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Dynamics of Motion in a Closed Universe

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Abstract

Inhabitants of Earth live at different angles, depending on where they are located on the sphere. The same concept applies in four dimensions to remote locations in a Universe with curved space-time. The premise of this paper is that the overall curvature of space-time in a non-flat universe causes the space-time at locations that are great distances apart to be at different angles. This necessitates that the speed of light at a remote location be different than the local speed of light. Since the constancy of the speed of light is at the heart of relativity theory, it is proposed that a small change is required to the relativistic equations of motion to accommodate this effect. This is done by modifying the g_{oo} term of the space-time metrics. The results provide some interesting effects. One, the modification brings the Hubble plot for very distant galaxies (with red-shifts up to 1.00) more in line with a universe that does not contain dark energy; two, the modification nicely explains the anomalous accelerations of the Pioneer spacecraft; and three, the modification provides an explanation for the orbital velocities and gravitational lensing of galaxies without the need for dark matter. The calculated value of the density parameter, Ω_{TOT} that provides the best fit to observations is about 1.04, which is within the Wilkinson Microwave Anisotropy Probe (WMAP) range of 1.02±0.02. However, this theory breaks down when applied to the orbits of the planets since it predicts there should be additional precessions that are big enough to be observed. An alternative explanation is proposed.

Key Words: Universe, curvature, redshift, relativity, geometry, Pioneer, dark matter

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1. Introduction

The shape of the universe is determined by a balance between the momentum of expansion and the pull of gravity. The rate of expansion is expressed by the Hubble Constant, H, while the strength of gravity depends on the density and pressure of the matter in the universe. If the density of the universe is equal to the "critical density", then the universe is flat, otherwise the universe is non-flat. Recent observations have shown that the universe is either flat, or very close to it (Verde, 2004).

Since Earth is a sphere, it is clear that residents of Italy live at a different angle than residents of Canada. Their “up” vectors are not parallel. The same concept must apply in four dimensions for remote locations in a Universe with curved space-time. The main thrust of this paper is that in a non-flat universe, observers in different locations necessarily exist at different space-time angles. Their time vectors in space-time would not be parallel. Also their light vectors, which travel locally at a space-time angle of 45º with a speed c, would not be parallel. Thus an observer at Position A in the crude sketch in Figure 1 below would observe that light has a slightly different speed at Position B than at Position A.

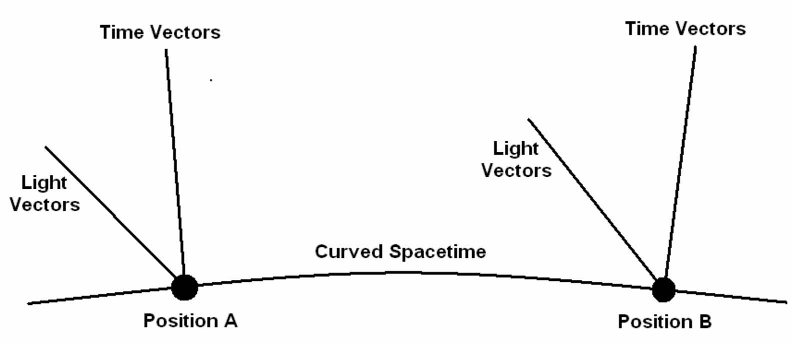


Figure 1

A variation of the speed of light was evident to Einstein when he discovered the general theory of relativity that explained gravity in terms of curved space-time. In the 1920 book “Relativity: the special and general theory” he wrote: . . . *according to the general theory of relativity, the law of the constancy of the velocity of light in vacuo, which constitutes one of the two fundamental assumptions in the special theory of relativity [...] cannot claim any unlimited validity.*

2. Calculating the Magnitude of the Change in Light Speed

Let’s assume that a photon is emitted towards us from a light source that is some distance away, r_o. The photon’s position at any time is r. As shown in the sketch in Figure 2 below, the light travels along the space-time geodesic that results from the geometry of the path. The photon reaches us traveling at the speed of light, thus the photon arrives at Point A with a space-time angle of 45°. In a flat Universe, a photon emitted at point A will travel the segment BA, a straight line in space-time, with a constant angle of 45°. However, the path that the photon traces in space-time in a non-flat universe is different than if the Universe is flat. In a non-flat universe, a photon will travel the segment CA, a curved line.

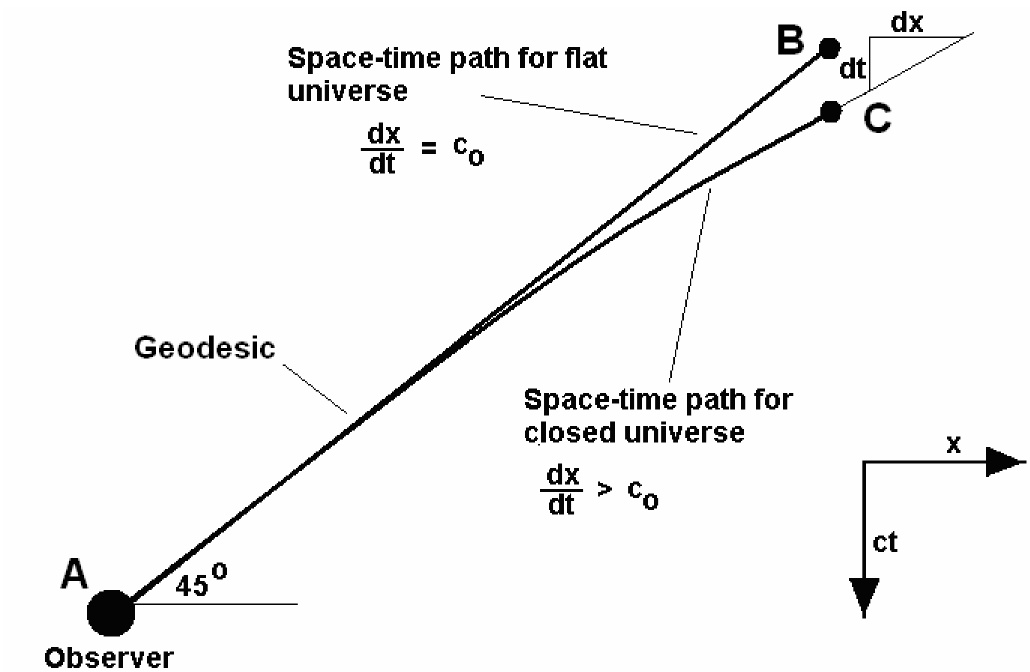


Figure 2

Since the angle of the geodesic changes over the course of the trip, then the speed of the photon in space relative to the observer is not constant. If the local speed of light for the observer is c_o, then the observed speed of light in a non-flat universe varies with distance such that $c(r) \neq c_o$. For a closed universe, the world line of the photon is slightly curved so that it arrives at a quicker time than r_o/c_o. One can see from the sketch that the vertical (time) distance from Point C to A is less than from B to A, so the CA trip must be shorter in time.

The radius of curvature of the curved path, R, can be calculated from the Friedmann equation given below:

$$R = \frac{c}{H\sqrt{\dfrac{\rho}{\rho_{crit}} - 1}} = \frac{c}{H\sqrt{\Omega_{TOT} - 1}} \qquad (2\text{-}1)$$

Where H is Hubble's parameter, ρ is the density of the Universe, ρ_{crit} is the critical density, and Ω_{TOT} is the density parameter, which is ρ/ρ_{crit}.

By considering the geometry, c can be calculated as a function of c_o, R and r, the distance from the observer, as follows:

$$c = c_o\sqrt{\frac{R^2}{\left(\dfrac{R}{\sqrt{2}} - r\right)^2} - 1} \qquad (2\text{-}2)$$

For r<<R and removing higher order terms;

$$c \approx c_o\left(1 + \frac{2\sqrt{2}\,r}{R}\right) \qquad (2\text{-}3)$$

This represents a change in the observed speed of light that is caused by the curvature of a closed universe. Since the constancy in the speed of light is at the heart of relativity theory, it is proposed that this small effect would not be included in the relativistic equations of motion.

3. Modifying the Space-time Metric

The proper speed of light, c, is the incremental change in proper length (dL) divided by the incremental change in proper time (dτ):

$$c = \frac{dL}{d\tau} = \frac{\sqrt{g_{\mu\nu} dx^u dx^\nu}}{d\tau} \qquad (3\text{-}1)$$

If we are to surmise that the speed of light changes with distance as shown earlier, then this must be reflected in the space-time metric. The space-time metric determines the geometry of space-time, as well as determining the geodesics of particles and light beams. The general formula for a diagonal space-time metric is:

$$ds^2 = g_{oo} c^2 dt^2 + g_{11} dx^2 + g_{22} dy^2 + g_{33} dz^2 \qquad (3\text{-}2)$$

The measurement of speed is a coordinate-dependent quantity, and is therefore somewhat ambiguous. To determine speed (distance moved/time taken) you must first choose some standards of distance and time, and different choices can give different answers. This is already true in special relativity; if you measure the speed of light in an accelerating reference frame, the answer will, in general, differ from *c*. Since the first term on the right hand side includes the speed of light, it is proposed here that the g_{oo} term be slightly modified to g_{oo}-prime in Equation 3-3 below to incorporate the change in c as described in Equation 2-3.

$$g_{00}^{'} = g_{00} \left(1 + \frac{2\sqrt{2}\, r}{R}\right)^2 \approx g_{00} \left(1 + \frac{4\sqrt{2}\, r}{R}\right) \quad (For\ r << R) \qquad (3\text{-}3)$$

4. Redshift From a Distant Galaxy With the Modified Metric

The supernova observations in the late 1990's brought about a revolution in cosmology. Measurements of the geometry and the matter contents of the universe from Type 1A supernovae have implied a dynamical age of the universe that accommodates the oldest known stellar objects. However, the geometry that best fits the observed redshifts has raised the need for a dark energy component, Ω_Λ, consisting of about 75% of the universe. This dark energy is not readily explained within the current particle physics theories. Figure 3 below shows the observed data for redshifts up to 1.0 (Knop et al., 2003). Currently the geometry that best fits the observed data has Ω_Λ equal to 0.75.

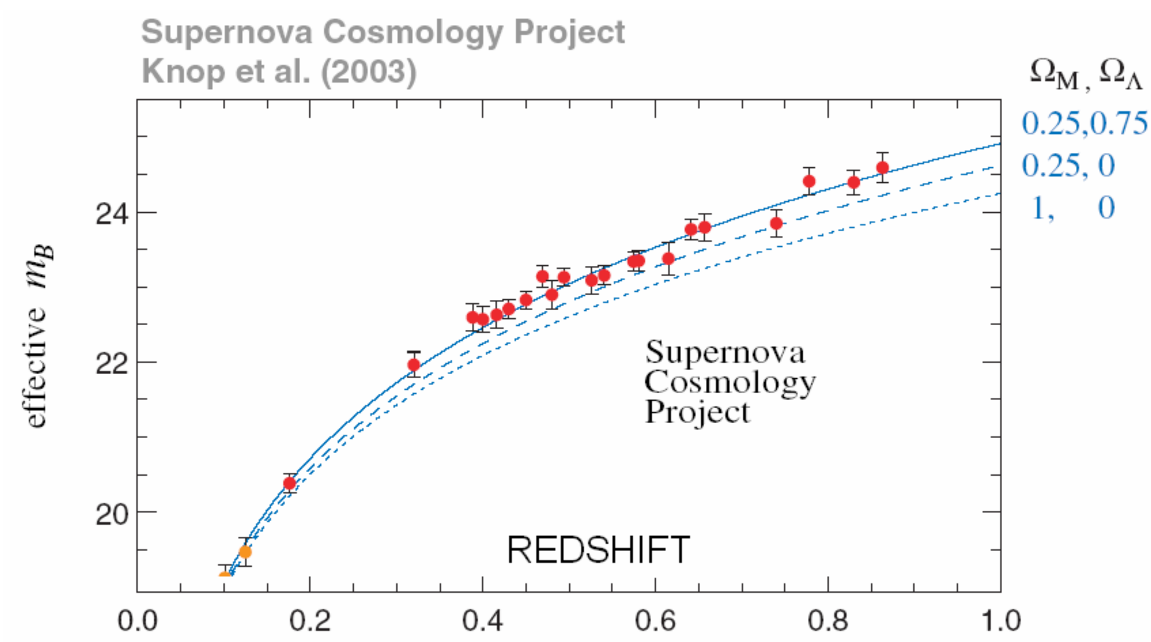


Figure 3

To determine the effect of the proposed modification on the redshifts from distant galaxies, the g_{oo} term of the Friedmann-Lemaître-Robertson-Walker (FLRW) metric is modified per Equation 3-3 to provide the following:

$$ds^2 = \left(1 + \frac{2\sqrt{2}\, r}{R}\right)^2 c_o^2 dt^2 - a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2\right) \qquad (4\text{-}1)$$

To conduct a consistent assessment of the change in redshift (ΔZ) of supernovae with the modified metric, below are the calculations of the redshift with and without the modification to the FLRW metric. Some assumptions have been made to ease the calculations.

For the Modified Metric	**For the Unmodified Metric**
For a light beam, ds=0 Also for dθ=dφ=0, and removing second order terms and higher of r/R, results in: $c_o \frac{dt}{a(t)} \approx \frac{dr}{1 + \frac{2\sqrt{2}\, r}{R}}$	$c_o \frac{dt}{a(t)} \approx dr$

For the Modified Metric | **For the Unmodified Metric**

For the Modified Metric

For $a_o \equiv 1.0$ at $r = 0$, and assuming $da/dt \approx H_o$

$$c_o \int_{a(t)}^{a_0} \frac{dt}{a(t)} \approx \int_r^0 \frac{dr}{1 + \frac{2\sqrt{2}r}{R}}$$

Integrating,

$$\ln\left(\frac{a_o}{a(t)}\right) = \frac{RH_o}{2\sqrt{2}\, c_o} \ln\left(1 + \frac{2\sqrt{2}\, r}{R}\right)$$

$$\frac{a_o}{a(t)} = 1 + Z = \left(1 + \frac{2\sqrt{2}\, r}{R}\right)^{\frac{RH_o}{2\sqrt{2}\, c_o}}$$

For $R = \dfrac{c_o}{H_o\sqrt{\Omega_{TOT} - 1}}$,

$$Z = \left(1 + \frac{2\sqrt{2}\, rH\sqrt{\Omega_{TOT} - 1}}{c_o}\right)^{\frac{1}{2\sqrt{2}\,\sqrt{\Omega_{TOT} - 1}}} - 1$$

For the Unmodified Metric

$$c_o \int_{a(t)}^{a_0} \frac{dt}{a(t)} \approx \int_r^0 dr$$

$$\ln\left(\frac{a_o}{a(t)}\right) = \frac{RH_o}{c_o} \ln r$$

$$\frac{a_o}{a(t)} = 1 + Z = e^{\frac{H_o r}{c_o}}$$

$$Z = e^{\frac{H_o r}{c_o}} - 1$$

The change in redshift due to the modification of the FLRW metric (ΔZ_{MOD}) is the difference between the two values for Z above, or

$$\Delta Z_{MOD} = \left(1 + \frac{2\sqrt{2}\, rH_o\sqrt{\Omega_{TOT} - 1}}{c_o}\right)^{\frac{1}{2\sqrt{2}\,\sqrt{\Omega_{TOT} - 1}}} - e^{\frac{H_o r}{c_o}} \qquad (4\text{-}2)$$

Figure 4 is a plot of the modified and unmodified values of Z and their difference, ΔZ, for a slightly closed universe with Ω_{TOT} equal to 1.04:

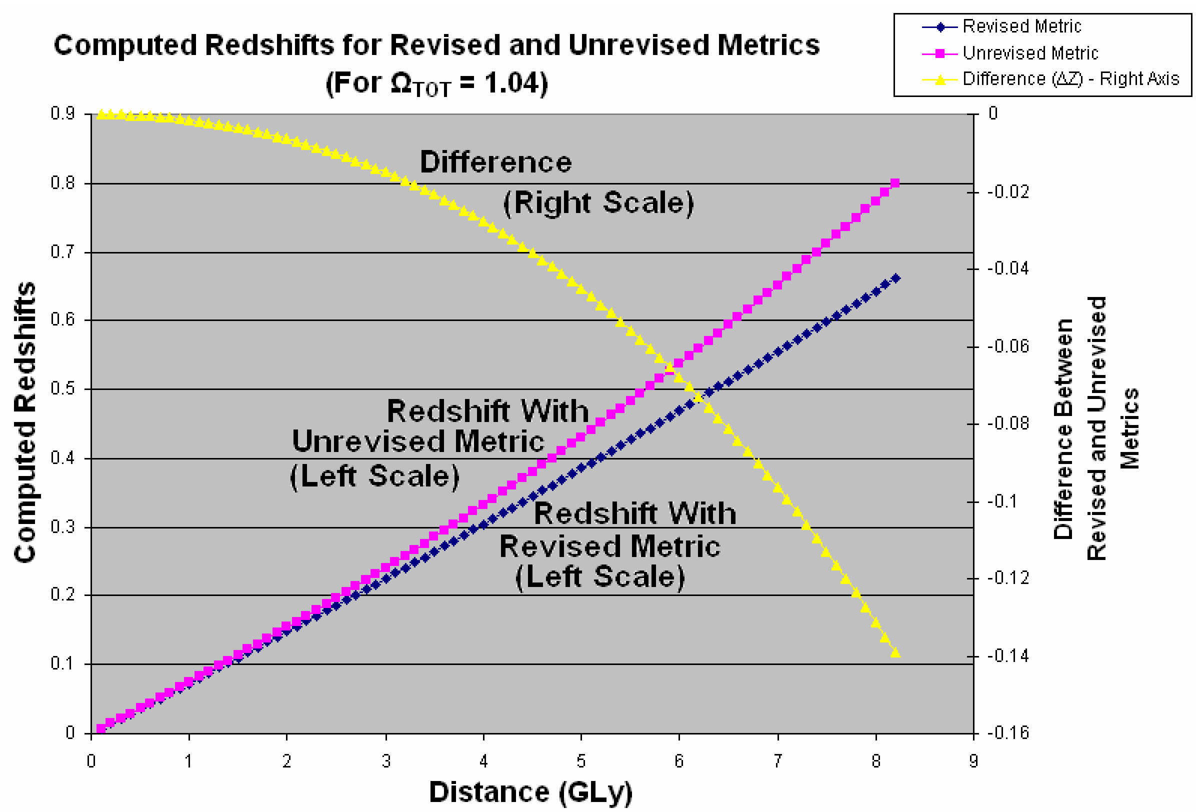


Figure 4

One can see that for Ω_{TOT}=1.04, the difference between the two different Z values is about -0.08 at the redshift of 0.50, which is a distance of about 6 GLy. If the observed hubble plot in Figure 3 is adjusted by this amount, it falls in line with the (Ω_M =1.0, Ω_Λ =0.0) hubble plot, instead of the (Ω_M =0.25, Ω_Λ =0.75) hubble plot. Thus if the corrections from the modified FLRW metric in a closed universe are included, the observed redshifts would point towards a geometry for a universe that does not contain dark energy.

It is noted that Ω_{TOT} equal to 1.04 is consistent with the Wilkinson Microwave Anisotropy Probe (WMAP) result, which is 1.02±0.2 (Verde, 2004).

5. Particles in Orbit with the Modified Metric

The Schwarzschild metric for particles in orbit around a large mass, M, is as follows:

$$ds^2 = (1 - \frac{2GM}{rc_o^2})\, c_o^2 dt^2 - \frac{dr^2}{1 - \frac{2GM}{rc_o^2}} - r^2 d\phi^2$$

This assumes that the θ component is constant at ½π (90°), so that dθ is zero and sinθ is unity.

To model changes in the orbits of particles with the proposed new effect, the g_{oo} term of the Schwarzschild metric is modified per Equation 3-3, which results in the following new metric:

$$ds^2 = (1 - \frac{2GM}{rc_o^2} + 2kr)\, c_o^2 dt^2 - \frac{dr^2}{1 - \frac{2GM}{rc_o^2}} - r^2 d\phi^2 \qquad (5\text{-}1)$$

Where: $$k = \frac{2\sqrt{2}}{R} \qquad (5\text{-}2)$$

To ease further calculations, the metric is written as:

$$c^2\ d\tau^2 = e^{v} c_o^2\ dt^2 - \frac{dr^2}{1 - \frac{2GM}{rc_o^2}} - r^2 d\phi^2 \qquad (5\text{-}3)$$

Where: $$e^{v} = (1 - \frac{2GM}{rc_o^2} + 2kr) = \gamma \qquad (5\text{-}4)$$

For the relativistic equations of motion;

$$\frac{d^2 x^\alpha}{d\tau^2} + \Gamma^\alpha_{\mu\nu} \frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0 \qquad (5\text{-}5)$$

Where,

$$\Gamma^\alpha_{\mu\nu} = \frac{1}{2\mathbf{g}_{\alpha\alpha}}\left(\frac{\partial \mathbf{g}_{\alpha\mu}}{\partial \mathbf{x}^\nu} + \frac{\partial \mathbf{g}_{\alpha\nu}}{\partial \mathbf{x}^\mu} - \frac{\partial \mathbf{g}_{\mu\nu}}{\partial \mathbf{x}^\alpha}\right) \qquad (5\text{-}6)$$

The appropriate Christoffel symbols are:

$$\Gamma^2_{12} = \Gamma^2_{21} = \frac{1}{2r^2}(2r) = \frac{1}{r} \qquad (5\text{-}7)$$

$$\Gamma^0_{10} = \Gamma^0_{01} = \frac{e^{-\nu}}{2}\nu' \, e^\nu = \frac{\nu'}{2} \qquad (5\text{-}8)$$

Where the prime (`) represents the partial differentiation with respect to r. Inserting these surviving Christoffel symbols into the relativistic equations of motion yields the following two equations,

$$\frac{d^2\phi}{d\tau^2} + \frac{2}{r}\frac{dr}{d\tau}\frac{d\phi}{d\tau} = 0 \qquad (5\text{-}9)$$

$$\frac{d^2 t}{d\tau^2} + \nu' \frac{dr}{d\tau}\frac{dt}{d\tau} = 0 \qquad (5\text{-}10)$$

Integrating these gives the following two equations,

$$r^2 \frac{d\phi}{d\tau} = h \qquad (5\text{-}11)$$

$$\frac{dt}{d\tau} = a\, e^{-\nu} = \frac{a}{\gamma} \qquad (5\text{-}12)$$

Where a and h are constants of integration. We can re-write the metric in Equation 5-3 as,

$$c^2 = \gamma\, c_o^2 \frac{dt^2}{d\tau^2} - e^{-v} \frac{dr^2}{d\tau^2} - r^2 \frac{d\phi^2}{d\tau^2} \qquad (5\text{-}13)$$

Substituting for dt/dτ, dr/dτ and dφ/dτ, from Equations 5-11 and 5-12, and using Equation 2-3, yields;

$$(1+2kr)\, c_o^2 = \frac{c_o^2\, a^2}{\gamma} - \frac{h^2}{\gamma} \frac{1}{r^2} \frac{dr^2}{d\phi^2} - \frac{h^2}{r^2} \qquad (5\text{-}14)$$

Multiplying through by γ, and converting for γ in Equation 5-4;

$$c_o^2 - \frac{2GM}{r} + 4c_o^2 kr = c_o^2\, a^2 - h^2 \left(\frac{1}{r^2}\frac{dr}{d\phi}\right)^2 - \frac{h^2}{r^2} + \frac{2GMh^2}{c^2 r^3} - \frac{kh^2}{r} \qquad (5\text{-}15)$$

Letting u ≡ 1/r and replacing;

$$c_o^2 - 2GMu + \frac{4c_o^2 k}{u} = c_o^2\, a^2 - h^2 \left(\frac{du}{d\phi}\right)^2 - h^2 u^2 + \frac{2GMh^2 u^3}{c^2} - kh^2 u \qquad (5\text{-}16)$$

Differentiating with respect to φ, dividing by $2h^2(du/d\varphi)$, and removing the last term on the right-hand-side, which is negligibly small for c>>v, yields;

$$\frac{d^2u}{d\phi^2} + u = \frac{GM}{h^2} + \frac{3GM\, u^2}{c^2} + \frac{2c_o^2 k}{h^2 u^2} \qquad (5\text{-}17)$$

Where; $$r^2 \frac{d\phi}{d\tau} = h \qquad (5\text{-}18)$$

Equation 5-17 is comparable to the equations of a Newtonian orbit;

$$\frac{d^2u}{d\phi^2} + u = \frac{GM}{h^2} \qquad (5\text{-}19)$$

Where; $$r^2 \frac{d\phi}{dt} = h \qquad (5\text{-}20)$$

The $3GMu^2/c^2$ term in Equation 5-17 above is the relativistic term that was used to successfully explain the observed anomaly of the precession of Mercury of 43 arc-seconds per century. The $2c^2k^2/h^2u^2$ term on the right hand side of Equation 5-17 is the additional term due to the Schwarzschild metric being modified.

6. The Pioneer Anomalous Acceleration

Launched about thirty years ago, Pioneer 10 and Pioneer 11 traveled through the solar system and are currently outside of the solar system moving at relatively constant velocities directly away from the sun. We have since lost communication with them. From earlier data, the Pioneer engineers have computed the spacecraft velocities and compared them to the expected values from known gravitational and other effects. After correcting for all possible effects, there is an unexplained constant acceleration ($\mathbf{a_p}$) towards the sun of $8.74 \pm 1.33 \times 10^{-10}$ m/s^2 (Anderson et al., 1998). It is proposed that the Pioneer orbits be re-calculated using the modified equation of motion, Equation 5-17, to assess whether this anomaly can be explained.

The Pioneer spacecraft are in a special orbit, essentially moving directly away from the sun. Equation 5-17 above can be used to calculate their motions, making the following substitution from Equation 5-18:

$$d\phi = \frac{h}{r^2} d\tau \approx \frac{h}{r^2} dt \qquad (6\text{-}1)$$

Thus Equation 5-17 becomes (with u=1/r):

$$\frac{r^4}{h^2} \frac{d^2\left(\frac{1}{r}\right)}{dt^2} - \frac{1}{r} = \frac{GM}{h^2} + \frac{3GM}{r^2 c^2} + \frac{2c_o^2 k}{h^2} r^2 \qquad (6\text{-}2)$$

Or, isolating the radial acceleration term,

$$\frac{d^2 r}{dt^2} = \frac{2}{r}\left(\frac{dr}{dt}\right)^2 - \frac{h^2}{r^3} - \frac{GM}{r^2} - \frac{3GMh^2}{r^4 c^2} - 2c_o^2 k \qquad (6\text{-}3)$$

The only new term due to the modified Schwarzschild metric is the last term on the right-hand-side. Therefore the difference in the Pioneer acceleration due to this new term (a_p) would be:

$$a_p = \Delta\left(\frac{d^2 r}{dt^2}\right) = -2c_o^2 k = -4\sqrt{2}\ H\ c_o \sqrt{\Omega_{TOT} - 1} \qquad (6\text{-}4)$$

This result implies that with the revised Schwarzschild metric in Equation 5-1, the pioneer spacecraft would undergo an additional constant deceleration that is opposite to the direction of motion.

To find the magnitude of the computed deceleration, the following values are substituted into Equation 6-4:

$c_o = 3 \times 10^8$ m/s

$H = 71$ km/s/Mpc $= 2.3 \times 10^{-18}\ s^{-1}$

$\Omega_{TOT} = 1.04$ (as found in Section 4)

This yields a value for the Pioneer acceleration of **-7.8×10^{-10} m/s^2**. This is in very good agreement with the observed acceleration of $-8.74 \pm 1.33 \times 10^{-10}$ m/s^2.

7. Galactic Orbits with the Modified Metric

The motions of stars within galaxies have been largely out of line with the expected motions based on gravitational forces from the visible matter within the galaxy. The rotational velocities of the stars that are outside of the galactic centre are much higher than those expected from gravitational forces from baryonic matter. This has required the advent of "dark matter" to explain the differences.

We can apply the modified orbital equation of motion, Equation 5-17, to galactic orbits to see the effect. For galactic distances, the relativistic term, which is inversely proportional to the distance, is negligible. If we also assume the galaxy is stable (i.e., not contracting or expanding and the stars have circular orbits with $d^2u/d\varphi^2 = 0$), then Equation 5-17 becomes:

$$u = \frac{GM_{Galaxy}}{h^2} + \frac{2c_o^2 k}{h^2 u^2} \qquad (7\text{-}1)$$

Where M_{Galaxy} is the mass of the galaxy within the orbit.

Substituting $h = r^2 d\varphi/dt = rv$, where v is the rotational velocity, and u=1/r yields,

$$v = \sqrt{\frac{GM_{Galaxy}}{r} + 2c^2 kr} \qquad (7\text{-}2)$$

The additional term in Equation 7-2 increases in proportion to r, whereas the gravitational term decreases in proportion to 1/r. Thus the new term becomes much more important for galaxy dynamics where the distances are much larger than for our solar system.

The "effective" mass of the galaxy (M_{eff}) that would be required to provide the rotational velocities for the modified metric can be calculated from Equation (7-2):

$$\frac{GM_{eff}}{r} = v^2 = \frac{GM_{Galaxy}}{r} + 2c^2 kr \qquad (7\text{-}3)$$

or,

$$M_{eff} = M_{Galaxy} + \frac{2c^2 kr^2}{G} = M_{Galaxy} + M_{ADD} \qquad (7\text{-}4)$$

The addition mass term on the RHS (M_{ADD}), which is a result of modifying the metric, could be interpreted to be the "dark matter". It can be calculated:

$$M_{ADD} = \frac{2c^2 kr^2}{G} = \frac{4\sqrt{2} Hcr^2 \sqrt{\Omega_{TOT} - 1}}{G} \qquad (7\text{-}5)$$

To find the magnitude of this component at the outskirts of our galaxy, the following values are inserted:

$H = 71$ km/s/Mpc $= 2.3 \times 10^{-18}\ s^{-1}$

$\Omega_{TOT} = 1.04$ (as found in Section 4)

r = 15,000 pc (the radius of our galaxy that contains the bulk of the mass, which is about 100,000 Ly diameter)

Inserting these values in Equation 7-5 yields a value of $M_{ADD} = 25 \times 10^{41}$ kg, compared to 3.6×10^{41} kg estimation for the mass of visible (baryonic) matter in the galaxy (about 180 billion solar masses). This gives a proportion of about 87% for M_{ADD} compared to the effective mass. If we interpret this to be the "dark matter" contribution, it is in good agreement with the proportion of dark matter in the galaxy found by other analyses (about 85-90% dark matter). This would also require that the dark matter distribution be roughly spherically symmetric (not flat), which has been observed.

Thus modifying the metric and applying it to the galactic motions provides an alternative for dark matter that could explain the observed rotational velocities of galaxies.

Equation 7-4 can be divided by the volume, $4/3\pi r^3$ and differentiated to yield the densities at a distance of r as follows:

$$\rho_{eff}(r) = \rho_{Galaxy}(r) + \frac{c^2 k}{\pi G}\left(\frac{1}{r}\right) \qquad (7\text{-}6)$$

Where ρ_{eff} is the "effective" density.

The density of the additional component is inversely proportional to the radius. If this is the dark matter component, it is consistent with observations that the density of the dark matter in a galaxy is a maximum in the centre and gradually decreases as it gets to the outermost part, but increases considerably the total size of the galaxy (Innova, 2009).

8. Gravitational Lensing

A gravitational lens is formed when the light from a very distant, bright source is "bent" around a massive object such as a galaxy, or cluster of galaxies, that is between the source object and the observer. The process is known as gravitational lensing, and is one of the predictions of Einstein's general theory of relativity.

With the revised Schwarzschild metric in Equation 5-1, the expected deflection of light is revised. The resulting angle of deflection is:

$$\theta = \frac{4GM_{Deflector}}{r_o c^2} + 8kr_o \qquad (8\text{-}1)$$

Where r_o is the distance of closest approach of the light to the centre of mass and $M_{Deflector}$ is amount of deflecting mass within the radius r_o. The first term on the RHS is from Einstein’s theory, which was famously confirmed during an eclipse of the sun. The second term in the RHS is the result of the modified metric. It is very small (immeasurable) for distances within our solar system, but becomes more important for distances within the galaxy and within clusters of galaxies.

Using the same concept as in Section 7 by calculating the effective mass (M_{eff}) that would cause the deflection, yields:

$$M_{eff} = M_{Deflector} + \frac{2c^2 k r_o^{\,2}}{G} \qquad (8\text{-}2)$$

The additional term on the RHS is the same as the M_{ADD} term found for the galaxy dynamics in Section 7, Equation 7-5. Thus this could also be interpreted to be the "dark matter" contribution to gravitational lensing for galaxy clusters. Using the following inputs for an average gravitational cluster:

$H = 71$ km/s/Mpc $= 2.3 \times 10^{-18}\ s^{-1}$

$\Omega_{TOT} = 1.04$ (as found in Section 4)

$r_o = 5.0 \times 10^{22}$m (radius of a cluster with diameter 10^{23}m)

yields a value for M_{ADD} of 7×10^{15} solar masses. This is about 7 times greater than the visible baryonic mass of about 10^{15} solar masses. Thus the M_{ADD} component (or dark matter) comprises about 87% of the total effective mass of the cluster. This is again consistent with observations. For example, Wikipedia states that in a typical cluster perhaps only 5% of the total mass is in the form of galaxies, maybe 10% is in the form of hot X-ray emitting gas, and the remainder is dark matter.

It should be noted that the angle of deflection for galaxy clusters using dark matter theory should vary with the square of the distance, r_o. This is because it must be assumed that the mass of the dark matter inside the cluster varies directly with volume, which varies with the radius cubed. That is:

$$\theta = \frac{4GM_{Deflector}}{r_o c^2} \propto \frac{r_o^3}{r_o} \propto r_o^2 \qquad (8\text{-}3)$$

However, as shown in Equation 8-1, the variation in the amount of deflection using the modified metric varies linearly with r_o, or:

$$\theta = 8kr_o \propto r_o \qquad (8\text{-}4)$$

Thus the changes of the deflection with the relative size of the cluster are not the same between the dark matter theory and this curvature theory, as shown in the plot below in Figure 5. For example, one would expect the deflection due to dark matter to be increased by a factor of four when the cluster size is doubled, whereas the deflection due to the modified metric would only increase by a factor of two. This should be verifiable either with different clusters, or with deflection measurements at different positions within the same cluster. This could provide verification of this theory.

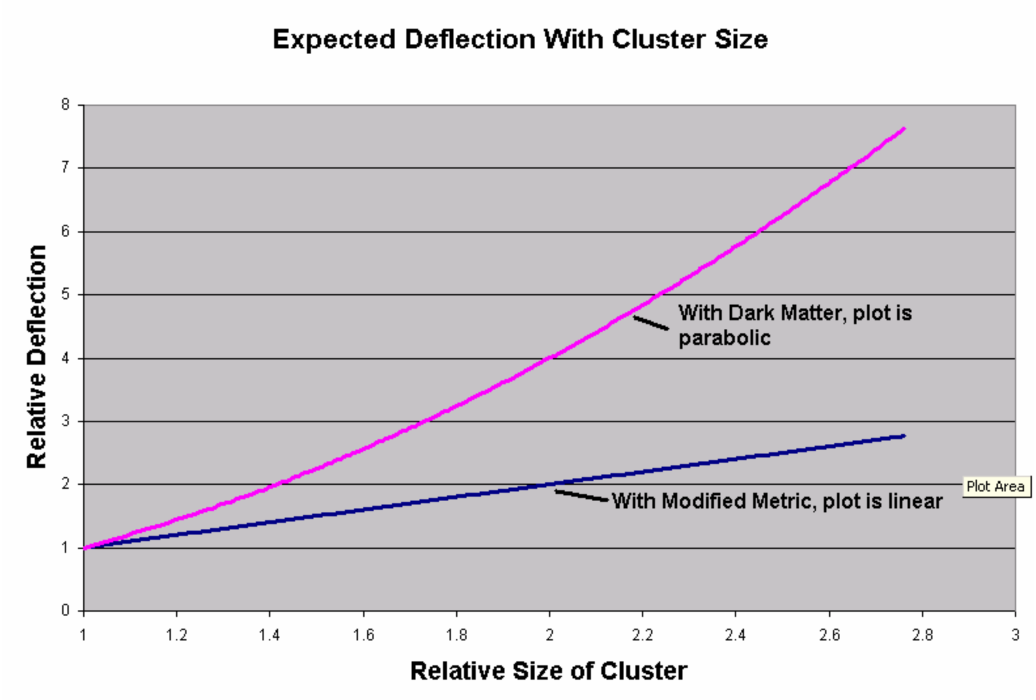


Figure 5

9. Curvature of the Universe

From the analyses in Sections 4 and 7, this paper proposes that dark matter and dark energy do not exist. However, this appears to conflict with the calculations of Ω_{TOT} of 1.04, as was done in Sections 4 and 6. Observations of visible matter in the universe indicate it constitutes only about 4% of the critical density. If so, it would seem that Ω_{TOT} should be lower than 1.00, and the universe structure would be open.

Equation 7-6 applies to a galaxy, but can be considered to extend to the universe in general to find ρ_{TOT} as a function of the density of matter and the additional curvature term as follows:

$$\rho_{TOT}(r) = \rho_{Matter}(r) + \frac{c^2 k}{\pi G}\left(\frac{1}{r}\right) \tag{9-1}$$

Where ρ_{Matter} is the density of matter in the Universe.

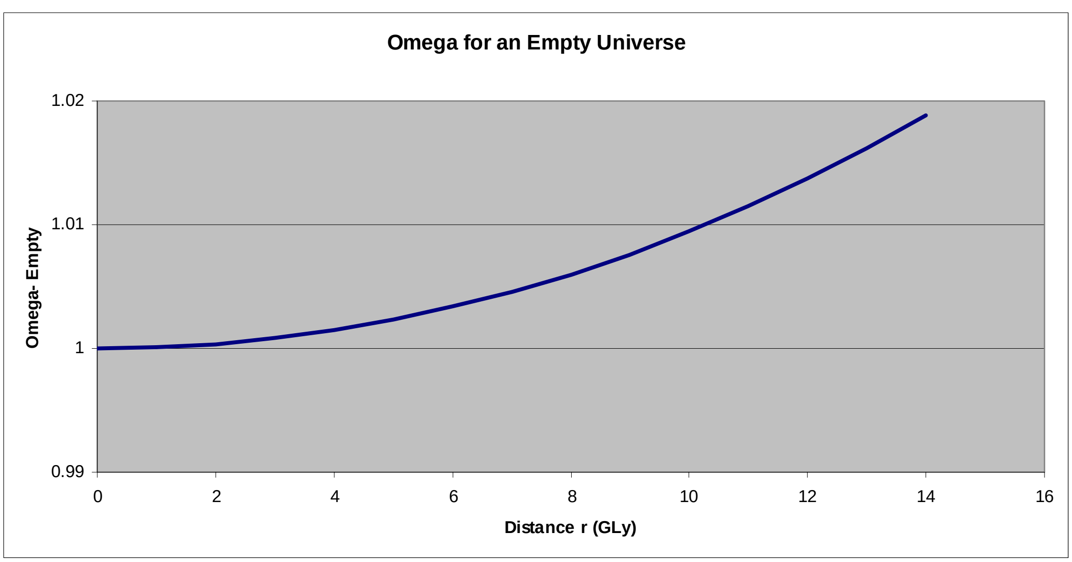
First we can calculate Ω_{TOT} for an empty universe (Ω_{Empty}) by setting ρ_{Matter} to zero and dividing by the critical density (ρ_{crit} =3H^2/8πG), and substituting for k to yield:

$$\frac{\rho_{Empty}}{\rho_{crit}} = \Omega_{Empty}(r) = \frac{16\sqrt{2}c\sqrt{\Omega_{Empty} - 1}}{3rH} \tag{9-2}$$

This becomes a quadratic of Ω_{Empty}:

$$\Omega^2_{Empty} - \frac{512c^2}{9r^2H^2}\Omega_{Empty} + \frac{512c^2}{9r^2H^2} = 0 \tag{9-3}$$

Using accepted values for c and H and solving the quadratic, below are the calculated values of Ω_{Empty} as a function of r:



Ω_{Empty} approaches one for small r and increases to about 1.02 at the observable edge of the universe (13.7 GLy). We must be careful not to extrapolate r too far, since the calculations in Equation 2-3 are based on r<<R, where R is about 70 GLy for Ω_{TOT} =1.04. If we propose that Ω_{TOT} for the current matter-filled universe is equal to:

$$\Omega_{TOT} = \Omega_{Matter} + \Omega_{Empty} \qquad (9\text{-}4)$$

Recent observations have estimated that the current visible matter of the universe constitutes about 4.6% of the total including dark matter and dark energy. This would mean that Ω_{Matter} is about 0.044 (0.46/1.04), putting Ω_{TOT} at about 1.044. This is close to the value of 1.04 found earlier in Sections 4 and 6.

10. Planetary Orbits with the Modified Metric

The new orbital equation from the revised Schwarzschild metric breaks down when applied to the motions of the planets in our solar system. The planetary motions have been observed for centuries and their orbits are known accurately.

To find the effect of this new term on the planetary orbits, we assume an almost circular orbit and set $u = GM/h^2 + \eta$, where η represents the slight deviation from the circular orbit. Inserting this formula into Equation 5-17 and neglecting the term η^2 (for $\eta \ll GM/h^2$) we find:

$$\eta(\phi) = C_1 \cos[(1-\beta)\phi + \delta] \qquad (10\text{-}1)$$

Where C_1 and δ are two arbitrary constants of integration.

From Equation 5-17;

$$\beta \approx 3\left(\frac{GM}{ch}\right)^2 - \frac{2c^2kh^4}{(GM)^3} \qquad (10\text{-}2)$$

The fact that β is non-zero means there is a precession in the orbit. The first term on the RHS is the famous relativistic correction for the precession of a planet's orbit. The second term is a new factor due to the modified Schwarzschild metric.

Table 1 below shows the approximate precessions for the first six planets using the value for Ω_{TOT} of 1.04 found in Sections 4 and 6:

Table 1

Planet	**Relativistic Precession (arc-seconds/century)**	**Additional Precession Due to Modified Metric (arc-seconds/century)**
Mercury	43.0	-10.5
Venus	8.7	-14.3
Earth	3.9	-16.8
Mars	1.4	-20.8
Jupiter	0.06	-38.4
Saturn	0.013	-52.6

The additional precession due to the modified metric increases the further the planets are from the sun. These new precessions are large enough to have been observed, and have not been.

Despite a great deal of effort, the writer cannot currently provide an explanation why the adjustment for the speed of light in a closed universe so nicely explains the Hubble plot and the motions of Pioneer and the galaxies, and then fails when applied to the planets.

One possible explanation offered here for the lack of precession is that the gravitational and universal curvature effects are not additive. Let's take a case where there is a large volume of the universe that contains no mass. In this case the local value of Ω_{TOT} would be near one (from Section 9), which would indicate from the Friedman equation that this space-time volume should be very flat. However, we could propose that the shape of the universe is *pervasive*, in other words it is consistent even in local volumes with no mass. If this is the case, then adding mass to the local volume shouldn't change its space-time shape until the added mass for a given radius, r, exceeds M_{ADD} as calculated in Section 7. This implies that the effect on the dynamics of motion of space-time curvature is not additive, but *substitutes*. The larger effect becomes the only contributor. Thus for areas within our solar system, the effect of the sun's gravitation is greater than any effect from the curvature of the universe, and therefore only the gravitational effect is observed. For large spaces like galaxies and beyond, the effect of curvature of the universe is greater than the gravitational effect, thus this effect wins out. This would explain the observed orbits of galaxies and red-shifts, and also the lack of effect on planetary dynamics. The agreement with the observed deceleration orbits of the pioneer spacecraft would have to be conceded to be coincidental. This may also be why some globular clusters show no evidence of any dark matter at all (Rejkuba et al., 2008) since the gravitational effect is generally larger than the effect of curvature for most globular clusters due to their small radius and high density.

11. Conclusions and Summary

This paper proposes that the curvature of space-time in a closed universe causes the speed of light to vary with distance from an observer. To accommodate this, a small change is made to the g_{oo} term in the relativistic metrics.

The results provide some interesting effects. One, the modification brings the Hubble plot for very distant galaxies (with red-shifts up to 1.0) more in line with a universe that does not contain dark energy; two, the modification nicely explains the anomalous accelerations of the Pioneer spacecraft; and three, the modification provides an explanation for the orbital velocities and gravitational lensing of galaxies without the need for dark matter.

The calculated value of Ω_{TOT} that provides the best fit to observations is about 1.04, which is within the Wilkinson Microwave Anisotropy Probe (WMAP) range of 1.02±0.02 (Verde, 2004).

However, this theory breaks down when applied to the orbits of the planets since it predicts there should be additional precessions that are big enough to be observed. An alternative explanation is proposed in Section 10.

12. References

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