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Physical Justification for the Einstein Real Gravity formulation of General Relativity

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Abstract

For diagonal metrics, the Einstein Real Gravity (RG) formulation of General Relativity (GR) has been developed by making use of an exponential parametrization of the metric tensor in terms of physical tensor potentials. But the metric is not an experimentally observable quantity and so this parametrization has yet to be physically justified, although, starting with Einstein's 1907 early treatment, there exist strong arguments in its favor. To address the issue, we RE-DEFINE the physical tensor potentials in terms of first order derivatives of the metric tensor, which are linearly related to the observable gravitational force field and the Christoffel symbols. Identifying the 00-component of the re-defined tensor potentials with Newton's gravitational potential, we show EXACT OBEDIANCE with (1) Newton's second law of motion from the geodesic equation for a point particle as well as with the (2) relativistic hydrostatic equilibrium relation for a perfect fluid from the Freud-Euler equation. These provide convincing physical justification for the Real Gravity formulation of General Relativity. Real gravitational and inertial forces are still linearly separated in the Christoffel symbols and the new definition coincides with the exponential parametrization for diagonal metrics. Non-diagonal stationary spacetimes or spacetimes allowing gravitational waves can now be treated consistently. Event horizons cannot develop in this theory.

1 The Structure of General Relativity

The general theory of relativity describes the laws of nature as seen from general reference frames, whether galilean or not. The equivalence principle assures us that the general motion of particles or material bodies under (loosely called) gravitational forces are equivalently described by observers in non inertial reference frames.

However, these loosely called gravitational forces covered by the equivalence principle are known not to be restricted to real (pure or actual) gravity. The equivalence principle in fact unifies the real gravitational force with other forces known as inertial forces such as the centrifugal and Coriolis forces of classical mechanics. All of these forces are traditionnally described as part of the gravitational force by general relativists.

Decades ago, attempts at describing a unified theory of gravitational and inertial forces has been presented by Rosen in the form of a bi-metric theory of gravity [1, 2, 3]. But the existence of two metrics in this theory is somewhat problematic.

In parallel, Yilmaz attempted to develop a theory dealing with strong gravitational fields. Based on 1907 arguments by Einstein [4], Yilmaz further developed these ideas by formulating an exponential parametrization for diagonal metrics, derived from a defining relationship between the Christoffel symbols and first-order derivatives of his so-called tensor potentials [5, 6, 7]. His theory also added an energy-momentum tensor for the gravitational field itself in the source term of Einstein's equations. Such an addition however implied that gravity is a self-generating force and that the vacuum of the theory can be a curved spacetime. This does not seem to agree with basic observations.

However, pending a linear constraint between Einstein and Yilmaz energy-momentum tensors [8, 9, 10, 11], Yilmaz theory can be shown to be completely equivalent to Einstein theory, if the latter is parametrized as well in terms of tensor potentials. This equivalence solved the so-called zero-pressure problem of the Yilmaz theory brought up by Misner [12, 13, 14, 15] for stars interior, and made the constrained Yilmaz theory free of self-generating gravitational forces, a welcome improvement.

Our strategy has then been to focus on the development of the original Einstein theory of General Relativity (GR), assumed to be also valid in the domain of strong gravitational forces. This necessarily required shifting the physical gravitational field from the metric tensor to the tensor potentials. So one has to define such physical gravitational potentials in a way that reproduces *exactly* Newton's laws and which simultaneously allows for the separation of the loosely defined gravitational force into so-called real (pure or actual) gravitational and inertial forces.

The real gravitational force is very different from the other inertial forces. Unlike the latters, the real gravitational force vanishes at infinity [16]. Furthermore, unlike the others, it *cannot be removed globally* by a change of reference frame. This property announces the *tensorial* nature of the real gravitational force. The other inertial forces on the other hand *do disappear when going to a galilean reference frame* (cartesian or *natural* coordinates [17]). For this reason, inertial forces are *non-tensorial*. We therefore expect the sum of both types of forces to be described by a *pseudo-tensor*, the Christoffel symbols.

The physical reason why the real gravitational force cannot be made to disappear is because it *curves spacetime* [16, 17, 18, 19], due to the presence of material sources carrying mass or energy. The other forces do not since they appear because of our choice of reference frame and coordinate system, although they remain unified with the real gravitational force through general relativity and the equivalence principle.

We thus see that the equivalence principle is a very wide principle which englobes inertial forces beyond the real spacetime curving gravitational force originating from matter energy-momentum. The description of real gravity is then cluttered by these frame-dependent forces which complicates the dynamics (and mathematics).

It is then reasonable to split general relativity into two parts, a part covering the flat spacetime inertial forces which should be incorporated into a generalization of special relativity, and another one dealing exclusively with curved spacetime phenomena, *i.e.* the real gravitational force.

As we now demonstrate, such a linear separation of forces is already structurally present in Einstein General Relativity, and so Rosen bi-metric theory [1, 2, 3] is not required.

The real gravitational force

Let us proceed by first making use of the following three fundamental relationships of Einstein General Relativity [16, 17, 18, 19],

(1) Einstein curvature field equation:

$$\begin{aligned} G_j^{\,k} &= R_j^{\,k} - \frac{1}{2}\,\delta_j^{\,k} R \;=\; \frac{8\pi k}{c^4}\,\tau_j^{(\mathrm{E})k} \\ R_j^{\,k} &= \frac{8\pi k}{c^4}\left(\tau_j^{(\mathrm{E})k} - \frac{1}{2}\delta_i^{\,j}\,\tau^{(\mathrm{E})}\right) , \end{aligned} \tag{1.1}$$

(2) Transformation law of the metric tensor:

$$\begin{aligned} ds^2 &= -\,g_{ij}\,dx^i dx^j \\ g_{ij} &= g'_{ab}\,\frac{\partial x'^a}{\partial x^i}\,\frac{\partial x'^b}{\partial x^j} , \end{aligned} \tag{1.2}$$

(3) Transformation law of the Christoffel symbols pseudo-tensor:

$$\Gamma^i{}_{kl} = \Gamma'^s{}_{rp}\,\frac{\partial x^i}{\partial x'^s}\,\frac{\partial x'^r}{\partial x^k}\,\frac{\partial x'^p}{\partial x^l} + \frac{\partial x^i}{\partial x'^s}\,\frac{\partial^2 x'^s}{\partial x^k \partial x^l} . \tag{1.3}$$

Much information is contained in the above three relationships. But to uncover their true contents, we must rely on the physical (observable) nature of the gravitational force they aspire to describe.

A point particle's motion in a general gravitational field is described by the geodesic equation,

$$\frac{du^i}{ds} = -\Gamma^i{}_{kl} u^k u^l , \tag{1.4}$$

with 4-velocity $u^i = dx^i/ds$ ($u_i u^i = -1$), 4-acceleration $du^i/ds = d^2x^i/ds^2$ and 4-force $-m\Gamma^i{}_{kl}u^k u^l$, with m the particle's mass.

As mentioned earlier, two types of forces act on this single particle, *i.e.* the inertial or coordinate-dependent force and the real gravitational force originating from a nearby matter source. Both forces are included in the Christoffel symbols $\Gamma^i{}_{kl}$ pseudo-tensor.

It is important to understand that the pseudo-tensor nature of the combined forces originates from its coordinate-dependent part. Without it, $\Gamma^i{}_{kl}$ should be describing a tensor force.

We therefore take a closer look at the transformation law (1.3) for the Christoffel symbols. Let us consider the case where x'^a describes a galilean frame of reference and so belongs to a system of cartesian coordinates, while x^i belongs to a general system of curvilinear coordinates and so describes a general non-inertial frame of reference.

Obviously, the second term in the rhs of eq. (1.3) is the non-tensor (coordinate-dependent) part responsible for the pseudo-tensor nature of $\Gamma^i{}_{kl}$. Such a term actually vanishes when x^i is chosen to describe a galilean reference frame.

Thus let us assume the following *linear* separation of the inertial and real gravitational forces in the Christoffel symbols [8, 9, 10, 11, 12, 13, 14, 15],

$$\Gamma^i{}_{kl} = S^i{}_{kl} + \Delta^i{}_{kl} \ , \tag{1.5}$$

in which the $S^i{}_{kl}$ are called the Inertial (Coordinates) symbols and the $\Delta^i{}_{kl}$ the (real) Gravitational symbols. Now in cartesian coordinates, there are no inertial forces acting on the single particle. Therefore we have,

$$\begin{aligned} S'^i{}_{kl} &= 0 \\ \Gamma'^i{}_{kl} &= \Delta'^i{}_{kl} \ . \end{aligned} \tag{1.6}$$

Inserting the decomposition (1.5) and the galilean values (1.6) into the transformation law (1.3) yields,

$$\Delta^i{}_{kl} + S^i{}_{kl} = \Delta'^s{}_{rp} \frac{\partial x^i}{\partial x'^s} \frac{\partial x'^r}{\partial x^k} \frac{\partial x'^p}{\partial x^l} + \frac{\partial x^i}{\partial x'^s} \frac{\partial^2 x'^s}{\partial x^k \partial x^l} \ . \tag{1.7}$$

Since $S^i{}_{kl}$ must describe the coordinate-dependent force, we find immediately,

$$\begin{aligned} S^i{}_{kl} &= \frac{\partial x^i}{\partial x'^s} \frac{\partial^2 x'^s}{\partial x^k \partial x^l} \\ &= g^{im}\, g'_{rs} \frac{\partial x'^r}{\partial x^m} \frac{\partial^2 x'^s}{\partial x^k \partial x^l} \ , \end{aligned} \tag{1.8}$$

where the second row originates from the transformation law (1.2) of the metric tensor. We therefore have,

$$\Delta^i{}_{kl} = \Delta'^s{}_{rp} \frac{\partial x^i}{\partial x'^s} \frac{\partial x'^r}{\partial x^k} \frac{\partial x'^p}{\partial x^l} \ , \tag{1.9}$$

which shows that the Gravitational symbols $\Delta^i{}_{kl}$ form a true tensor and so describe the real gravitational force.

The above tensor property remains valid for *any* curvilinear coordinate systems. That the real gravitational force is a true tensor force is absolutely necessary to allow for the existence of propagating gravitational waves carrying energy across spacetime as this requires localizability of real gravitational energy. This is the answer to the old energy localizability problem studied over a century ago by Schrodinger [20], Bauer [21] as well as Einstein himself [22].

Note that when x'^a describes an arbitrary curvilinear coordinate system, the transformation law of the Inertial symbols becomes, quite generally,

$$S^i_{\ kl} = S'^s_{\ rp} \frac{\partial x^i}{\partial x'^s} \frac{\partial x'^r}{\partial x^k} \frac{\partial x'^p}{\partial x^l} + \frac{\partial x^i}{\partial x'^s} \frac{\partial^2 x'^s}{\partial x^k \partial x^l} \ , \tag{1.10}$$

which of course reduces to eq. (1.8) for x'^a galilean ($S'^s_{\ rp} = 0$).

Now returning to the geodesic equation (1.4), we notice that the 4-acceleration du^i/ds, unlike the 4-velocity u^i, is not a vector. Recalling however the separation (1.5), we define [8, 9, 11] the following *generalized 4-acceleration* a^i,

$$a^i = \frac{\mathcal{D}u^i}{ds} \equiv \frac{du^i}{ds} + S^i_{\ kl} u^k u^l = -\Delta^i_{\ kl} u^k u^l \ . \tag{1.11}$$

Because $\Delta^i_{\ kl} u^k u^l$ and u^i are tensors, the above generalized 4-acceleration is a vector under general coordinate transformations. It is related to the so-called *proper 4-acceleration* α^i as follows,

$$\alpha^i = a^i + \Delta^i_{\ kl} u^k u^l \ , \tag{1.12}$$

where,

$$\alpha^i = \frac{du^i}{ds} + \Gamma^i_{\ kl} u^k u^l \ . \tag{1.13}$$

A geodesic is a particle trajectory with zero proper acceleration ($\alpha^i = 0$) and the generalized 4-acceleration a^i is the contravariant acceleration vector experienced by material particles under the influence of the real gravitational force described by the tensorial Gravitational symbols $\Delta^i_{\ kl}$ and generated by other matter or radiation sources.

When other forces such as the electromagnetic force are present, the tensor equation (1.11) is modified as follows,

$$a^i = -\Delta^i_{\ kl} u^k u^l + \frac{e}{mc^2} F^i_{\ k} u^k \ . \tag{1.14}$$

In the absence of matter sources, the real gravitational force vanishes and so $\Delta^i_{\ kl} = 0$. Eq. (1.14) then describes the tensor equation of motion of a particle in flat spacetime, but in curvilinear coordinates, and so not from the viewpoint of a galilean reference frame,

$$a^i = \frac{\mathcal{D}u^i}{ds} = \frac{du^i}{ds} + S^i_{\ kl} u^k u^l = \frac{e}{mc^2} F^i_{\ k} u^k \ . \tag{1.15}$$

Even though spacetime is flat, inertial forces still occur when making use of curvilinear coordinates, *i.e.* from a non-inertial reference frame. Eq. (1.15) therefore represents a generalization of Special Relativity (SR) to non-inertial frames in the absence of real gravity, thanks to the principle of equivalence which is still at work even in flat spacetime.

Finally, the separation of the Christoffel symbols as a sum of Coordinates (Inertial) and Gravitational symbols in turns enables us to likewise separate the Riemann curvature tensor $R^i{}_{klm}$ as follows [8, 9, 11],

$$R^i{}_{klm} \;\equiv\; R^i{}_{klm}[\partial\hat{\Gamma}, \hat{\Gamma}\times\hat{\Gamma}] \;=\; R^i{}_{klm}[\partial\hat{S}, \hat{S}\times\hat{S}] + R^i{}_{klm}[\mathcal{D}\hat{\Delta}, \hat{\Delta}\times\hat{\Delta}] \;, \tag{1.16}$$

in which we defined,

$$R^i{}_{klm}[\partial\hat{\Gamma}, \hat{\Gamma}\times\hat{\Gamma}] \;\equiv\; \partial_l \Gamma^i{}_{km} - \partial_m \Gamma^i{}_{kl} + \Gamma^i{}_{nl}\Gamma^n{}_{km} - \Gamma^i{}_{nm}\Gamma^n{}_{kl} \;, \tag{1.17}$$

$$R^i{}_{klm}[\partial\hat{S}, \hat{S}\times\hat{S}] \;\equiv\; \partial_l S^i{}_{km} - \partial_m S^i{}_{kl} + S^i{}_{nl}S^n{}_{km} - S^i{}_{nm}S^n{}_{kl} \;, \tag{1.18}$$

and,

$$\begin{aligned} R^i{}_{klm}[\mathcal{D}\hat{\Delta}, \hat{\Delta}\times\hat{\Delta}] \;&\equiv\; \mathcal{D}_l \Delta^i{}_{km} - \mathcal{D}_m \Delta^i{}_{kl} + \Delta^i{}_{nl}\Delta^n{}_{km} - \Delta^i{}_{nm}\Delta^n{}_{kl} \\ &=\; R^i{}_{klm}[\partial\hat{\Delta}, \hat{\Delta}\times\hat{\Delta}] + R^i{}_{klm}[0, (\hat{S}\times\hat{\Delta}) + (\hat{\Delta}\times\hat{S})] \;, \end{aligned} \tag{1.19}$$

where the "covariant derivative" $\mathcal{D}_l$ (wrt $S^i{}_{kl}$) is defined as,

$$\mathcal{D}_l \Delta^i{}_{km} \;\equiv\; \partial_l \Delta^i{}_{km} - S^n_{kl}\Delta^i{}_{nm} - S^n_{ml}\Delta^i{}_{kn} + S^i{}_{nl}\Delta^n{}_{km} \;. \tag{1.20}$$

Now we notice that $R^i{}_{klm}[\mathcal{D}\hat{\Delta}, \hat{\Delta}\times\hat{\Delta}]$ is a tensor while $R^i{}_{klm}[\partial\hat{S}, \hat{S}\times\hat{S}]$ *is not*. Since their sum $R^i{}_{klm}[\partial\hat{\Gamma}, \hat{\Gamma}\times\hat{\Gamma}]$ is itself the full Riemann curvature tensor, we then must have,

$$R^i{}_{klm}[\partial\hat{S}, \hat{S}\times\hat{S}] \;=\; 0 \;, \tag{1.21}$$

and so we find,

$$R^i{}_{klm} \;=\; R^i{}_{klm}[\mathcal{D}\hat{\Delta}, \hat{\Delta}\times\hat{\Delta}] \;. \tag{1.22}$$

This situation is analogous to bi-metric theories [1, 2, 3], although there is only one metric in this problem. The Ricci tensor and scalar curvature are then given as,

$$R_{ik} \;=\; R^l{}_{ilk}[\mathcal{D}\hat{\Delta}, \hat{\Delta}\times\hat{\Delta}] \;=\; R_{ik}[\mathcal{D}\hat{\Delta}, \hat{\Delta}\times\hat{\Delta}] \;, \tag{1.23}$$

$$R \;=\; g^{ik}R_{ik}[\mathcal{D}\hat{\Delta}, \hat{\Delta}\times\hat{\Delta}] \;=\; R[\mathcal{D}\hat{\Delta}, \hat{\Delta}\times\hat{\Delta}] \;, \tag{1.24}$$

leading to the following expression for the Einstein tensor,

$$G_{ik} \;\equiv\; R_{ik} - \frac{1}{2}g_{ik}R \;=\; G_{ik}[\mathcal{D}\hat{\Delta}, \hat{\Delta}\times\hat{\Delta}] \;. \tag{1.25}$$

2 Defining the Physical Tensor Potentials

In his Nuovo Cimento article [5], Yilmaz defined his tensor potentials $\phi_j^{(\mathrm{Y})k}$ decribing the relativistic gravitational tensor field from its linear relationship with the Christoffel symbols in the following manner,

$$\partial_l \phi_m^{(\mathrm{Y})n} \;=\; -\frac{1}{4}g^{nr}\,(\,\Gamma_{rml} + \Gamma_{mrl}\,) + \frac{1}{4}\delta_m^{\,n}\,\Gamma^s_{sl} \;, \tag{2.1}$$

or equivalently,

$$\begin{aligned} \partial_l \bar{\phi}_m^{(Y)n} &= -\frac{1}{4} g^{nr} \left(\Gamma_{rml} + \Gamma_{mrl} \right) \\ \bar{\phi}_m^{(Y)n} &= \phi_m^{(Y)n} - \frac{1}{2} \delta_m^{\,n} \phi^{(Y)} , \end{aligned} \tag{2.2}$$

where $\phi^{(Y)} \equiv \phi_l^{(Y)l}$ is the trace of the Yilmaz tensor potentials. For diagonal metrics, these definitions are equivalent to an exponential parametrization of the metric tensor $g_{ij} = \{\exp[-4\bar{\phi}^{(Y)}]\}_i^k \eta_{kj}$ [5, 6, 7, 23, 24, 25].

However such a definition is really arbitrary and, furthermore, when treating interior problems beyond the so-called Newton gauge [15], the 00-component of the Yilmaz tensor potentials is no longer solely identified with the newtonian gravitational potential Φ. It is instead *shifted* as follows (in cartesian coordinates) [15],

$$\phi_0^{(Y)0} - \phi_\alpha^{(Y)\alpha} = -\frac{\Phi}{c^2} . \tag{2.3}$$

Beyond the Newton gauge, an alternative definition has been given by the author [15] which enables to lock the Newton gravitational potentiel on the 00-component of the new tensor potentials. It is given as follows (in cartesian coordinates) [15],

$$\phi_j^k \equiv -2\, \bar{\phi}_j^{(Y)k} \quad ; \quad \phi_0^0 = \frac{\Phi}{c^2} , \tag{2.4}$$

which is in simple relation with the definition (2.1) by Yilmaz. Although a degree of arbitrariness is seemingly lifted by this new definition which locks the newtonian potential to the 00-component, a more profound physical justification remains to be given for it.

Fortunately, such a justification is readily found in the derivation of the so-called Freud-Euler equation [15, 26, 27, 28] for a prefect relativistic fluid.

The Freud-Euler equation

Making use of the Bianchi identity as well as the rule for covariant derivatives of symmetric tensors [16], we get the following relationship for the Einstein tensor $G_j^{\,k}$,

$$D_k G_j^{\,k} = \frac{1}{\sqrt{-g}}\, \partial_k (\sqrt{-g}\, G_j^{\,k}) - \frac{1}{2}\, \partial_j g_{kl}\, G^{kl} = 0 . \tag{2.5}$$

We then find,

$$\begin{aligned} \partial_k (\sqrt{-g}\, G_j^{\,k}) &= \frac{1}{2} \left(g^{km} \partial_j g_{kl} \right) \sqrt{-g}\, G_m^{\,\,l} \\ &= \frac{1}{2} \left(\mathbf{g}^{km} \partial_j \mathbf{g}_{kl} \right) \sqrt{-g}\, R_m^{\,\,l} , \end{aligned} \tag{2.6}$$

with the **rescaled metric tensor** defined as follows,

$$\mathbf{g}^{ij} \equiv \sqrt{-g} g^{ij} \;\; ; \;\; \mathbf{g}_{ij} \equiv \frac{1}{\sqrt{-g}} g_{ij} . \tag{2.7}$$

Now in general curvilinear coordinates, Einstein General Relativity is described by the following set of relationships [8, 9, 10, 11, 12, 14, 15],

$$F_j^{\ k} = \frac{8\pi k}{c^4}\,(\tau_j^{(\mathrm{E})k} + \tilde{t}_j^{(\mathrm{E})k}) \quad ; \quad E_j^{\ k} = \frac{8\pi k}{c^4}\,\tilde{t}_j^{(\mathrm{E})k}$$
$$G_j^{\ k} = F_j^{\ k} - E_j^{\ k} = \frac{8\pi k}{c^4}\,\tau_j^{(\mathrm{E})k} \quad ; \quad R_j^{\ k} = \frac{8\pi k}{c^4}\left(\tau_j^{(\mathrm{E})k} - \frac{1}{2}\delta_j^{\ k}\tau^{(\mathrm{E})}\right) \ , \tag{2.8}$$

supplemented by the so-called Freud identity [3, 5, 15, 29],

$$\partial_k(\sqrt{-g}F_j^{\ k}) = 0 \quad \to \quad \partial_k[\ \sqrt{-g}(\tau_j^{(\mathrm{E})k} + \tilde{t}_j^{(\mathrm{E})k})\ ] = 0\ , \tag{2.9}$$

with $F_j^{\ k}$ and $E_j^{\ k}$ the Freud and Einstein-Pauli pseudo-tensors respectively, as well as the matter energy-momentum tensor $\tau_j^{(\mathrm{E})k}$ and the gravitational energy-momentum pseudo-tensor $\tilde{t}_j^{(\mathrm{E})k}$.

Inserting the above relationships into the expression (2.6) for the Bianchi identity, we arrive at the following general forms for the Freud-Euler equation,

$$\begin{aligned}\partial_k(\sqrt{-g}\,\tau_j^{(\mathrm{E})k}) &= \frac{1}{2}\,(g^{km}\partial_j g_{kl})\,\sqrt{-g}\,\tau_m^{(\mathrm{E})l} \\ &= \frac{1}{2}\,(\mathbf{g}^{km}\partial_j \mathbf{g}_{kl})\,\sqrt{-g}\left(\tau_m^{(\mathrm{E})l} - \frac{1}{2}\delta_m^{\ l}\tau^{(\mathrm{E})}\right) \\ &= -\,\partial_k(\sqrt{-g}\,\tilde{t}_j^{(\mathrm{E})k})\ .\end{aligned} \tag{2.10}$$

The Freud-Euler equation (2.10) is a universal matter dynamics equation in general curvilinear coordinates, taking into account the effects of the real gravitational and inertial forces present in the physical system, as well as the energy-momentum conservation law (2.9). The form (2.10) is especially revealing as it provides a natural **defining physical relation** for the tensor potentials $\phi_j^{\ k}$ in arbitrary curvilinear coordinates. Such a defining physical relation is therefore taken as follows,

$$\partial_j\phi_l^{\ m} \ \equiv\ \frac{1}{2}\,g^{km}\partial_j g_{kl}\ , \tag{2.11}$$

yielding,

$$\partial_j\bar{\phi}_l^{\ m} \ =\ \frac{1}{2}\,\mathbf{g}^{km}\partial_j \mathbf{g}_{kl}\ , \tag{2.12}$$

where,

$$\bar{\phi}_l^{\ m} \ \equiv\ \phi_l^{\ m} - \frac{1}{2}\delta_l^{\ m}\ln\sqrt{-g} \quad ; \quad \phi \ \equiv\ \phi_l^{\ l}\ . \tag{2.13}$$

Now since,

$$\frac{1}{2}\,g^{km}\partial_j g_{km} \ =\ \partial_j\ln\sqrt{-g} \ =\ \Gamma^l_{\ jl}\ , \tag{2.14}$$

we find,

$$\phi = \ln\sqrt{-g} \quad \to \quad \sqrt{-g} = e^{\phi} = e^{-\bar{\phi}}\ . \tag{2.15}$$

The Freud-Euler equation (2.10) is then re-written as [15],

$$\begin{aligned}\partial_k(\sqrt{-g}\,\tau_j^{(\mathrm{E})k}) &= \sqrt{-g}\,\tau_m^{(\mathrm{E})l}\,\partial_j\phi_l^{\ m} \\ &= \sqrt{-g}\left(\tau_m^{(\mathrm{E})l} - \frac{1}{2}\delta_m^{\ l}\tau^{(\mathrm{E})}\right)\partial_j\bar{\phi}_l^{\ m} \\ &= -\,\partial_k(\sqrt{-g}\,\tilde{t}_j^{(\mathrm{E})k})\ .\end{aligned} \tag{2.16}$$

It is a simple exercise to show that the **defining physical relation** (2.11) coincides with the author's previous definition (2.4).

Recalling eq. (2.2) for the Yilmaz potentials $\bar{\phi}_m^{(\mathrm{Y})n}$, the author's old definition (2.4) becomes,

$$\partial_l \phi_m^n \;=\; \frac{1}{2} g^{nr} \left(\Gamma_{rml} + \Gamma_{mrl} \right) \;. \tag{2.17}$$

Making use of the well-known relation,

$$\Gamma_{mkl} = \frac{1}{2} \left(\partial_l g_{mk} + \partial_k g_{ml} - \partial_m g_{kl} \right) , \tag{2.18}$$

the formula (2.17) is finally re-written as follows,

$$\partial_l \phi_m^n \;=\; \frac{1}{2}\, g^{nr} \partial_l g_{rm} \;, \tag{2.19}$$

which readily agrees with the defining physical relation (2.11).

Linear separation of the physical tensor potentials

Making use of the following set of identities for the metric tensor [5, 15],

$$\begin{aligned} \partial_m g_{lj} &= 2 g_{kj} \partial_m \phi_l{}^k \;=\; 2 g_{kl} \partial_m \phi_j{}^k \\ \partial_m g^{ik} &= -2 g^{li} \partial_m \phi_l{}^k \;=\; -2 g^{lk} \partial_m \phi_l{}^i \;, \end{aligned} \tag{2.20}$$

as well as a similar set for the rescaled metric tensor,

$$\begin{aligned} \partial_m \mathbf{g}_{lj} &= 2 \mathbf{g}_{kj} \partial_m \bar{\phi}_l{}^k \;=\; 2 \mathbf{g}_{kl} \partial_m \bar{\phi}_j{}^k \\ \partial_m \mathbf{g}^{ik} &= -2 \mathbf{g}^{li} \partial_m \bar{\phi}_l{}^k \;=\; -2 \mathbf{g}^{lk} \partial_m \bar{\phi}_l{}^i \;, \end{aligned} \tag{2.21}$$

which can be deduced from the defining physical relation (2.11), and recalling the following formula for the Christoffel symbols,

$$\Gamma^i_{kl} = g^{im} \Gamma_{mkl} \;, \tag{2.22}$$

we find linear relationships between the Christoffel symbols and the derivatives of the physical tensor potentials [15],

$$\begin{aligned} \Gamma^i{}_{kl}[\partial \hat{\phi}] &= 2 \partial_{(l} \phi^i_{k)} \;-\; g^{im} g_{j(l} \partial_m \phi^j_{k)} \\ &= 2 \partial_{(l} \phi^i_{k)} \;-\; \mathbf{g}^{im} \mathbf{g}_{j(l} \partial_m \phi^j_{k)} \;. \end{aligned} \tag{2.23}$$

Now, recalling eq. (1.5) we have seen that the Christoffel symbols linearly separate into the Inertial (Coordinates) symbols $S^i{}_{kl}$ and the (real) Gravitational symbols $\Delta^i{}_{kl}$. Eq. (2.23) therefore tells us that a similar separation occurs for the physical tensor potentials themselves. So we have,

$$\phi_j{}^k \;=\; \varphi_j{}^k + \chi_j{}^k \quad ; \quad \bar{\phi}_j{}^k \;=\; \bar{\varphi}_j{}^k + \bar{\chi}_j{}^k \quad , \tag{2.24}$$

with (real) gravitational potentials $\varphi_j{}^k$ and inertial (coordinates) potentials $\chi_j{}^k$.

We thus find [15],

$$\begin{aligned} \Delta^i{}_{kl}[\partial \hat{\varphi}] &= 2 \partial_{(l} \varphi^i_{k)} \;-\; g^{im} g_{j(l} \partial_m \varphi^j_{k)} \\ &= 2 \partial_{(l} \varphi^i_{k)} \;-\; \mathbf{g}^{im} \mathbf{g}_{j(l} \partial_m \varphi^j_{k)} \;, \end{aligned} \tag{2.25}$$

and,

$$\begin{aligned} S^i_{\ kl}[\partial\hat{\chi}] &= 2\partial_{(l}\chi^{\ i}_{k)} - g^{im}g_{j(l}\partial_m\chi^{\ j}_{k)} \\ &= 2\partial_{(l}\chi^{\ i}_{k)} - \mathbf{g}^{im}\mathbf{g}_{j(l}\partial_m\chi^{\ j}_{k)} \ . \end{aligned} \tag{2.26}$$

3 Justifying the Defining Physical Relation

The defining physical relation (2.11) must now yield concrete physical results in order to justify itself and to make the whole Einstein Real Gravity formulation as the correct physical interpretation of the equations of General Relativity. Here we discuss two very simple examples which reproduce exactly known physical laws. Finally we derive the equation for gravitational radiation and show its agreement with traditional General Relativity in the weak gravitational field approximation.

Exact relativistic hydrostatic equilibrium

An isotropic diagonal metric in spherical coordinates $(ct, r, \vartheta, \varphi)$ is given as follows [15],

$$\begin{aligned} ds^2 &= e^{\nu}c^2dt^2 - e^{\lambda}\,\eta_{\alpha\beta}[\hat{\chi}]\,dx^\alpha dx^\beta \\ &= e^{2\Phi/c^2}c^2dt^2 - e^{2\Psi/c^2}dl^2 \ , \end{aligned} \tag{3.1}$$

where the interval dl^2 is given as,

$$\begin{aligned} dl^2 &= \eta_{\alpha\beta}[\hat{\chi}]\,dx^\alpha dx^\beta \\ &= dr^2 + r^2d\vartheta^2 + r^2\sin^2\vartheta d\varphi^2 \ , \end{aligned} \tag{3.2}$$

and the physical gravitational tensor potentials components given as follows,

$$\varphi_0^{\ 0} \equiv \frac{\Phi}{c^2} = \frac{\nu}{2} \ ; \ \varphi_\alpha^{\ \beta} \equiv \frac{\Psi}{c^2}\delta_\alpha^{\ \beta} = \frac{\lambda}{2}\delta_\alpha^{\ \beta} \ , \tag{3.3}$$

with Φ and Ψ describing the Newton and spatial potentials respectively ($\chi_0^{\ 0} = 0\,;\,\eta_{00}[\hat{\chi}] = \eta_{00} = -1$), and to which correpond the following characteristics,

$$\begin{aligned} \chi_0^{\ 0} &= \chi_1^{\ 1} \ = \ 0 \\ \chi_2^{\ 2} &= \tfrac{1}{2}\ln(r^2) \\ \chi_3^{\ 3} &= \tfrac{1}{2}\ln(r^2\sin^2\vartheta) \\ \chi &= \chi_k^{\ k} \ = \ \tfrac{1}{2}\ln(r^4\sin^2\vartheta) \\ \eta^{ij}[\hat{\chi}] &= (\,-e^{-2\chi_0^{\ 0}},\,\eta^{\alpha\sigma}(e^{-2\hat{\chi}})_\sigma^{\ \beta}\,) \\ &= (\,-1\,,1\,,e^{-\ln(r^2)}\,,e^{-\ln(r^2\sin^2\vartheta)}\,) \\ \sqrt{-\eta[\hat{\chi}]} &= e^{\chi} \ = \ e^{\frac{1}{2}\ln(r^4\sin^2\vartheta)} \ , \end{aligned} \tag{3.4}$$

where the metric determinant is expressed as follows,

$$\begin{aligned} \sqrt{-g[\hat{\phi}]} &\equiv \sqrt{-\eta[\hat{\chi}]}\,\sqrt{-g_c[\hat{\varphi}]} \\ &= e^{\chi+\varphi} \\ &= e^{\chi+(\Phi+3\Psi)/c^2} \ . \end{aligned} \tag{3.5}$$

Note that $\sqrt{-g_c[\hat{\varphi}]}$ corresponds to the metric determinant $\sqrt{-g[\hat{\phi}]}$ in cartesian coordinates where we have $\chi_j^{\ k} = \chi = 0$.

Let us now find some general relations for the potentials (Φ,Ψ) in the case of a spherically symmetric spacetime-dependent perfect pascalian fluid described by the following energy-momentum tensor,

$$\sqrt{-g}\,\tau_j^{(\mathrm{E})k} = \sqrt{-g}\,[\,\varepsilon\, u_j u^k + p_j^k\,]\ , \tag{3.6}$$

with pressure stress-tensor p_j^k ($p_0^0 = p_\alpha^0 = p_0^\beta = 0$; $p_\alpha^\beta = p\,\delta_\alpha^\beta$), energy density ε and 4-velocity u^i. Here we shall further restrict ourselves to the case of a *slow moving fluid* ($\frac{|v|}{c} \ll 1\,;\, u^\alpha \to 0\,;\, u_0 u^0 \to -1$).

Making use of the Freud-Euler equation (2.16), we develop as follows,

$$\begin{aligned} \partial_k(\ln\sqrt{-g})\,\tau_j^{(\mathrm{E})k} + \partial_k\tau_j^{(\mathrm{E})k} &= \tau_m^{(\mathrm{E})l}\,\partial_j\phi_l^{\ m} \\ \to\ \ \partial_k(\chi+\varphi)\,\tau_j^{(\mathrm{E})k} + \partial_k\tau_j^{(\mathrm{E})k} &= \tau_m^{(\mathrm{E})l}\,\partial_j(\varphi_l^{\ m} + \chi_l^{\ m})\ . \end{aligned} \tag{3.7}$$

In spherical coordinates we have $\chi_0^{\ 0} = \chi_1^{\ 1} = 0$ and so the $j = 0$ and $j = 1$ components yield respectively,

$$\begin{aligned} \varepsilon\,\partial_0\Big(\frac{\Phi+3\Psi}{c^2}\Big) + \partial_0\varepsilon &= \varepsilon\,\partial_0\Big(\frac{\Phi}{c^2}\Big) - 3p\,\partial_0\Big(\frac{\Psi}{c^2}\Big) \\ p\,\partial_r\Big(\frac{\Phi+3\Psi}{c^2}\Big) + \partial_r p &= -\varepsilon\,\partial_r\Big(\frac{\Phi}{c^2}\Big) + 3p\,\partial_r\Big(\frac{\Psi}{c^2}\Big)\ , \end{aligned} \tag{3.8}$$

which finally leads to,

$$\begin{aligned} 3\,\partial_0\Big(\frac{\Psi}{c^2}\Big) &= -\ \frac{\partial_0\varepsilon}{(\varepsilon+p)} \\ \partial_r\Big(\frac{\Phi}{c^2}\Big) &= -\ \frac{\partial_r p}{(\varepsilon+p)}\ \ , \end{aligned} \tag{3.9}$$

in agreement with the results of Landau and Lifchitz [16].

Given an equation of state $p = p(\varepsilon)$, these equations can be integrated as follows [15, 16],

$$\begin{aligned} 3\Big(\frac{\Psi}{c^2}\Big) &= -\int\frac{d\varepsilon}{(\varepsilon+p)} + f_1(r) \\ \Big(\frac{\Phi}{c^2}\Big) &= -\int\frac{dp}{(\varepsilon+p)} + f_2(t)\ \ , \end{aligned} \tag{3.10}$$

with arbitrary functions $f_1(r)$ and $f_2(t)$.

While the first equation of (3.9) reveals that a gain or loss in the fluid energy density is accompanied by a change of the (radial) size of the fluid (Ψ acting as a spatial scale), the second equation is nothing but the exact and familiar relativistic hydrostatic equation relating variations of the Newton potential (radial gravitational force) to radial variations of the fluid pressure.

Exact second law of motion

The trajectory of a particle of mass m in a gravitational field is given by the geodesic equation (1.4),

$$\frac{du^i}{ds} = -\Gamma^i_{\ kl}u^k u^l\ . \tag{3.11}$$

At velocities much smaller than the speed of lignt, the 4-velocity u^i is expressed as follows,

$$u^0 = \frac{1}{\sqrt{-g_{00}}} \quad ; \quad u^\alpha \simeq 0 \, , \tag{3.12}$$

and so the particle's acceleration in the gravitational field is given by,

$$\frac{d^2x^\alpha}{ds^2} = -\Gamma^\alpha_{00} u^0 u^0 = -\frac{\Gamma^\alpha_{00}}{(-g_{00})} \, . \tag{3.13}$$

In cartesian coordinates, we have $\Gamma^i{}_{kl} = \Delta^i{}_{kl}$ and $\phi_j{}^k = \varphi_j{}^k$ ($\chi_j{}^k = 0$). Making use of the relationship (2.25) we then get,

$$\frac{d^2x^\alpha}{ds^2} = -\frac{\Delta^\alpha_{00}}{(-g_{00})} = \frac{1}{g_{00}} \left[2\partial_0 \varphi_0{}^\alpha - g^{\alpha m} g_{j0} \partial_m \varphi_0{}^j \right] . \tag{3.14}$$

Now for an isotropic non-stationary (non-rotating) and constant metric ($g_{0\gamma} = 0$), we have $\varphi_0{}^0 = \Phi/c^2$ with Φ the newtonian gravitational potential. We therefore arrive at the following expression,

$$\begin{aligned} \frac{d^2x^\alpha}{ds^2} &= \frac{1}{g_{00}} (-g_{00}) \, \partial^\alpha \varphi_0{}^0 \\ &= -\partial^\alpha \left(\frac{\Phi}{c^2} \right) , \end{aligned} \tag{3.15}$$

which is nothing but Newton's second law of motion in a gravitational field.

Real gravity equation for gravitational radiation

It is well known that the Einstein tensor $G_i{}^k$ can be decomposed in terms of the Freud $F_i{}^k$ and Einstein-Pauli $E_i{}^k$ pseudo-tensors as follows [3, 5, 8, 9, 10, 11, 15, 29],

$$\begin{aligned} \sqrt{-g} G_i{}^k &= \sqrt{-g} \left(R_j{}^k - \frac{1}{2} \delta_j{}^k R \right) \\ &= \sqrt{-g} \left(F_i{}^k - E_i{}^k \right) , \end{aligned} \tag{3.16}$$

with the Einstein-Pauli pseudo-tensor given as [3, 5, 15],

$$\sqrt{-g} \, E_j{}^k = \sqrt{-g} \left(W_j{}^k - \frac{1}{2} \delta_j{}^k W \right) \quad ; \quad W = W_k{}^k \, , \tag{3.17}$$

where,

$$\begin{aligned} \sqrt{-g} \, W_j{}^k &\equiv \frac{1}{2} \left(\Gamma^k_{\ rs} \partial_j \mathbf{g}^{rs} - \Gamma^s_{\ rs} \partial_j \mathbf{g}^{rk} \right) \\ &= -2 \, \mathbf{g}^{ks} \left(\partial_j \bar{\phi}_l{}^m \partial_m \bar{\phi}_s{}^l - \frac{1}{2} \partial_j \bar{\phi}_l{}^m \partial_s \bar{\phi}_m{}^l + \frac{1}{4} \partial_j \bar{\phi} \partial_s \bar{\phi} \right) , \end{aligned} \tag{3.18}$$

and the Freud pseudo-tensor expressed in the following manner,

$$\sqrt{-g} F_j{}^k \equiv \partial_l (\sqrt{-g} B_j{}^{kl}) \, , \tag{3.19}$$

with the antisymmetric super-potential $B_j^{\;kl}$ given by [3, 5, 15],

$$\sqrt{-g}B_j^{\;kl} \;=\; \sqrt{-g}B_j^{\;[kl]} \;=\; -\,\frac{1}{2}\Big[\,\delta_j^{\;k}(\mathbf{g}^{rs}\Gamma^l_{\;rs} - \mathbf{g}^{lr}\Gamma^s_{\;rs}) + \delta_j^{\;l}(\mathbf{g}^{kr}\Gamma^s_{\;rs} - \mathbf{g}^{rs}\Gamma^k_{\;rs}) + (\mathbf{g}^{lr}\Gamma^k_{\;jr} - \mathbf{g}^{kr}\Gamma^l_{\;jr})\Big]\;. \tag{3.20}$$

The anti-symmetry of the super-potential leads directly to the so-called Freud identity [3, 5, 15, 29],

$$\partial_k(\sqrt{-g}F_j^{\;k}) \;=\; 0\;, \tag{3.21}$$

which is related to energy-momentum conservation in general relativity.

Making use of eq. (2.23) for the Christoffel symbols, the Freud pseudo-tensor becomes, after some algebra,

$$\begin{aligned}\sqrt{-g}F_j^{\;k} \;&=\; -\sqrt{-g}\,\Box\,\bar\phi_j^{\;k} + \partial_l\big[\,\mathbf{g}^{mk}\partial_m\bar\phi_j^{\;l} + (\delta_p^{\;k}\delta_j^{\;l} - \delta_p^{\;l}\delta_j^{\;k})\mathbf{g}^{mr}\partial_m\bar\phi_r^{\;p}\,\big]\\ &\equiv\; -\sqrt{-g}\,\Box\,\bar\phi_j^{\;k} \;+[\mathrm{GT}]_j^{\;k}\;,\end{aligned} \tag{3.22}$$

where the *gauge term* $[\mathrm{GT}]_j^{\;k}$ is defined as,

$$\begin{aligned}[\mathrm{GT}]_j^{\;k} \;&\equiv\; [\mathrm{gt}]_j^{\;k} - \frac{1}{2}\delta_j^k\,[\mathrm{gt}]\\ [\mathrm{gt}]_j^{\;k} \;&\equiv\; \partial_l\big[\,\mathbf{g}^{mr}\,(\delta_r^{\;k}\partial_m\bar\phi_j^{\;l} + \delta_j^{\;l}\partial_m\bar\phi_r^{\;k})\,\big]\\ [\mathrm{gt}] \;&=\; [\mathrm{gt}]_j^{\;k}\delta_k^{\;j} \;=\; 2\,\partial_l(\,\mathbf{g}^{mr}\,\partial_m\bar\phi_r^{\;l}\,)\;,\end{aligned} \tag{3.23}$$

and the curved spacetime d'Alembertian given by,

$$\sqrt{-g}\,\Box \;\equiv\; \partial_l(\,\mathbf{g}^{lm}\partial_m\,)\;. \tag{3.24}$$

Making use of the explicit expression (3.22) for the Freud pseudo-tensor in terms of the tensor potentials and recalling Einstein's equations (2.8), we arrive at the desired field equation [15],

$$-\sqrt{-g}\,\Box\,\bar\phi_j^{\;k} \;=\; \frac{8\pi k}{c^4}\sqrt{-g}\,(\tau_j^{(\mathrm{E})k} + \tilde t_j^{(\mathrm{E})k}) \;-\; [\mathrm{GT}]_j^{\;k}\;. \tag{3.25}$$

To proceed further, we go to the so-called *harmonic gauge* [16] defined as follows,

$$\mathbf{g}^{rs}\Gamma^k_{\;rs} \;=\; -\partial_r\mathbf{g}^{kr} \;=\; \partial_r\bar\phi_j^{\;r} \;=\; 0\;. \tag{3.26}$$

In such a gauge, the gauge term (3.23) and d'Alembertian (3.24) simply become,

$$\begin{aligned}[\mathrm{GT}]_j^{(\mathrm{H})k} \;&=\; -2\,\mathbf{g}^{rm}\,\partial_l\bar\phi_r^{\;k}\partial_m\bar\phi_j^{\;l} \;=\; -2\,\mathbf{g}^{rk}\,\partial_l\bar\phi_r^{\;m}\partial_m\bar\phi_j^{\;l}\\ \sqrt{-g}\,\Box \;&=\; \mathbf{g}^{lm}\,\partial_l\partial_m\;.\end{aligned} \tag{3.27}$$

Insertion into (3.25) yields,

$$-\,\mathbf{g}^{lm}\,\partial_l\partial_m\,\bar\phi_j^{\;k} \;=\; \frac{8\pi k}{c^4}\sqrt{-g}\,(\tau_j^{(\mathrm{E})k} + \tilde t_j^{(\mathrm{E})k}) \;-\; [\mathrm{GT}]_j^{(\mathrm{H})k}\;. \tag{3.28}$$

In a weak gravitational field, we need only to keep terms linear in the tensor potentials $\bar\phi_j^{\;k}$. So quadratic terms such as the gravitational energy-momentum pseudo-tensor $\tilde{t}_j^{(\mathrm{E})k}$, because of its relation (2.8) with the Einstein-Pauli pseudo-tensor $E_j^{\ k}$ of eqs. (3.17)-(3.18), as well as the gauge term (3.27) in the harmonic gauge drop out.

Identifying the Landau-Lifchitz gravitational wave field $\psi_j^{(\mathrm{LL})k}$ as follows,

$$\frac{1}{2}\,\psi_j^{(\mathrm{LL})k} \;\equiv\; \bar{\phi}_j^{\ k} \;, \tag{3.29}$$

the field equation (3.28) finally becomes, in the weak gravitational field approximation,

$$-\,\frac{1}{2}\,\eta^{lm}\,\partial_l\partial_m\,\psi_j^{(\mathrm{LL})k} \;=\; \frac{8\pi k}{c^4}\,\tau_j^{(\mathrm{E})k} \;, \tag{3.30}$$

which is the Landau-Lifchitz wave equation for gravitational radiation from a matter source [16, 30, 31].

4 Concluding Remarks

In this work, we tried to re-organize the logical structure of the derivation of the Einstein Real Gravity formulation of General Relativity in a way which emphasizes its fundamental physical basis. It's connection to newtonian theory is very natural, contrary to the traditional approach to General Relativity which connects to Newton's theory only in a weak gravitational field situation [16, 17, 18, 19, 30, 31].

Both formulations are algebraically completely equivalent. But the introduction *à la* Yilmaz [5, 15] of the tensor potentials into the game changes things fundamentally. The most striking consequence is the complete elimination of event horizons and singular objects from General Relativity.

Einstein Real Gravity is a strong gravitational field theory, contrary to the traditional formulation, which runs very fast into troubles when continued to the strong fields domain. Although the Schwarzschild solution seems to be a solution of Real Gravity, it does not describes black holes because the physical gravitational field is shifted to the tensor potentials. Furthermore, even in the traditional formulation context, Mitra [32, 33] already argued that the only consistent Schwarzschild solution is the extreme zero mass case, $i.e.$ flat Minkowski spacetime.

In view of the recent literature on astronomical black holes [34, 35, 36, 37] and gravitational waves [38, 39, 40, 41, 42], one may ask if not black holes, what are the densed black objects observed from a variety of sources?

Observationally, one is able to see that they are heavy and dense objects characterized by very high redshift. However no event horizon has ever been observed and none ever will. Event horizons are unobservable in principle.

The answer to the question may very well lie in a new theoretical class of astrophysical objects called eternally collapsing (ECOs) or magnetic eternally collapsing objects (MECOs). The theory of ECOs has originally been developed by Mitra [43, 44] as a new class of radiation pressure Eddington balanced highly redshifted gravitationally collapsed objects.

The theory of MECOs by Robertson and Leiter [45] added an intrinsic magnetic field to the properties of these objects and identified whole classes of astronomical objects such as galactic black hole candidates (GBHCs) and active galactic nuclei (AGNs) as potential MECOs candidates. Being magnetic, as seemingly observed from astrophysical data [46, 47, 48], they cannot be black holes. Black holes do not have an intrinsic magnetic field.

Additional works by Rudolph Schild from the Harvard-Smithsonian Center for Astrophysics came in support of the MECO theory [49, 50, 51].

So as can be appreciated, there ARE interesting viable alternatives to traditional astrophysical theories or models of astronomical objects and astrophysicists are encouraged to keep an objective and open mind before prematurely identifying such astronomical objects.

It is in the domain of strong gravitational fields that the Real Gravity theory separates itself from traditional General Relativity. It is such a domain that astrophysicists and astronomers must be exploring with greater emphasis.

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