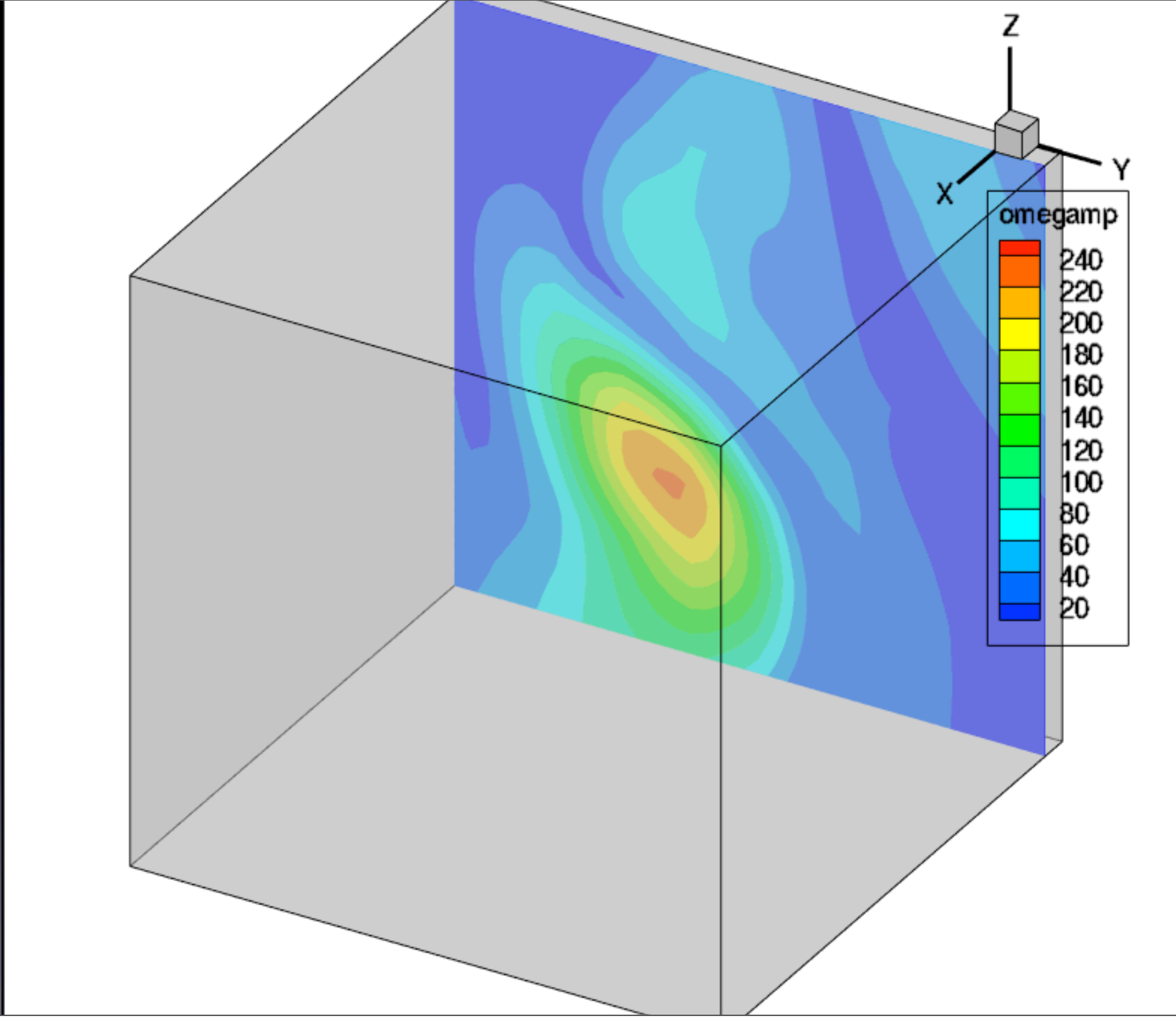
High Amplitude ω'

Box size $(32\Delta)^3 = (68\eta)^3 = (1.7\lambda)^3$



Intense ω' Region (zoom in)
Box size $(9\Delta)^3 = (19\eta)^3 = (0.47\lambda)^3$




Z
Y
X
omegamp
240
220
200
180
160
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120
100
80
60
40
20

PDF for a single structure - $P_S(\omega)$

Model I - Exponential decay of circulation

$$\omega_x = \frac{\Gamma}{\pi\sigma_r^2} exp(-r^2/\sigma_r^2) exp(-x^2/\sigma_x^2)$$

Let $S = r^2/\sigma_r^2 + x^2/\sigma_x^2 \rightarrow S = log(\frac{\omega_{max}}{\omega})$

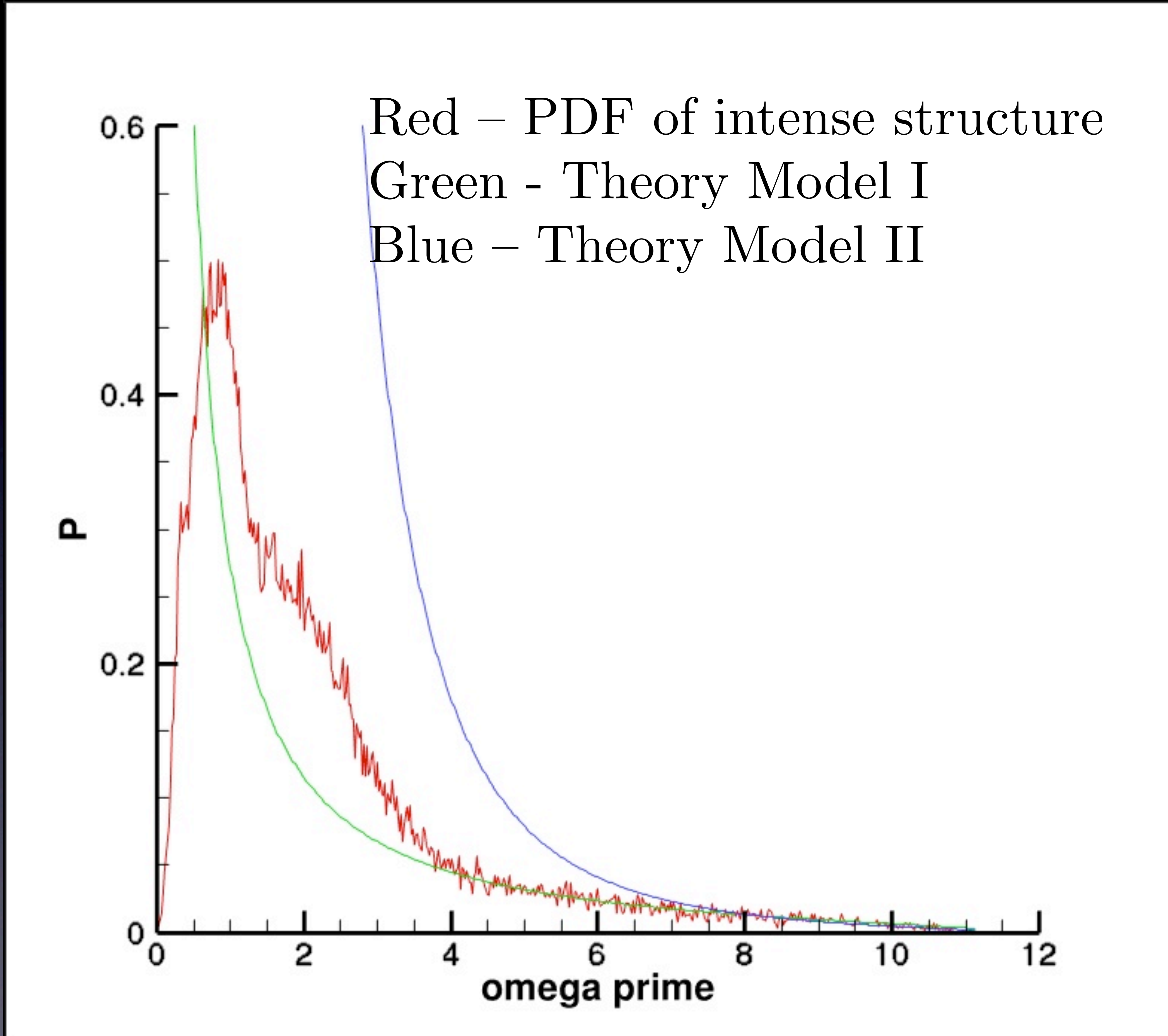
Volume inclosed by $S \rightarrow V(S) = \frac{4}{3}\pi\sigma_r^2\sigma_x S^{3/2}$

$$P_V(V) = \frac{dV}{V_T} \rightarrow \boxed{P_S(\omega)d\omega = C[log(\frac{\omega_{max}}{\omega})]^{1/2}\frac{d\omega}{\omega}}$$

Model II - Constant circulation, expanding vortex core

$$\omega_x = \frac{\Gamma}{\pi\sigma_r^2(x)} exp(-r^2/\sigma_r^2(x))$$

with $\sigma_r^2(x) = \sigma_o^2(1 + x^2/\sigma_x^2)$


Red – PDF of intense structure
Green - Theory Model I
Blue – Theory Model II
0.6
0.4
0.2
0
P
0
2
4
6
8
10
12
omega prime

Structure evolution in time
(preliminary)

Assume the following form for ω_x

$$\omega_x = \frac{\Gamma(t)}{\pi\sigma_r^2(t)} exp(-r^2/\sigma_r^2(t)) exp(-x^2/\sigma_x^2(t))$$

Consider the vorticity transport equation for ω_x

with leading order straining flow -

$$\begin{aligned}\mathbf{U} = & (a(t)x + b(t)(x^2 - r^2/2) + c(t)(x^3/3 - xr^2/2)\mathbf{e}_x \\ & +(-a(t)r/2 - b(t)xr - c(t)(x^2/2 - r^3/8)\mathbf{e}_r\end{aligned}$$

Exact equation for $\Gamma(t)$ (Kelvin's Theorem)

$$\frac{d\Gamma(t)}{dt} = -2\nu/\sigma_x^2(t)$$

Structure evolution in time
(continued, preliminary)

Compute $\int dV$, and $\int r^2 dV$ moments of the transport equation with the result

$$\frac{d\sigma_x^2(t)}{dt} = 2a(t)\sigma_x^2(t) + c(t)\sigma_x^4(t) + 4\nu$$

$$\frac{d\sigma_r^2(t)}{dt} = -a(t)\sigma_r^2(t) - \frac{c(t)}{2}\sigma_r^4(t) + 4\nu$$

with ω_r term and local induced velocities argued to be small. The straining flow proportional to $b(t)$ simply shifts the location of the maximum ω_x along the x axis.

Quasi-steady solution for the geometry if $c < 0$

$$\sigma_x^2(t) \approx -2\frac{a(t)}{c(t)}$$

$$\sigma_r^2(t) \approx 2\frac{\nu}{a}$$

$\Gamma(t)$ decays exponentially as given above.

Summary
Intense vortex structures

- Geometry of intense structures, vorticity distribution

  Cigar shaped, aspect ratio $\approx$ 6.5, gaussian distribution of circulation along axis

- Connection between geometry and PDF

  Vorticity distribution yields pdf for a single structure

- Evolution in time

  Equations for gaussian widths in radius and axial direction and for circulation interms of local strainrate field, longtime survival possible