Beyond Collisionless Dark Matter:
From a Particle Physics Perspective

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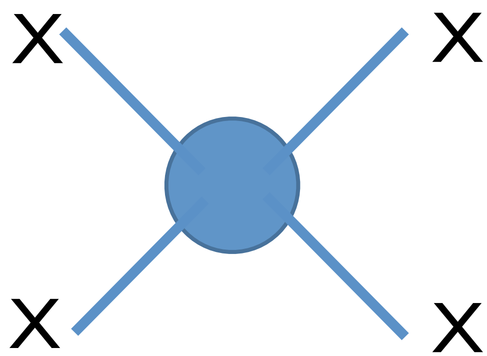


Harvard SIDM workshop 08/07-08/09

Collisionless VS. Collisional

- Large scales: Great!
- Small scales (dwarf galaxies, subhalos)?

  cusp vs. core problem
  "too big to fail" problem

See yesterday`s talk

- These anomalies can be solved if DM is sufficiently self-interacting

Spergel, Steinhardt (1999)

Recent simulations

Harvard group: Vogelsberger, Zavala, Loeb (2012); Zavala, Vogelsberger, Walker (2012)
UCI group: Rocha, Peter, Bullock, Kaplinghat, Garrison-Kimmel, Onorbe, Moustakas (2012); Peter, Rocha, Bullock, Kaplinghat (2012)

Astrophysics Summary

- Evidence for DM self-interactions on dwarf galaxy scales

$\sigma/m_X \sim 0.1\text{–}10\ \mathrm{cm^2/g}$ for $v \sim 10\text{-}30$ km/s; $\Gamma = n\sigma v \sim H$

- Constraints: elliptical halo shapes; evaporation of subhalos; core collapse; the Bullet Cluster

Peter, Rocha, Bullock, Kaplinghat (2012)

$\sigma/m_X < 1\ \mathrm{cm^2/g}$ for $v \sim 300$ km/s (group)
and $v \sim 3000$ km/s (the Bullet Cluster)

Challenges

$\sigma \sim 1\mathrm{cm^2}\,(m_X/\mathrm{g}) \sim 2\times10^{-24}\ \mathrm{cm^2}\,(m_X/\mathrm{GeV})$

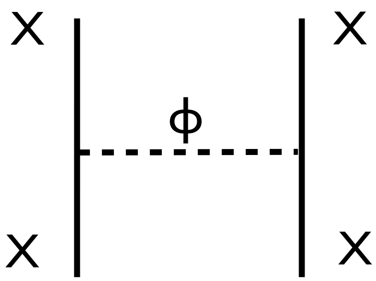
strong interaction
cross section

- A really large scattering cross section!

For WIMPs $\sigma_{EW} \sim 10^{-36}\ \mathrm{cm^2}$

- Avoid constraints

A Light Force Carrier



- A light force mediator is necessary

$$\sigma \approx 5 \times 10^{-23}\,\mathrm{cm}^2 \left(\frac{\alpha_X}{0.01}\right)^2 \left(\frac{m_X}{10\,\mathrm{GeV}}\right)^2 \left(\frac{10\,\mathrm{MeV}}{m_\phi}\right)^4$$

in the perturbative and small velocity limit

SIDM predicts a scale much below the weak scale ~100 GeV

~100 GeV

Go beyond usual WIMPs

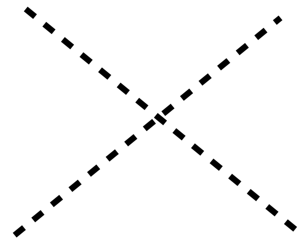
~sub-GeV

In many DM models that are well-motivated for other reasons, there are light mediators and DM candidates can be self-interacting

Scalar Dark Matter

- Scalar DM with self-coupling

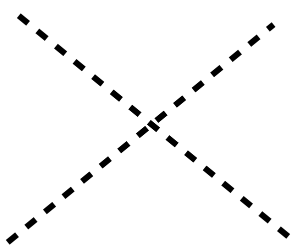
Peebles (2000); Goodman (2000)



- The simplest DM model: add a SM singlet scalar field

$$\frac{1}{2}m^2X^2 + \frac{1}{4}\eta X^4 + \frac{1}{4}\rho X^2(\varphi^\dagger\varphi) - \frac{1}{2}\mu^2\varphi^\dagger\varphi + \frac{1}{4}\lambda(\varphi^\dagger\varphi)^2$$

Silveira, Zee (1985)



$\sim\eta$

$\sigma \sim \eta^2/m^2$

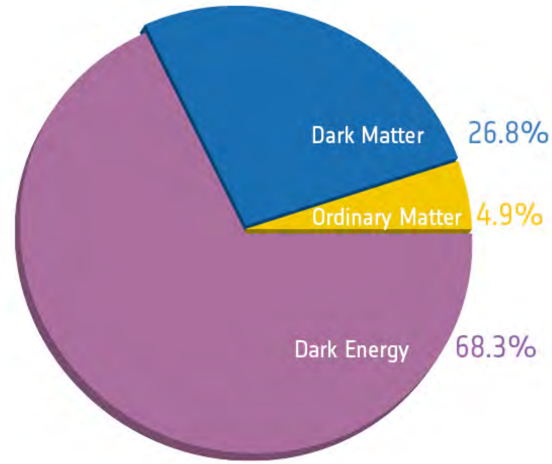
$\sim\rho^2/\lambda$

$\sigma \sim (\rho^2/\lambda)^2/m^2$

Assume O(1) coupling, the DM mass is ~10 MeV

Bento, Bertolami, Rosenfeld, Teodor (2000); Burgess, Pospelov, Veldhuis (2000); Holz, Zee (2001)

Asymmetric Dark Matter



$$\Omega_X/\Omega_B \sim 5$$

- Baryon number asymmetry

$$\eta_B = (n_B - n_{\bar{B}})/n_\gamma \sim 6 \times 10^{-10}$$

- Dark matter asymmetry

$$\eta_X = (n_X - n_{\bar{X}})/n_\gamma \sim \eta_B$$

$$m_X \sim 5 m_B \sim 5 \text{ GeV}$$

Nussinov (1985);Kaplan (1992);Kaplan, Luty, Zurek (2009); Shelton, Zurek (2011); Buckley, Randall (2011); Morrissey, Sigurdson, Tulin (2010)...

DM candidates carry dark strong interactions

Mirror matter as self-interacting DM

Visible world	Mirror world
Baryons	Baryons'

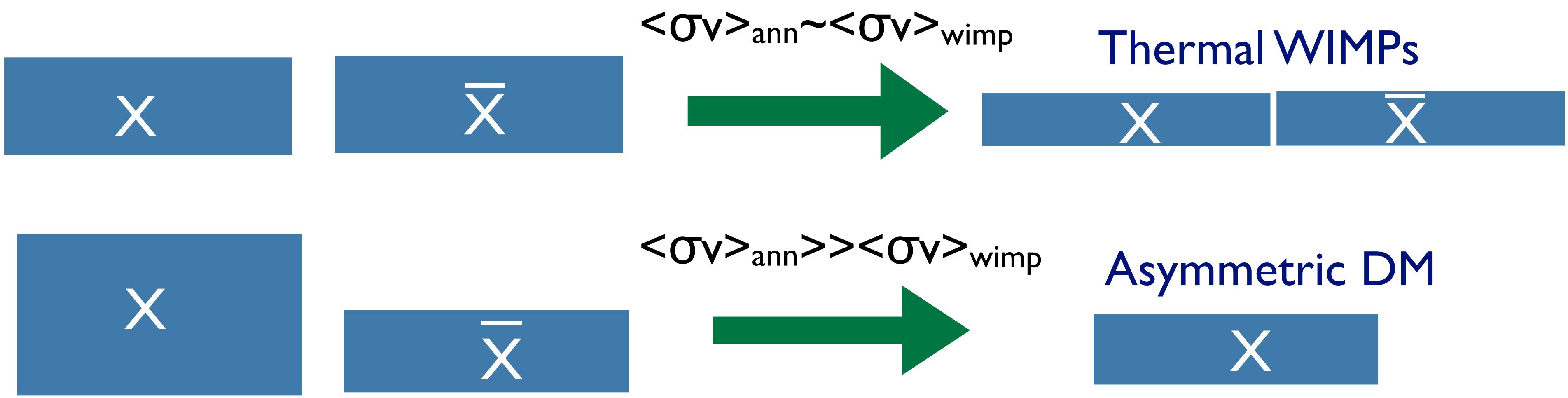
Mohapatra, Teplitz (2000); Mohapatra, Nussinov, Teplitz (2001); Foot (2001); Foot, Volkas (2003)...

↓

use the transverse cross section

Asymmetric Dark Matter

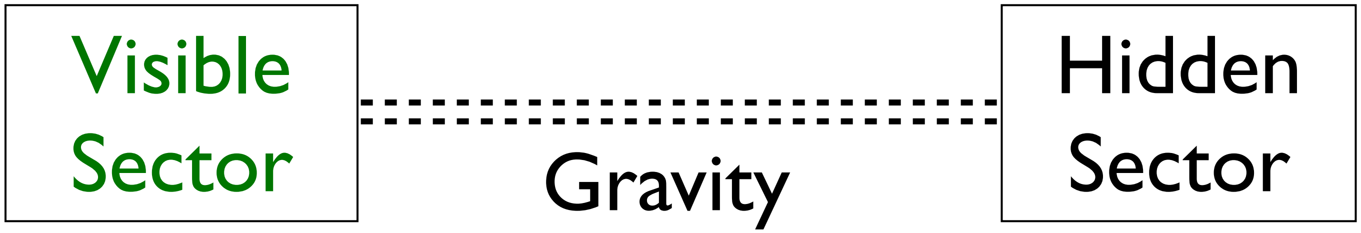
- ADM favors light mediators $\quad \eta_X = (n_X - n_{\bar{X}})/n_\gamma \sim \eta_B$



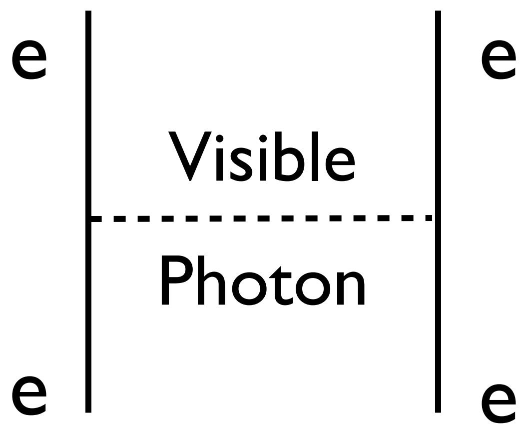
Need a large annihilation cross section

Buckley (2011); Lin, HBY, Zurek (2011)

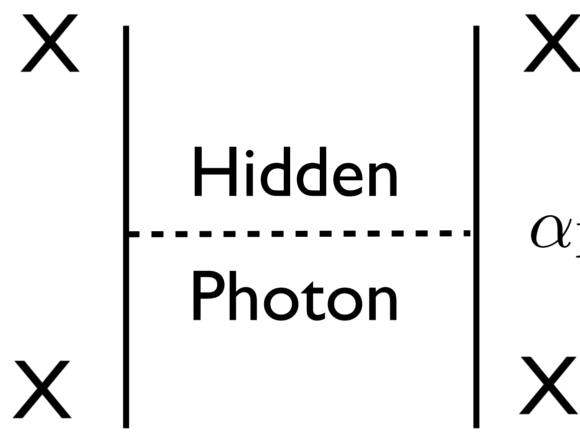
Hidden Charged Dark Matter



- An example: hidden charged dark matter



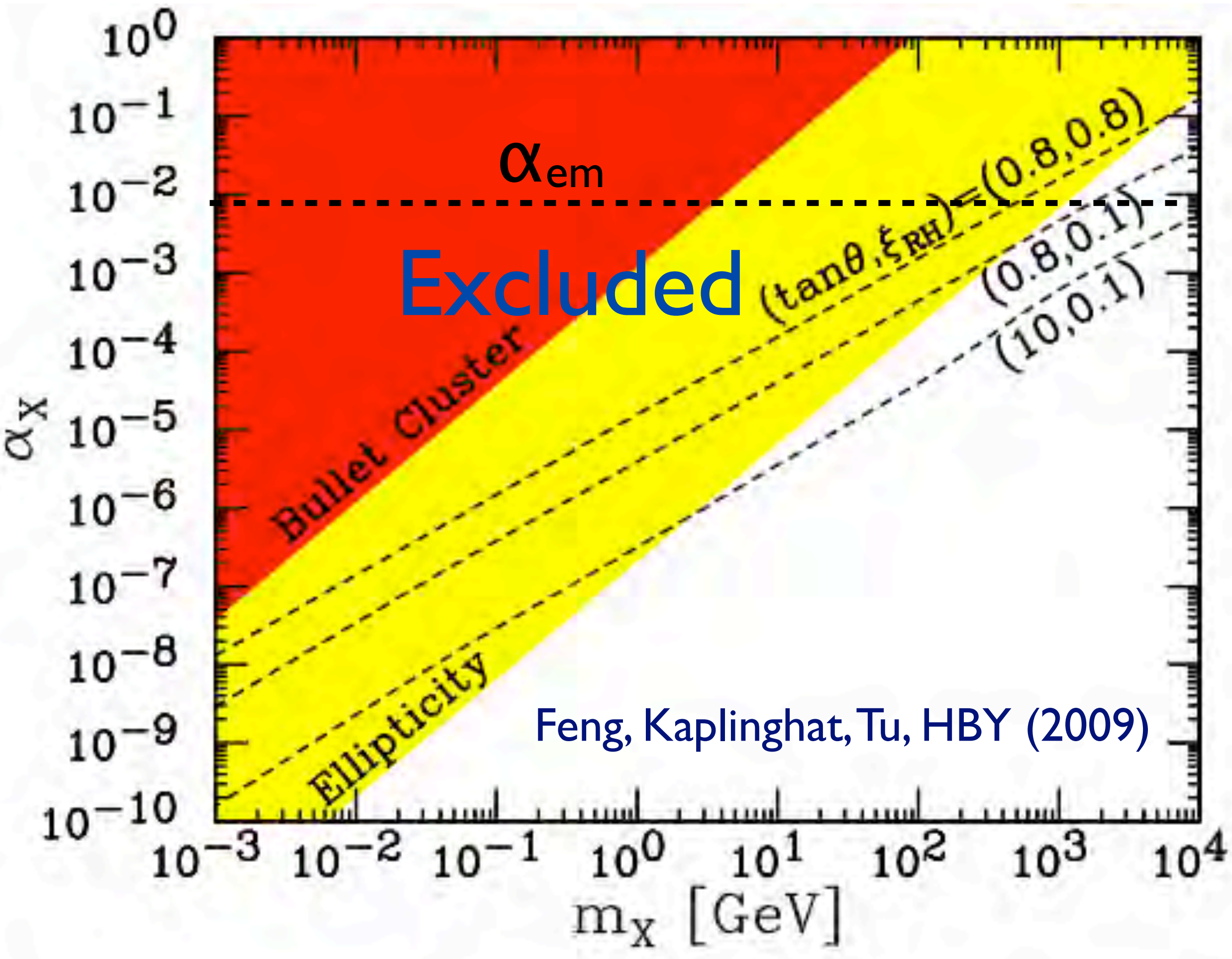
$$\alpha_{\rm em} = \frac{1}{137}$$

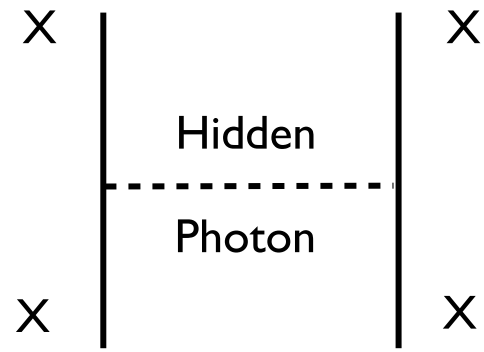


$$\alpha_X = ?$$

Feng, Tu, HBY (2008); Ackerman, Buckley, Carroll, Kamionkowski (2008); Feng, Kaplinghat, Tu, HBY (2009)

Hidden Charged Dark Matter





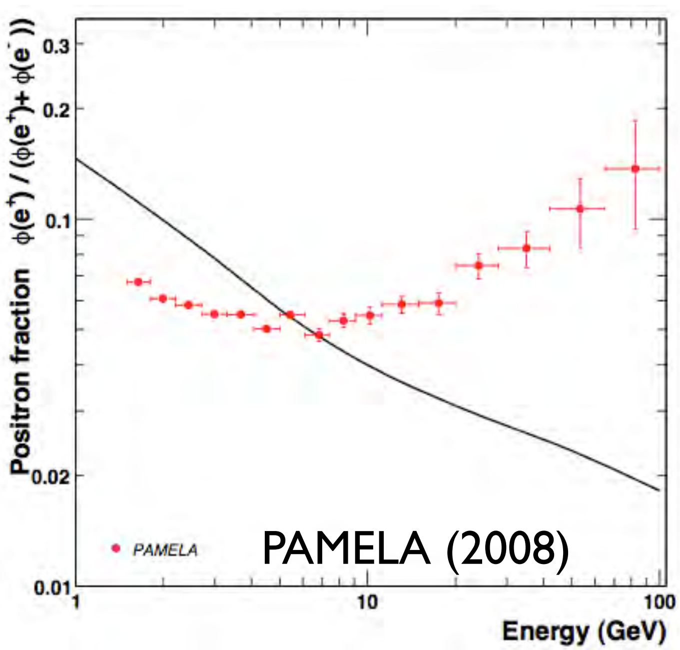
$$\alpha_X = ?$$

$$\sigma \sim \frac{\alpha_X^2}{m_X^2 v^4}$$

v~3000 km/s (Bullet Cluster)
v~200 km/s (NGC720)
Note: v-dependence

- The SS paper assumed a constant self-scattering cross section
- With a light mediator, velocity-dependence is a general feature of scattering

Models Motivated by PAMELA



To fit the data

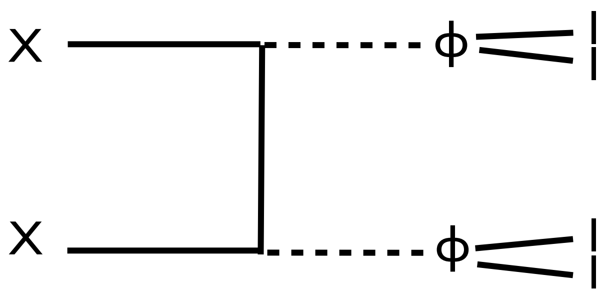
$\langle\sigma v\rangle_{ann} \sim 100\text{-}1000 \langle\sigma v\rangle_{wimp}$

But the relic density is too small

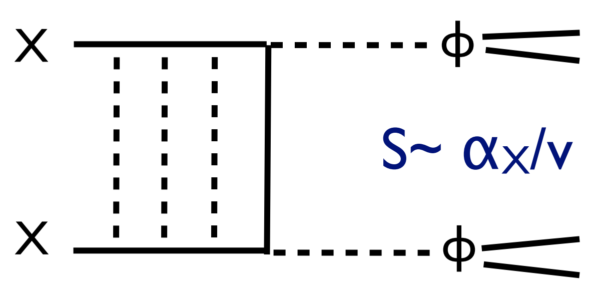
$\Omega_X \approx 0.27 \langle\sigma v\rangle_{wimp} / \langle\sigma v\rangle_{ann}$

Boost DM annihilation now, but not in the early Universe

Arkani-Hamed, Finkbeiner, Slatyer, Weiner (2008); Pospelov, Ritz (2008)

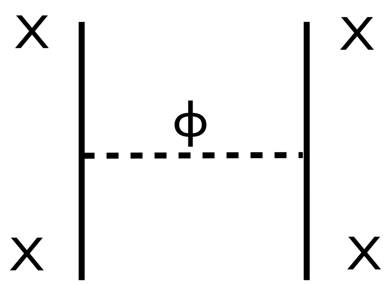


If $m_\phi \sim$ sub-GeV



$S \sim \alpha_X / v$

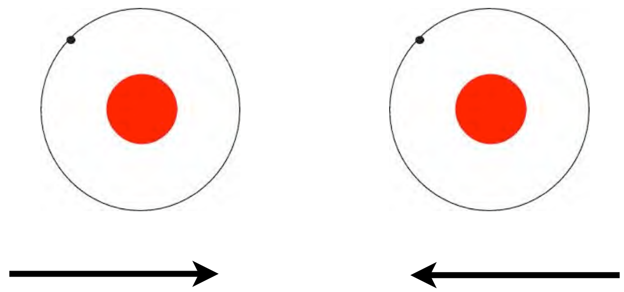
The same ϕ can mediate DM self-interactions



Feng, Kaplinghat, HBY (2009); Buckley, Fox (2009); Loeb, Weiner (2010)

Atomic Dark Matter

- Dark atoms can have self-interactions (not too dissipative)



Old history (related to ADM): mirror hydrogen atoms

Explain DAMA

Foot (2003); Kaplan, Krnjaic, Rehermann, Wells (2009)
Cline, Liu, Wei Xue (2012)

Interesting cosmology: dark CMB; dark acoustic oscillation; late kinetic decoupling

Feng, Kaplinghat, Tu, HBY (2009)

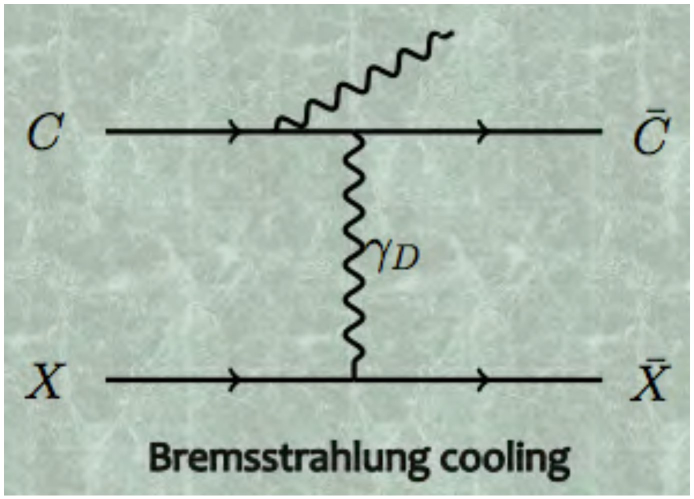
Francis-Yan Cyr-Racine, Kris Sigurdson (2013)

Francis-Yan' talk; Kris's talk

Double-Disk Dark Matter

- 5% DM is dissipative and forms a dark disk (3DM)

Fan, Katz, Randall, Reece (2013)   McCullough,Randall (2013)



“e”

“p”

From Fan`s slides

From Fan`s slides

Implications for direct/indirect detections
See Lisa`s talk for details

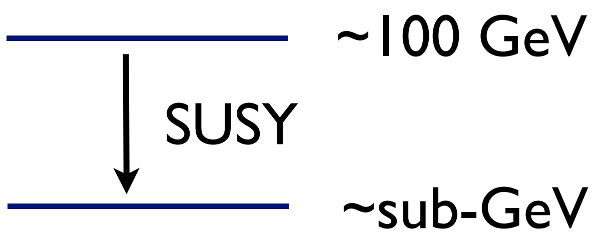
Partially self-interacting DM!

What Supersymmetry Can do?

- In typical SUSY models, neutralinos are DM candidates
- They have negligible self-interactions

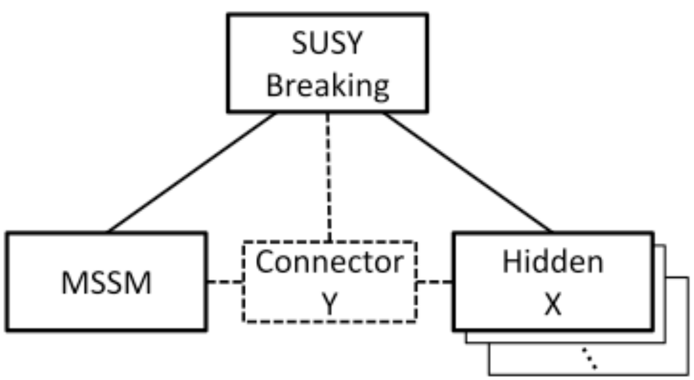
But SUSY can still play an important role for SIDM

- Generate the sub-GeV scale naturally



Katz, Sundrum (2009); Cheung, Ruderman, Wang, Yavin; Morrisey, Poland, Zurek (2009)

- Keep the WIMP miracle in the dark sector



Kumar, Feng (2008)

$$\Omega_X \sim \frac{1}{\langle \sigma v \rangle} \sim \frac{m_X^2}{\alpha_X^2} \sim \frac{m_W^2}{\alpha_W^2}$$

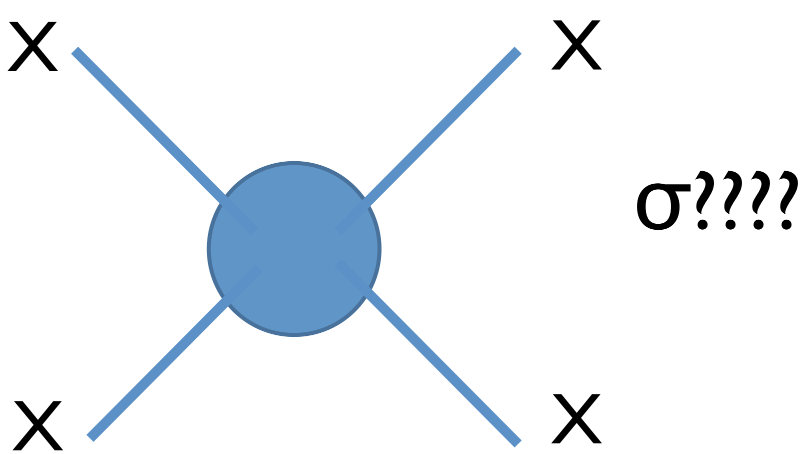
e.g. GMSB; AMSB

We even do not need to sacrifice the WIMP miracle

Feng, Tu, HBY (2008)

A Short Summary

- In many well-motived DM models, DM candidates are self-interacting
- How to calculate the DM self-scattering cross section given particle physics parameters ?

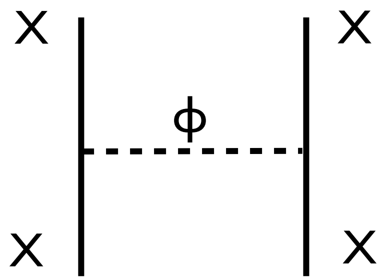


$$V(r) = \pm \frac{\alpha_X}{r} e^{-m_\phi r}$$

Feng, Kaplinghat, Tu, HBY (2009) JCAP
Feng, Kaplinghat, HBY (2009) PRL
Lin, HBY, Zurek (2011) PRD
Tulin, HBY, Zurek (2012) PRL
Tulin, HBY, Zurek (2013) PRD

- Cold DM is non-relativistic, and the usual Born approximation breaks down in most cases

A Simplified Model



$$\mathscr{L}_{\rm int} = \begin{cases} g_X \bar{X} \gamma^\mu X \phi_\mu & \text{vector mediator} \\ g_X \bar{X} X \phi & \text{scalar mediator} \end{cases}$$

More examples: Bellazzini, Cliche, Tanedo (2013)

A Yukawa potential

$$V(r) = \pm \frac{\alpha_X}{r} e^{-m_\phi r}$$

$$\alpha_X = g_X^2/(4\pi)$$

$$\sigma_T = \int d\Omega \, (1 - \cos\theta) \, \frac{d\sigma}{d\Omega}$$

regulate forward scattering

Map out the parameter space (m_X, m_ϕ, α_X)

- Solve small scale anomalies (small v)
- Avoid constraints on large scales (large v)
- Get the relic density right

Scattering with a Yukawa Potential

$$V(r) = \pm \frac{\alpha_X}{r} e^{-m_\phi r}$$

DM self-scattering

- **Perturbative (Born) regime** ✓
  $\alpha_X m_X / m_\phi \ll 1$

  Feng, Kaplinghat, HBY (2009)

  Lin, HBY, Zurek (2011)

- **Nonperturbative regime**
  $\alpha_X m_X / m_\phi \gtrsim 1$
  - **Classical regime**
    $m_X v / m_\phi \gg 1$
  - **Resonant regime**
    $m_X v / m_\phi \lesssim 1$

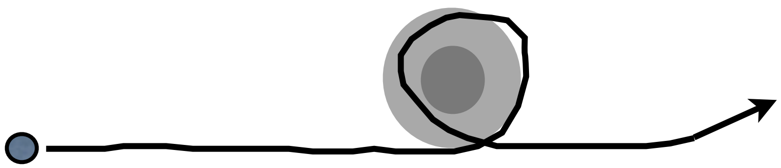
Exception: $m_\phi = 0$

Feng, Kaplinghat, Tu, HBY (2009)

Classical Regime

- Classical approximation from plasma physics

Khrapak et al. (2003) (2004)



Charged-particle scattering in plasma

$$\pm \frac{\alpha_X}{r} e^{-m_\phi r} \qquad \alpha_X = \alpha_{\rm EM}$$

$$m_\phi = \text{Debye photon mass}$$

$$\sigma_T^{\rm clas} \approx \begin{cases} \frac{4\pi}{m_\phi^2}\beta^2 \ln\left(1+\beta^{-1}\right) & \beta \lesssim 10^{-1} \\ \frac{8\pi}{m_\phi^2}\beta^2 / \left(1+1.5\beta^{1.65}\right) & 10^{-1} \lesssim \beta \lesssim 10^3 \\ \frac{\pi}{m_\phi^2}\left(\ln\beta + 1 - \frac{1}{2}\ln^{-1}\beta\right)^2 & \beta \gtrsim 10^3 \end{cases}$$

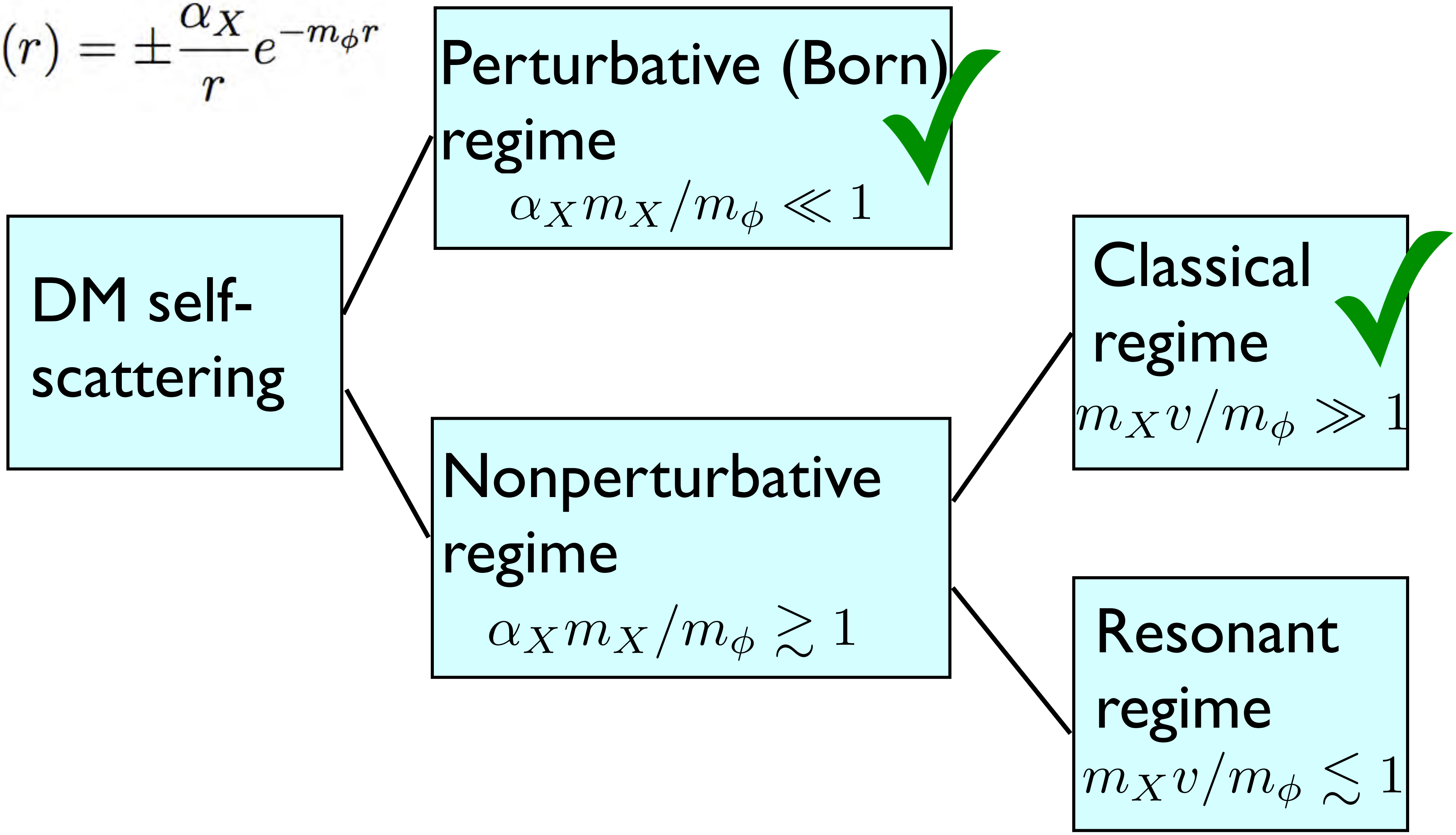
$$\beta \equiv 2\alpha_X m_\phi / (m_X v^2)$$

$\sigma_T \sim v^{-4}$ at large v
σ_T ~const at small v (saturated)

Apply to DM: σ_T is enhanced on dwarf scales compared to larger scales
Feng, Kaplinghat, HBY (2009); Loeb, Weiner (2010); Vogelsberger, Loeb, Zavala (2012)...

Beyond Perturbation

$$V(r) = \pm \frac{\alpha_X}{r} e^{-m_\phi r}$$



Numerical Approach

- Quantum mechanics 101-partial wave analysis

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR_\ell}{dr}\right)+\left(k^2-\frac{\ell(\ell+1)}{r^2}-2\mu V(r)\right)R_\ell=0$$

- Transfer cross section

See also: Buckley, Fox (2009)

$$\frac{d\sigma}{d\Omega}=\frac{1}{k^2}\left|\sum_{\ell=0}^{\infty}(2\ell+1)e^{i\delta_\ell}P_\ell(\cos\theta)\sin\delta_\ell\right|^2 \qquad \sigma_T = \int d\Omega\,(1-\cos\theta)\,\frac{d\sigma}{d\Omega}$$

$$\frac{\sigma_T k^2}{4\pi}=\sum_{\ell=0}^{\infty}[(2\ell+1)\sin^2\delta_\ell-2(\ell+1)\sin\delta_\ell\sin\delta_{\ell+1}\cos(\delta_{\ell+1}-\delta_\ell)]$$

Rearrange ell→ell+1

$$\frac{\sigma_T k^2}{4\pi}=\sum_{\ell=0}^{\infty}(\ell+1)\sin^2(\delta_{\ell+1}-\delta_\ell)$$

✓

Both formulas are identical in the limit of ell→∞

But the second one converges much faster

Numerical Approach

- Partial wave analysis

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR_\ell}{dr}\right) + \left(k^2 - \frac{\ell(\ell+1)}{r^2} - 2\mu V(r)\right)R_\ell = 0$$

- Boundary conditions $r \to \infty$

$$R_\ell(r) \to \sin(kr - \pi\ell/2 + \delta_\ell)/r$$

$$R_\ell(r) \to \cos\delta_\ell j_\ell(kr) - \sin\delta_\ell n_\ell(kr)$$ ✓

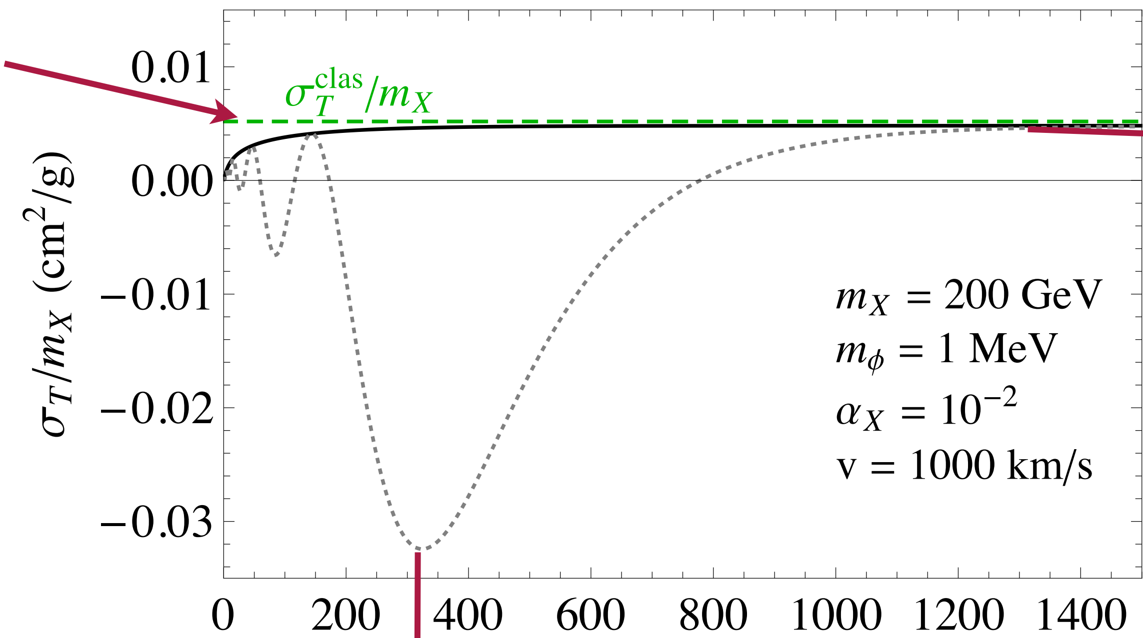
The second one is much more efficient

Numerical Approach

- Classical regime

Tulin, HBY, Zurek (2013)

Plasma



$$\sum_{\ell=0}^{\infty}(\ell+1)\sin^2(\delta_{\ell+1}-\delta_\ell)$$

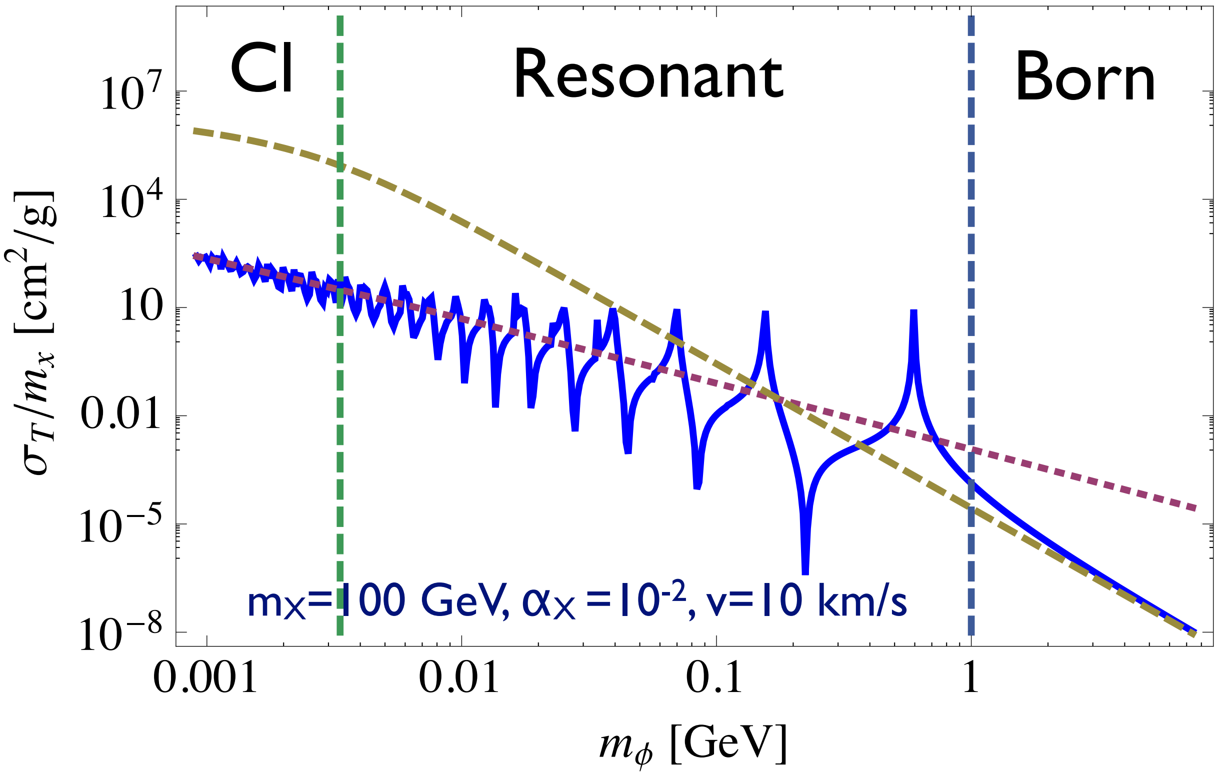
$\ell_{\max}$

$$\sum_{\ell=0}^{\infty}[(2\ell+1)\sin^2\delta_\ell - 2(\ell+1)\sin\delta_\ell\sin\delta_{\ell+1}\cos(\delta_{\ell+1}-\delta_\ell)]$$

We have confirmed the analytical formula from plasma physics

Numerical Approach

- All regimes



Solid: numerical; Dashed: Born; Dotted: plasma

In the resonant regime, the cross section can be enhanced or suppressed

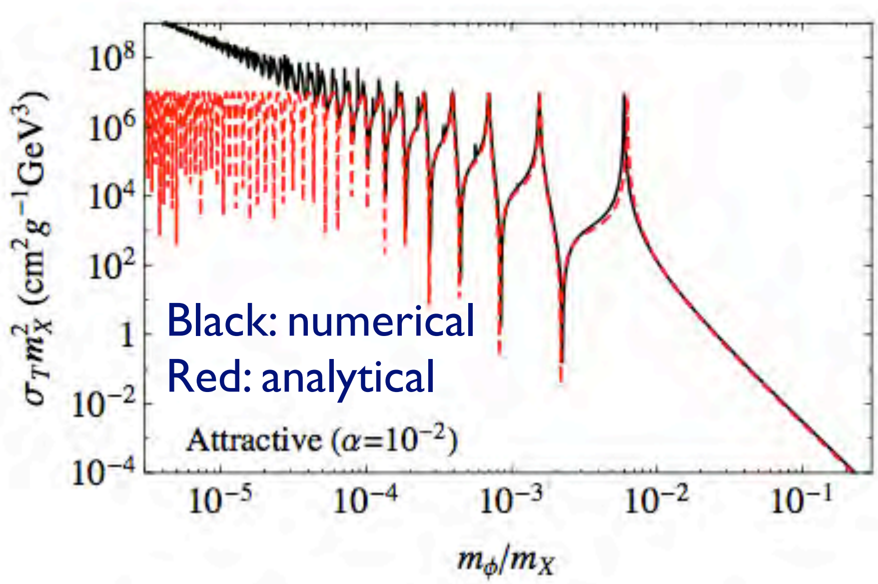
Analytical Approach

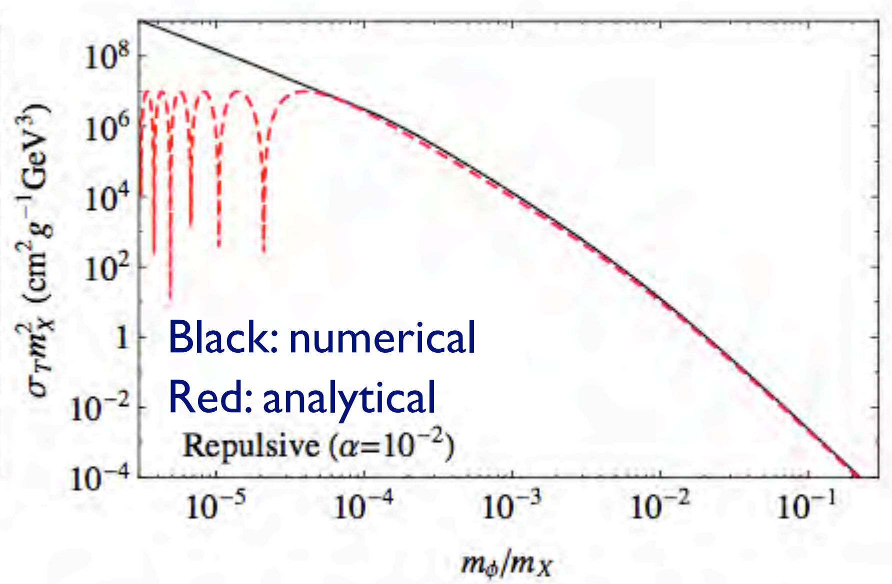
$$V(r) = \pm \frac{\alpha_X}{r} e^{-m_\phi r} \longrightarrow V(r) = \pm \frac{\alpha_X \delta\, e^{-\delta r}}{1 - e^{-\delta r}} \qquad \delta = \kappa m_\phi \qquad \kappa \simeq 1.6$$

Hulthén potential

The Schrödinger equation is solvable analytically for ell=0

$$\sigma_T^{\text{Hulthén}} = \frac{16\pi}{m_X^2 v^2} \sin^2 \delta_0 \qquad \delta_0 = \arg\left(\frac{i\,\Gamma\left(\frac{i m_X v}{\kappa m_\phi}\right)}{\Gamma(\lambda_+)\Gamma(\lambda_-)} \right), \quad \lambda_\pm \equiv \begin{cases} 1 + \frac{i m_X v}{2\kappa m_\phi} \pm \sqrt{\frac{\alpha_X m_X}{\kappa m_\phi} - \frac{m_X^2 v^2}{4\kappa^2 m_\phi^2}} & \text{attractive} \\ 1 + \frac{i m_X v}{2\kappa m_\phi} \pm i\sqrt{\frac{\alpha_X m_X}{\kappa m_\phi} + \frac{m_X^2 v^2}{4\kappa^2 m_\phi^2}} & \text{repulsive} \end{cases}$$

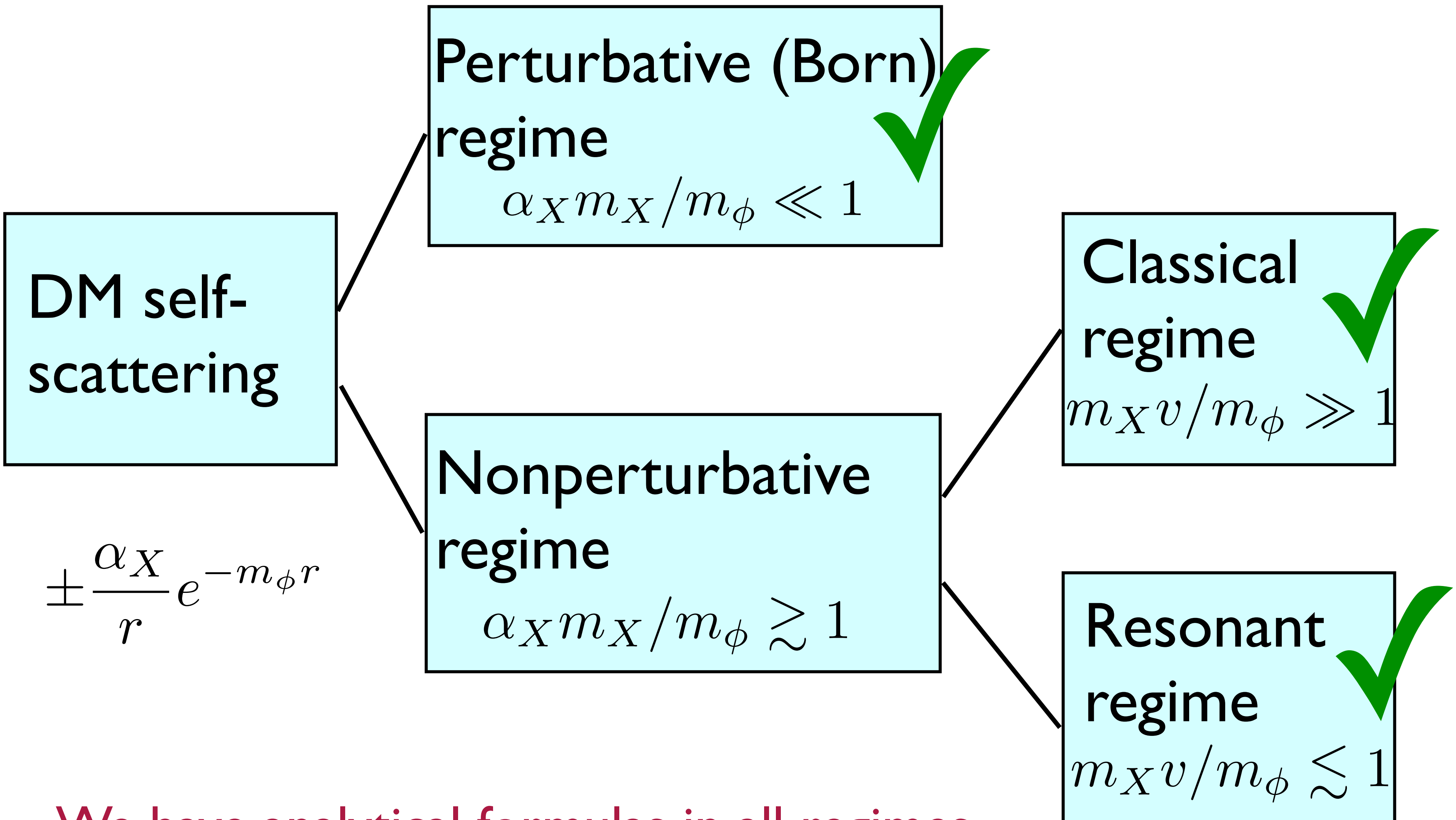




It is useful for simulations

Tulin, HBY, Zurek (2013)

Beyond Perturbation



We have analytical formulas in all regimes

Velocity Dependence

- σ_T has a rich structure

Tulin, HBY, Zurek (2012)

Born regime: σ_T~const below MW scales

Classical regime: σ_T increases on small scales

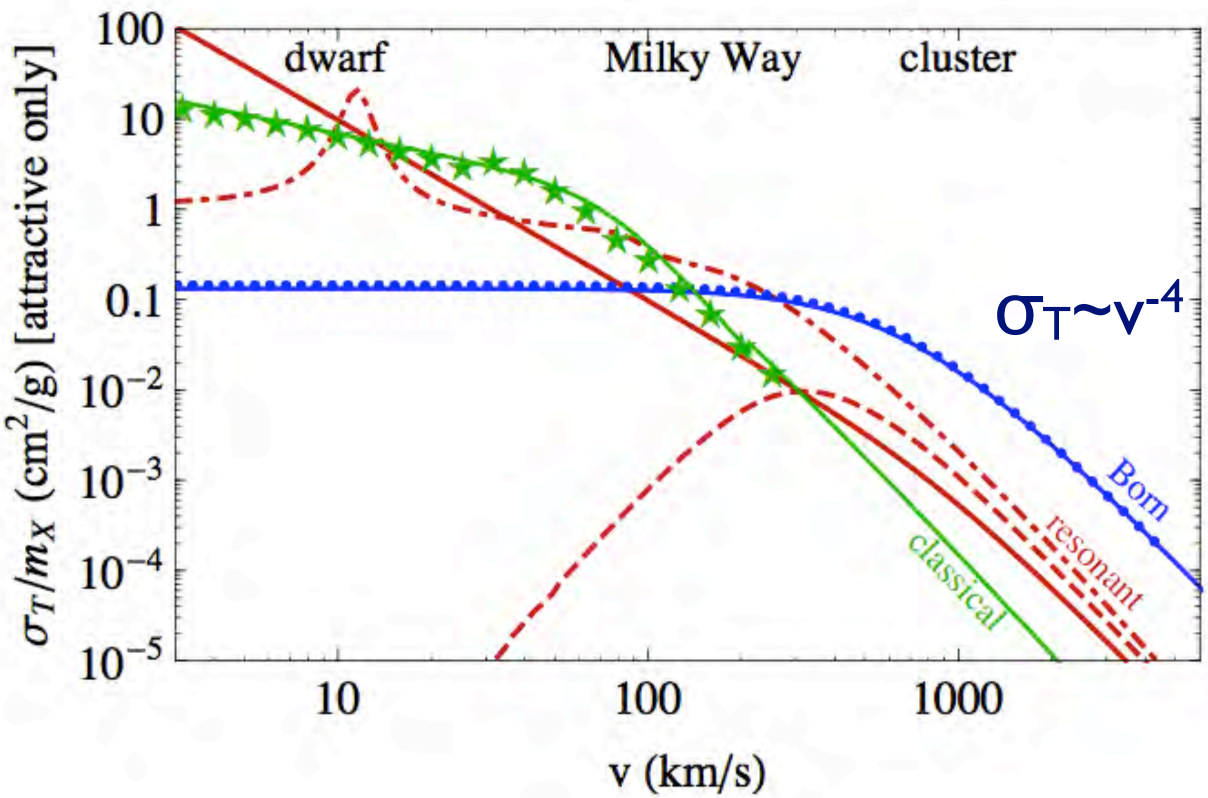
★: numerical

Resonant regime:

s-wave: $\sigma_T \sim v^{-2}$

p-wave

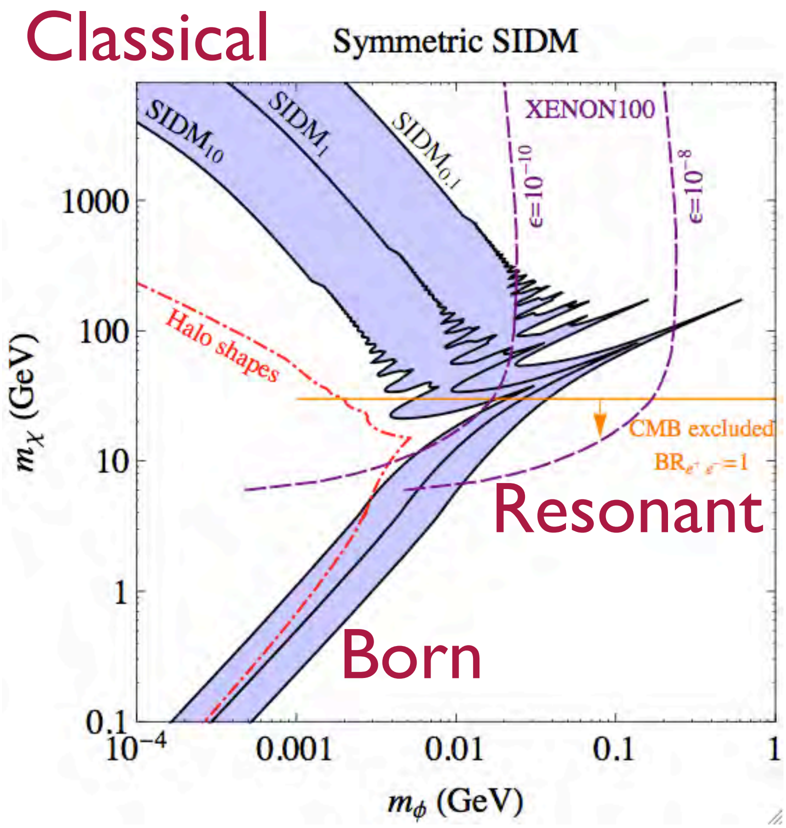
anti-resonance

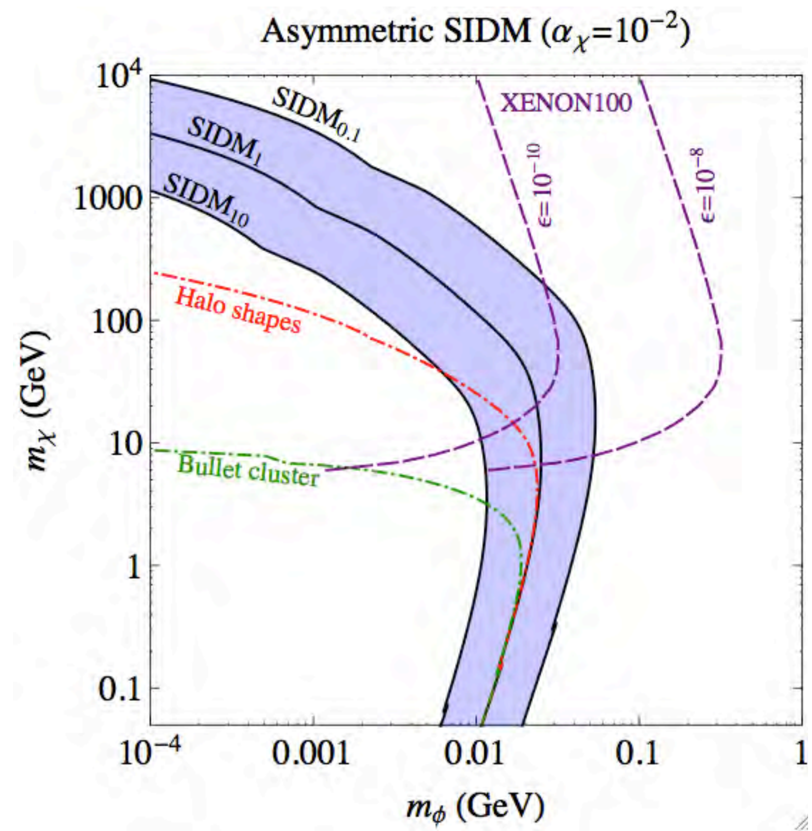


$\sigma_T \sim v^{-4}$

- In many cases, σ_T is enhanced on dwarf scales
- This helps us avoid constraints on MW and cluster scales

Dark Force Parameter Space



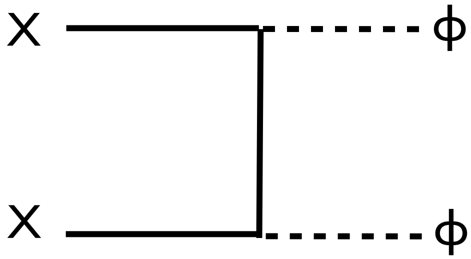


dw: dwarf (30 km/s)
halo shapes: (300 km/s)
cl: cluster (3000 km/s)

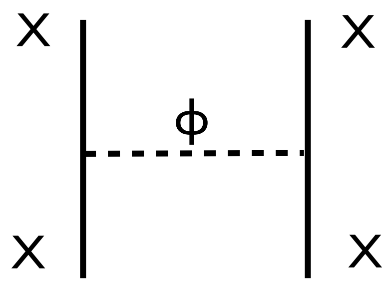
Fix α_X by $\Omega_X \approx 0.27$

shaded region: explain small scale anomalies
heavy SIDM: σ has a strong v-dependence
light SIDM: constant cross section limit

A Super Model

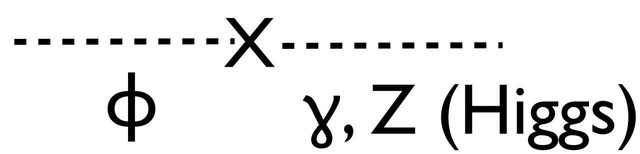


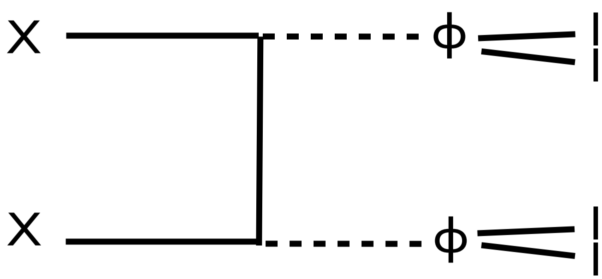
Relic density



Self-interactions

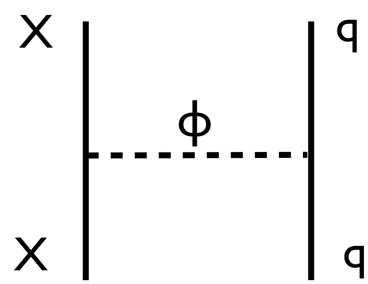
When ϕ couples to the SM sector





Indirect detection

Manoj`s talk



Direct detection

Sean`s talk

Implications

- Indirect detection

Particle physics

$$\frac{d\Phi(b,\ell)}{dE} = \frac{\langle \sigma_{\rm A} v \rangle}{2} \frac{J(b,\ell)}{J_0} \frac{1}{4\pi m_\chi^2} \frac{dN_\gamma}{dE}$$

$$J(b,\ell) = J_0 \int d\,x\ \rho^2(r_{\rm gal}(b,\ell,x))$$

Astrophysics

Implications

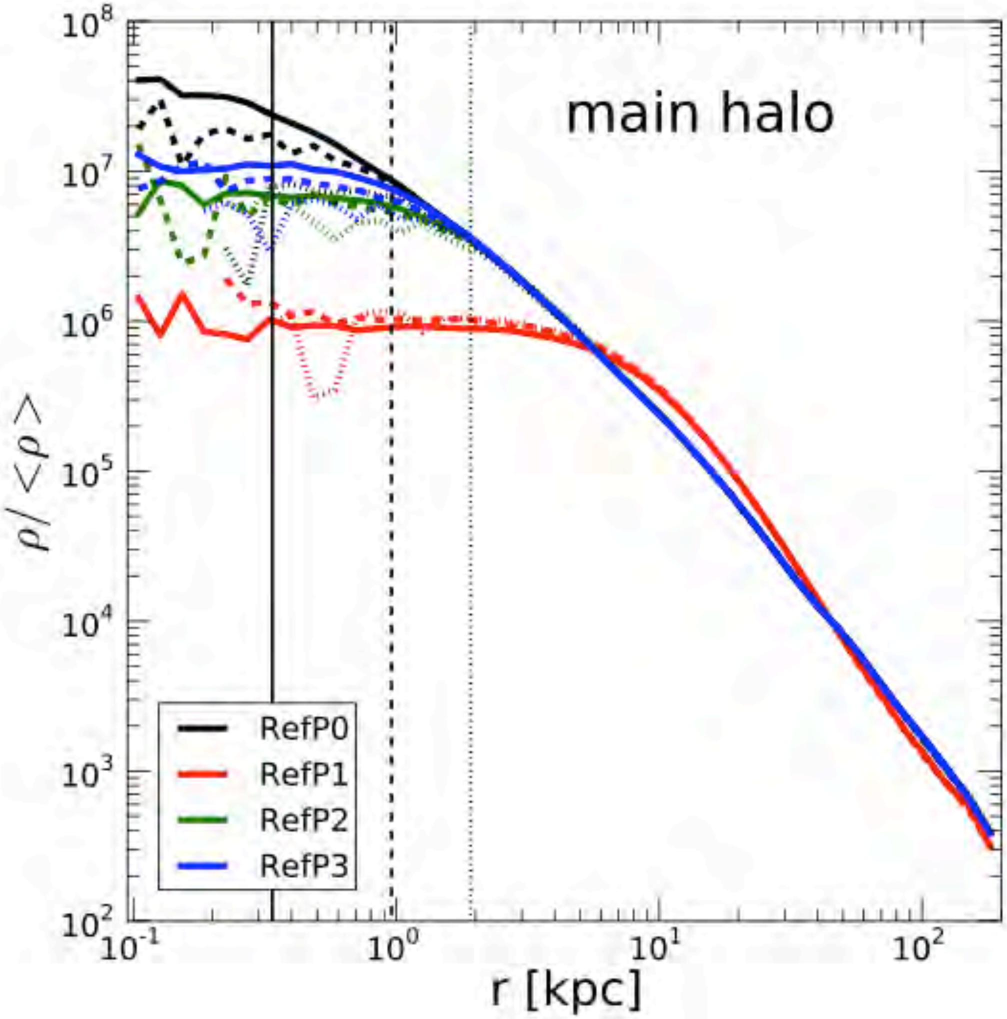
- Indirect detection

Name	Type	$\sigma_T^{\max}/m_\chi$ [cm^2 g^{-1}]	$v_{\max}$ [km s^{-1}]
RefP0	CDM	/	/
RefP1	SIDM (ruled out)	10	/
RefP2	vdSIDM (allowed)	3.5	30
RefP3	vdSIDM (allowed)	35	10

$$J(b,\ell) = J_0 \int d\,x\ \rho^2(r_{\rm gal}(b,\ell,x))$$

also depends on particle physics parameters (m_χ, m_ϕ, α_χ)

Vogelsberger, Zavala, Loeb (2012)

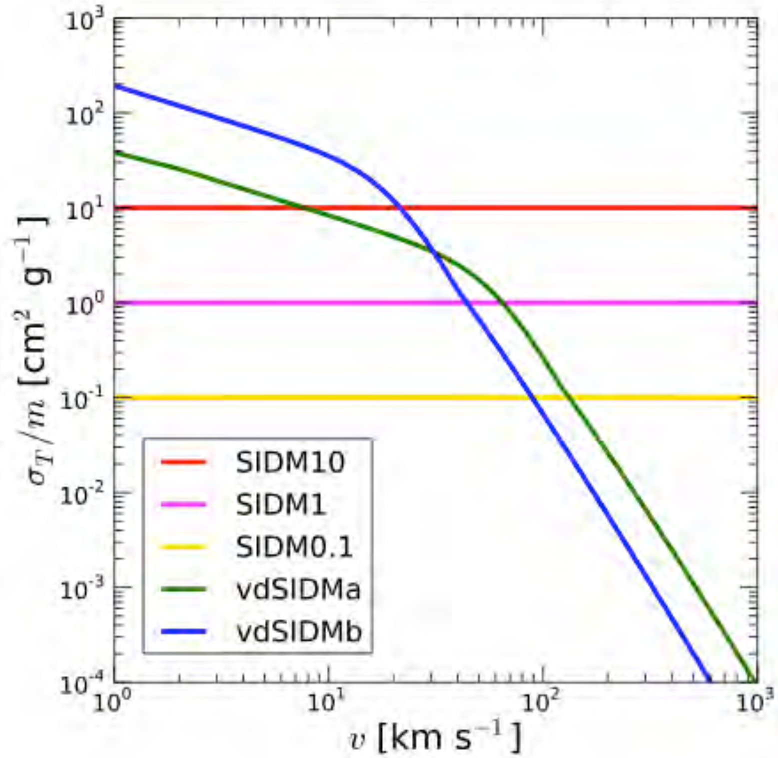


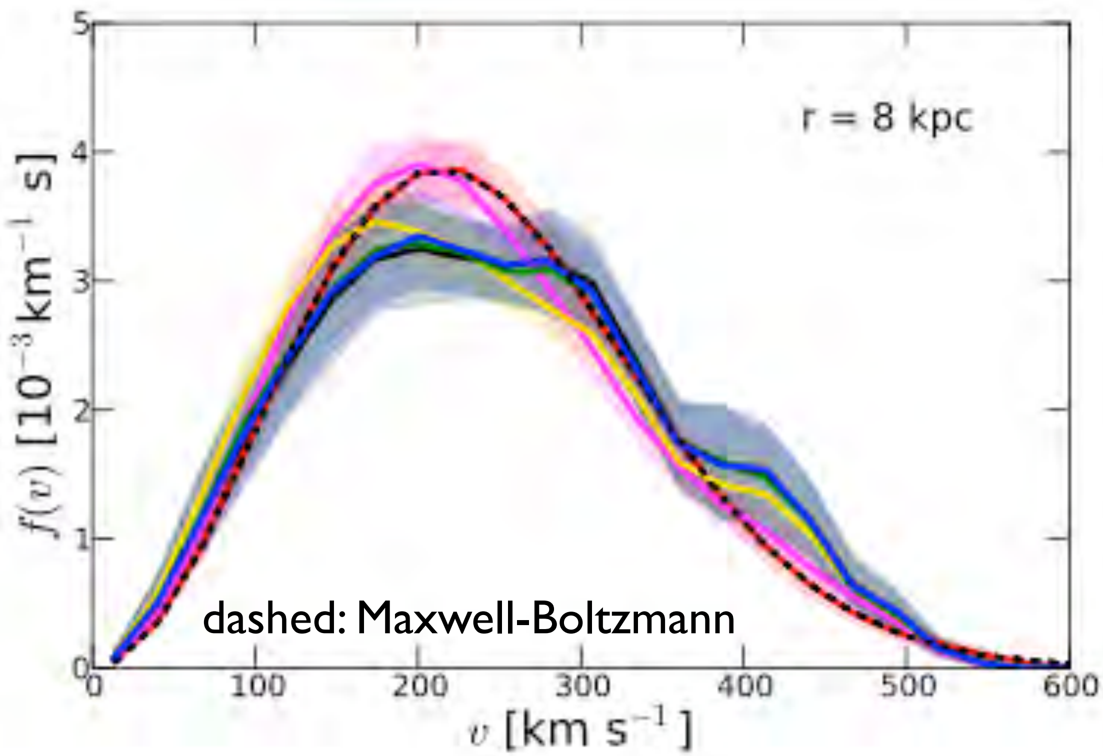
We need add baryons!

Implications

- Direct detection

Vogelsberger, Zavala (2013)





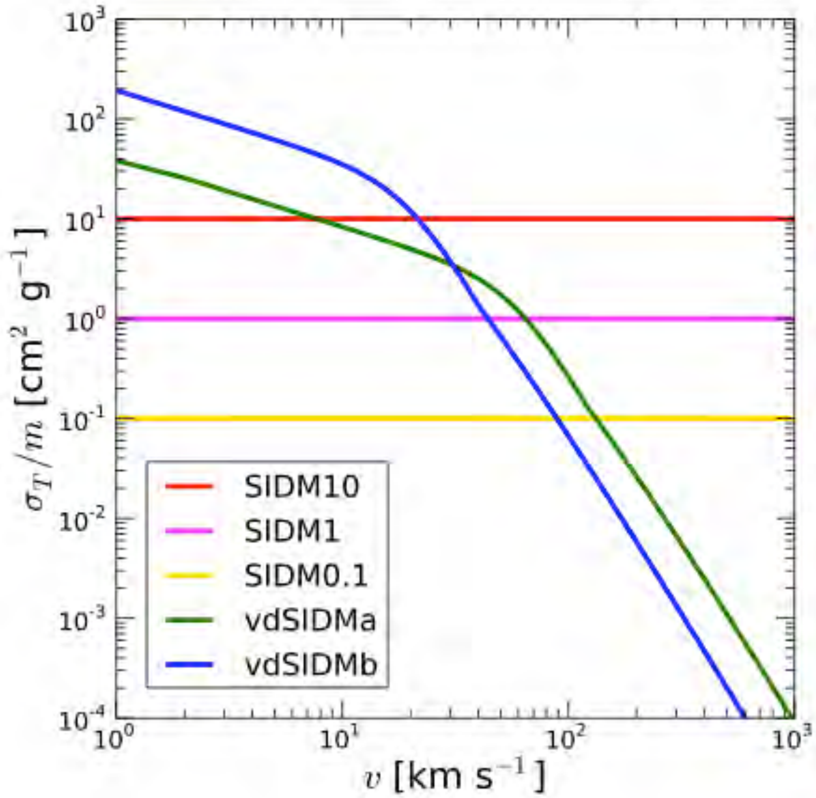
$$T(v_{\min}(E),t)=\int_{v_{\min}(E)}^{\infty}\frac{f_v(t)}{v}\,\mathrm{d}v,\; v_{\min}(E)=\left(\frac{E\,(m_\chi+m_N)^2}{2m_\chi^2 m_N}\right)^{1/2},$$

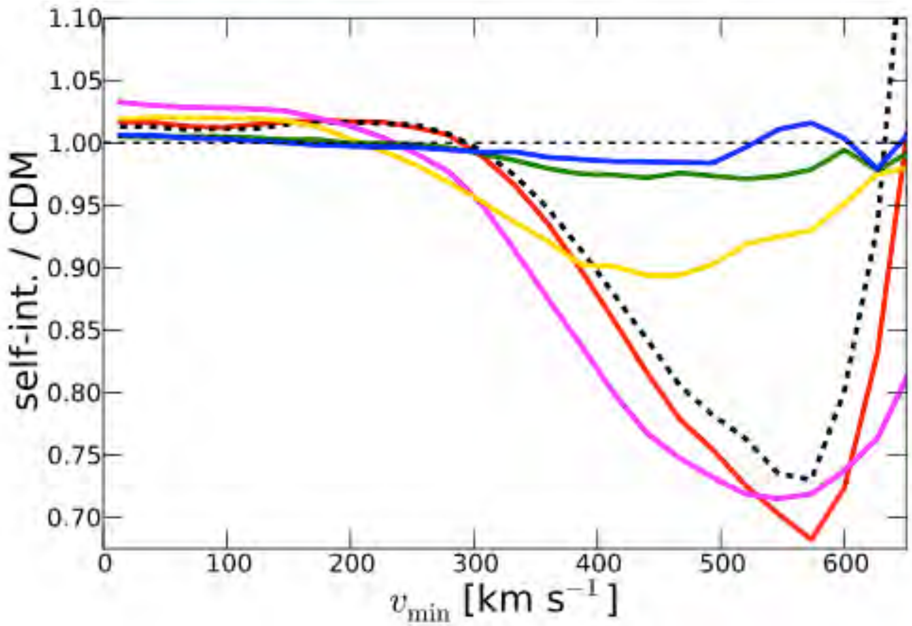
DM self-interactions drive the DM phase space distribution towards to a Maxwell-Boltzmann distribution

Implications

- Direct detection

$$T(v_{\min}(E),t)=\int_{v_{\min}(E)}^{\infty}\frac{f_v(t)}{v}\mathrm{d}v,\ v_{\min}(E)=\left(\frac{E\,(m_\chi+m_N)^2}{2m_\chi^2 m_N}\right)^{1/2},$$



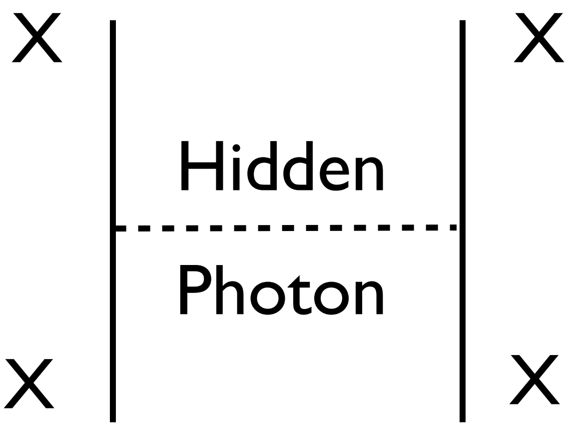


Vogelsberger, Zavala (2013)

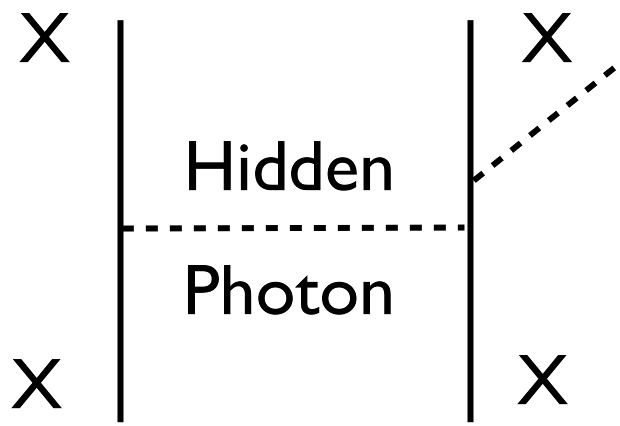
DM self-interactions drive the DM phase space distribution towards to a Maxwell-Boltzmann distribution

A Few More Comments

- SIDM can be non-dissipative

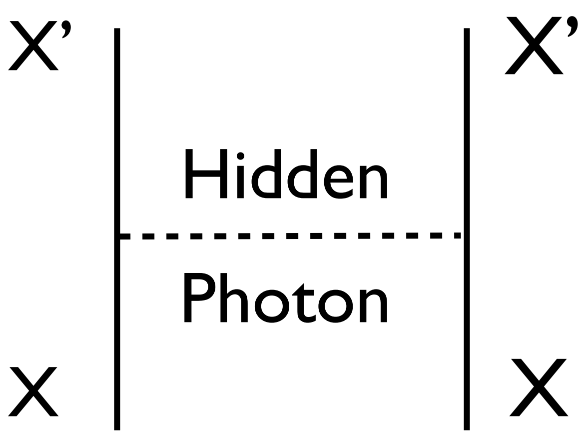


$\Gamma = n\sigma v \sim H$



$\Gamma = \alpha_X n\sigma v << H$ as long as $\alpha_X < 1$

- Inelastic DM



up scattering: bad

$\Delta m << m_X v^2$

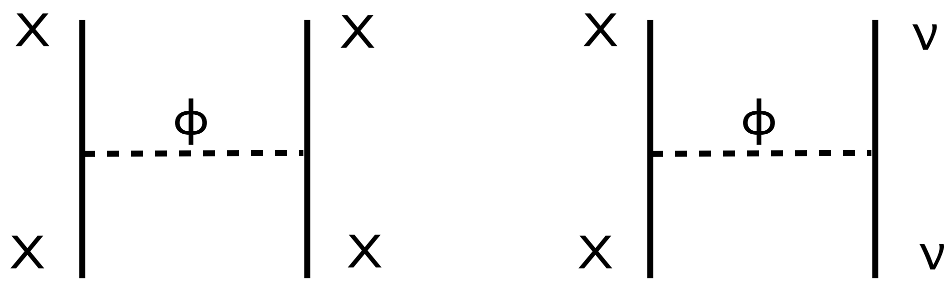
But v~10-30 km/s in dwarfs

For TeV DM, $\Delta m < 1$-10 keV

down scattering: good

A Few More Comments

- "Warm" SIDM?

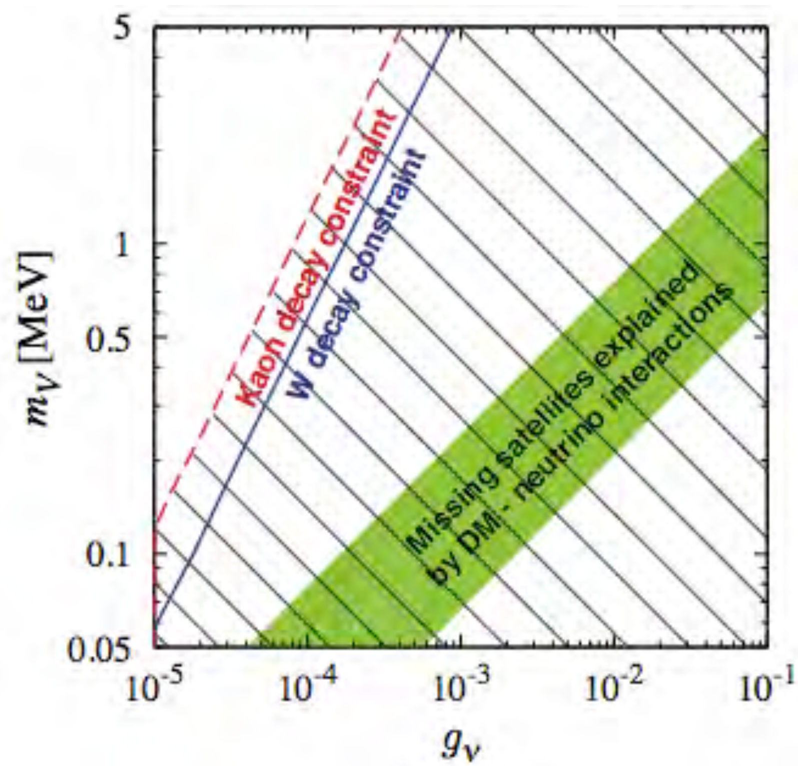


Make DM "warm"

late kinetic decoupling

suppress power spectrum

van den Aarsen, Bringmann, Pfrommer (2012)



Heavy "warm" DM

The original model was killed because it is not SU(2) invariant

Laha, Dasgupta, Beacom (2013)

Conclusions

- No reason to believe DM has to be collisionless
- We have solved the scattering problem with a Yukawa potential completely
- With a light dark force (with one coupling α_X)
  - Explain anomalies on dwarf galaxy scales
  - Satisfy bounds on larger scales
  - Provide the correct DM relic density
- Implications for indirect/direct detection