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Resolving the Hubble Tension by Taking Gravitationally Bound Space Into Account

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Abstract

The Hubble tension arises from the difference between direct measurements of the Hubble constant and indirect measurements, given a cosmological model. Measurements have been confirmed with increasing precision, pointing to an issue with the cosmological model. However, the simplest Lambda cold dark matter model provides a good fit for a large span of cosmological data. In this paper, we keep the Lambda cold dark matter model but modify it to consider the possible effect of gravitationally bound space. Modeling shows that as large gravitationally bound structures — namely, galaxy clusters — develop and gravitationally bind the space they enclose, their impact on the universe's rate of expansion resolves the Hubble tension.

KEY WORDS:

Hubble constant, Cosmological parameters, Dark energy, Cosmological constant, Galaxy clusters, Cosmological models.

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1. Introduction

The increasing precision of cosmological measurements has revealed a discrepancy known as the Hubble tension (see Abdalla et al. 2022 for a review)[1]. The Hubble tension refers to the difference between direct measurements of the Hubble constant (H_0) and indirect measurements, given a cosmological model. This tension reaches 5σ between the values obtained using the cosmic microwave background (CMB) data from Planck for the Lambda cold dark matter (ΛCDM) model (Planck Collaboration 2020)[2] and from the Cepheid-calibrated Type Ia supernovae of the SH0ES project (Riess et al. 2022)[3].

While systematic errors are considered a possible cause for the tension, the high precision and consistency of the data at both ends — late universe measurements, such as the Cepheid-calibrated Type Ia supernovae, and early universe measurements from the CMB — make this unlikely (for a review of different measurements, see Abdalla et al. 2022)[4]. In particular, for late universe measurements, recent JWST observations provide the strongest evidence yet that systematic errors in Hubble Space Telescope Cepheid photometry do not play a significant role in the present Hubble tension (Riess et al. 2023)[5].

Thus, there is growing interest in the possibility that this tension points to a model problem (Abdalla et al. 2022)[6]. However, the simplest ΛCDM model provides a good fit for a large span of cosmological data, so significant alterations are not appropriate.

Fundamentally, the CMB data necessitate that the universe expand by a certain amount so that our current universe's large-scale clustering of galaxies matches the CMB imprint of the structure after forward extrapolation with the ΛCDM model. This expansion is produced by a ΛCDM model with a Hubble constant H_0 of 67.4 ± 0.5 km s^{-1} Mpc^{-1} (Planck Collaboration 2020, 2021)[7][8]. On the other hand, direct local measurements employing parallax and extended measurements — for example, using Type Ia supernovae — as far as 10 billion years back are best fit by the ΛCDM model with an H_0 of 73 ± 1 km s^{-1} Mpc^{-1} (Riess et al. 2022)[9]. Herein, we will call these two models respectively ΛCDM67 and ΛCDM73.

The cosmological constant, Λ, in the ΛCDM model was added to account for the accelerated expansion of the universe required to fit the late universe measurements of the Cepheid-calibrated Type Ia supernovae. Originally proposed by Einstein for a different purpose, this Λ is believed to be due to as yet unknown dark energy in space that has a constant energy density and thus negative pressure, causing space to expand (Ryden 2018, 66)[10]. There is a body of work that tries to understand how bound structures can affect cosmology and whether the cosmological constant may need modification. However, a consensus has yet to be developed (for example see Sikora et al. 2021 and Buchert et al. 2015)[11][12]. Galaxy clusters are the largest gravitationally bound systems in the universe (Hong, Han, & Wen 2016)[13]. While these clusters occupy a small percentage of the space in the universe today, when the universe was smaller, they occupied a larger portion. The ΛCDM model does not consider that as structure develops, this gravitationally bound space, which varies with time, may not contribute to the overall expansion of the universe.

In this paper, we explore a modification to the ΛCDM model that considers gravitationally bound space, herein referred to as Λ_fCDM. In the theory section, we derive the modification to the standard ΛCDM model. In the simulation parameters section, we discuss with what parameters the new Λ_fCDM model is run to explore its impact on the Hubble tension. In the results section, we discuss the results of the model runs, from which we draw conclusions.

2. Theory

The CDM model was derived from Einstein's field equations; later, a cosmological constant denoted by Λ was introduced. We explore the modification to the model with Newtonian mechanics because for an isotropic, spherical, expanding universe, it has been shown that the key aspects of the solution can be understood with purely Newtonian dynamics, as it generates almost the identical Friedmann equation (Ryden 2018, Ch. 4 and 5)[14]. The only objective is to find a modification to the Λ term for use in the normal ΛCDM model. In General Relativity, the universe and space expand together; in the Newtonian treatment, we imagine a homogeneous sphere of matter expanding isotropically into existing empty Euclidian space. The sphere has an edge, a center of symmetry, and a fixed mass.

The acceleration of the outside edge of a sphere is given by Eq. 1 (Ryden 2018, 53; Harrison 2000, 331)[15][16]:

$$\ddot{r} = -\frac{GM}{r^2} = -\frac{G\rho V}{r^2} = -\frac{4\pi G\rho r}{3}, \tag{1}$$

where G is the gravitational constant, M is the mass of the sphere (which is enclosed in radius r), ρ is the density, V is the volume of the sphere, and ρr^3 is a constant. To Eq. 1, a cosmological constant denoted by lambda was added —

originally to cancel the gravitational deceleration and make the universe static, and recently to provide a positive acceleration component to the universe, which would become dominant at larger r values, as shown in Eq. 2:

$$\ddot{r} = -\frac{4\pi G\rho r}{3} + \frac{\Lambda r}{3}\,. \tag{2}$$

The physical interpretation of Λ is that it acts on all space. However, what if it cannot act on gravitationally bound space? Then this space does not contribute to the expansion of the universe. We step back to the version of Eq. 2 with volume in it and substitute the volume lambda impacts with only the volume that expands, V_e, as in Eq. 3:

$$\ddot{r} = -\frac{G\rho V}{r^2} + \frac{\Lambda r V_e}{3V}\,. \tag{3}$$

In the normal ΛCDM model, V_e is equal to V.

We define V_{GC} as the total volume of bounded space at any given time (which, as we shall see, is predominantly from galaxy clusters). The volume that expands is given by Eq. 4:

$$\mathrm{V_e = V - V_{GC}}\ . \tag{4}$$

Dividing by volume, Eq. 4 becomes

$$\mathrm{\frac{V_e}{V} = (1 - \frac{V_{GC}}{V})}, \tag{5}$$

and the effective Λ_f is given by

$$\Lambda_\mathrm{f} = \Lambda\,(1 - \frac{\mathrm{V_{GC}}}{\mathrm{V}}). \tag{6}$$

Hence, Eq. 3 becomes identical to Eq. 2, with Λ_f substituted for Λ as shown in Eq. 7:

$$\ddot{\mathrm{R}} = -\frac{4\pi\mathrm{G}\rho\mathrm{r}}{3} + \frac{\Lambda_\mathrm{f}\mathrm{r}}{3}. \tag{7}$$

We thus use the standard ΛCDM model with Λ_f instead of Λ. At the beginning of expansion, Λ_f is equal to Λ, as there are no gravitationally bound structures. As the universe expands, galaxies, then galaxy clusters, develop and close off certain space, and Λ_f becomes significantly smaller than Λ. Since cluster formation slows down as the acceleration of the universe increases, Λ_f trends back asymptotically (as V gets larger) to Λ. During this time, the number and size of clusters change with time, so in our modeling, we numerically integrate the ΛCDM equations with Λ_f substituted for Λ and call this universe the result of a Λ_fCDM model.

3. Simulation parameters

In our simulation, we would like to match the Λ_fCDM model to late universe and early universe observations. However, as a proxy for these measurements, we will use their matched ΛCDM models — that is, a ΛCDM67 model for the early universe results and a ΛCDM73 model for the late universe results.

Thus, we create the parameters for the Λ_fCDM model as follows (see **Table 1**).

Table 1

Universe Simulation Parameters

Parameter for universe	ΛCDM73 (km s^{-1} Mpc^{-1})	ΛCDM67 (km s^{-1} Mpc^{-1})	Λ_fCDM (km s^{-1} Mpc^{-1})	Comments on Λ_fCDM values
Hubble constant H_0	73	67.4	73	Match ΛCDM73 at current time
Critical density	Calculated (from H_0)	Calculated (from H_0)	Same as ΛCDM67	Match ΛCDM67 at early time

Scale factor	Set to 10	Scaled to ΛCDM73	Same as ΛCDM67	Match ΛCDM67 stretch
Matter density parameter	0.315	0.315	0.315	Match ΛCDM67 universe at early time
Dark energy density parameter	0.685	0.685	0.685	Match ΛCDM67
Present dark energy	As calculated from above parameters	As calculated from above parameters	Same as for ΛCDM73	Match current ΛCDM73 value
References ΛCDM73	H_0: (1) Densities: (2) Some recent results from ref. (3) indicate a lower matter density parameter (0.308), also modeled.			
References ΛCDM67	2			

References. (1) Riess et al. 2022[17]; (2) Planck Collaboration 2020, 2021[18][19]; (3) Dainotti et al. 2021[20].

The Hubble constant is set to the value of 73, as we currently observe. The critical density is set to that of ΛCDM67 because it and the matter density parameter are dominant in the early universe, and we want that match at that time. The scale factor (which is arbitrarily normalized to 10 for the ΛCDM73 universe) is set to that of the ΛCDM67 universe, as we need that full-scale factor to match the current structure to the CMB. The matter density parameter is obtained from the Planck results for the ΛCDM67 universe, and Λ_fCDM is matched to that — again, trying to retain the early universe conditions, which are matter dominated. Since we are not focused on the pre-CMB universe, we only model matter and dark energy based on the Planck data for both ΛCDM67 and ΛCDM73 universes. However, in the late universe, we want Λ_fCDM to behave as ΛCDM73, so we match its present dark energy. Note that Λ_fCDM therefore has a higher dark energy than the ΛCDM67 universe and is thus not “flat,” although that term is hard to define now that the effective lambda term is varying over time due to the impact of galaxy clusters.

The above matching maintains a ΛCDM67 early universe and its full scale to today while forcing today’s universe to have an H_0 of 73 and a dark energy term corresponding to that H_0. Note that we can pick a lower matter density parameter for ΛCDM73 for the simulation, as its value is not as well established in the references as that from the Planck data for ΛCDM67. We will comment on this in the results section.

Table 1 provides the key parameters; but in our universe, we also need to model bounded structure. Let’s start with the picture today (see **Table 2**).

Table 2
Cluster Parameters

Parameter	Value	Comments	References
Density parameter for clusters today Ω_{c0}	0.2		(1) (135)
Cluster mass M_c	5x10^{14} M_0	Use middle of range of 10^{14} to 10^{15} solar masses.	(2) (279)
Cluster radius, which dictates cluster volume V_c	Fit model at ~4 Mpc	Visible extent 1–5 Mpc. Pick midpoint of 3 or 1.5 for radius. Add dark matter halo extent of gravitational bounding of 2.5–3x visible radius.	Visible extent: (2) (279), 1 (134), (3) Dark matter halo: galaxy: (4) (26–28) NFW general profile: (5), (6) (L35–40)
Portion of non-expanding universe today	~0.034		Calculated from above values and critical density per **Table 1** $\frac{V_{GC0}}{V_0} = \Omega_{c0}\ \rho_{crit}\ V_c / M_c$

References. (1) Ryden 2018[21]; (2) Lang 2013, 279[22]; (3) White 2015[23]; (4) Sparke & Gallagher 2007[24]; (5) Navarro 1997[25]; (6) Okabe et al. 2013[26].

As shown in **Table 2**, we have estimates for the portion of matter contained in clusters, as well as the mass and size of the clusters. Cluster visible extent is well explored, but gravitationally bound space is much larger due to the majority of the cluster mass being in the form of dark matter (Gonzalez et al. 2013)[27]; so we estimate the overall cluster radius from the dark matter halo extent, as it has been found that the Navarro–Frenk–White (NFW) model (Navarro, Frenk, & White

1997)[28] is an excellent fit to a sample of 50 galaxy clusters at $0.15<z<0.3$ (Okabe et al. 2013)[29]. Note that if the estimate for the portion of matter contained in clusters is lower or higher than we are using, an opposite change in the cluster radius (as the cube root) will yield identical results.

With these parameters, we are able to calculate V_{gc0}/V_0 (where the zero subscript denotes the value today) at a few percent, as shown in the last row of **Table 2**.

At least for a few billion years back, due to the stability of clusters, we can calculate V_{gc}/V simply by scaling the value now upwards as the universe shrinks. At earlier times we also need to take into account changes in the number density and size of clusters.

Simulations with ΛCDM expect the very first stars to emerge some 50–100 million years after the Big Bang and the first galaxies a few hundred million years later, then cosmic mergers take place on progressively larger and larger scales. By the time a few billion years have gone by, we expect the universe to be rich in groups and clusters of galaxies, with clusters growing larger, richer, and more evolved as time goes on. About six billion years ago, dark energy became the dominant factor in the expansion of the universe, ensuring a swift drop in cluster growth and in mergers between clusters and leading to a stable cluster population not too different from today (Ryden 2018, Ch. 11)[30].

However, it is important to clarify that the ΛCDM model does not predict the clustering of the galaxy field directly. Instead, it provides a framework for predicting the density field of the dark matter following epochs of gravitational instability, settling eventually into the dark matter "haloes" (Navarro 1997)[31] that ultimately act as the sites of galaxy formation. As these haloes formed preferentially in locations where the initial density fluctuations were large, they are considered tracers of the underlying density field (Hernández-Aguayo et al. 2023)[32].

Thus, our challenge is that we need the ΛCDM model to estimate clusters at any given time, but we are trying to modify that model because it leads to tensions, including the so-called S_8 tension (see Abdalla et al. 2022 for a review)[33], that directly relate to structure formation. Further, some observations suggest that the formation of large structures took place earlier than expected in the ΛCDM model — for example, the collision velocity of the interacting galaxy cluster El Gordo (Asencio et al. 2021)[34].

Fortunately, we also have some significant observations to rely on. In the last decade or so, owing to the wide-area sky surveys performed with Sunyaev–Zeldovich (SZ) telescopes (Carlstrom et al. 2011; Fowler et al. 2007; Planck Collaboration et al. 2016)[35][36][37], it has become possible to detect clusters out to redshifts $z \sim 1.8$ (i.e., 10 billion years ago) with a simpler selection function — namely, the SZ signal tightly correlates with mass (Bocquet et al. 2019; Planck Collaboration et al. 2014)[38][39].

A sampling of relevant results and sources is provided in **Table 3**.

Table 3
Cluster Development

Event	Time	Observation	References
Early galaxies	After ~200 million years	Detected 87 galaxies that may have been the first to appear in the universe	(1)
Early proto-clusters	z=7.88	JWST early proto-galaxy cluster	(2)
	$z \sim 3.3$ (11.8 bya)	A massive proto-supercluster	(3)
Cluster abundance	$z \sim 1.8$ (~10 bya)	Detected clusters	(4), (5)
~50% clusters relaxed early	~10 bya	Half of clusters are stable starting ~10 bya	(6)
	$z=1.16$ (~8.5 bya)	Distant, dynamically relaxed, cool core cluster	(7)
	z=1.2 (~8.7 bya)	Evidence of relaxed clusters stable until z=1.2	(8)
Most clusters consistent	To z=1 (~8 bya)	Almost no difference in the X-ray luminosity functions (XLF) for clusters z>0.3 and z<0.3	(9)

		XLF at 0.3<z<0.6 consistent with the local XLF	(10)
		Cluster size does not change significantly in range 0.3<z<0.9	(11), (12)
Cluster number evolution	Constant to z=0.35 (4 bya), ~half to a third by z=0.5 (5.2 bya), drops to ~15% by z=0.7 (6.5 bya)		(13)
	Mild evolution in observed cluster abundance from z=0.5 to 1, half at z=0.5, and 1/6 at z=1		(14)

References. (1) Yan et al. 2023[40]; (2) Morishita et al. 2023[41]; (3) Forrest et al. 2023[42]; (4) Planck Collaboration et al. 2014[43]; (5) Ghirardini et al. 2021[44]; (6) McDonald 2017[45]; (7) Calzadilla et al. 2023[46]; (8) Darragh-Ford et al. 2023[47]; (9) Lewis et al. 2002[48]; (10) Ellis & Jones 2002[49]; (11) Khullar et al. 2022[50]; (12) Muzzin et al. 2012[51]; (13) Planck Collaboration et al. 2016[52]; (14) White 2015[53].

Based on these data, we model two "bookends" for cluster number and size: *early cluster* development, with number and size constant to ~8.6 bya, then decreased linearly to *no clusters* by ~10 bya. This is aggressive but will illustrate the effect of Λ_f clearly. For late cluster development, keep size constant and number decreasing as follows, steady at 1 (times current value) to z=0.35 (4 bya), decreasing linearly to 0.4 current value at z=0.53 (5.4 bya) and decreasing linearly to 0.15 at z=0.7 (6.5 bya) (Planck Collaboration et al. 2016)[54], then decreasing linearly to no clusters ~9 bya. The volume occupied by clusters is kept constant since, as shown in **Table 3**, the clusters are very consistent in size; this is because when early irregular and lumpy cluster shapes grow and become more massive, their radii increase only slowly, as most of the new mass concentrates in the core of the cluster (Sparke & Gallagher 2007, 294)[55].

We do not include any effect of galaxies in our modeling. About 5–10% of galaxies live in gravitationally bound clusters (Sparke & Gallagher 2007, 292)[56] versus alone or in groups. Clusters have hundreds to thousands of galaxies (Lang 2013, 279)[57]. Thus, for every cluster, there are ~10^4 unbound galaxies, but the cluster radius is 100x the galaxy radius (i.e., Mpc vs. tens of kpc). Thus, the gravitationally bound space of galaxies is 1/100th that of clusters today, and at z ~1 it is ~1/10th (assuming cluster number density drops to ~1/10 by z ~1). Galaxies have a significant bound space at earlier times, when the universe is much smaller, but at that time the universe is so matter dominated that small changes in Λ_f don't change the conclusions herein.

4. Results

Figure 1 shows the universes' scale factor versus time. The Λ_fCDM universes perform as set up(e denotes early cluster development and l denotes late cluster development to distinguish the Λ_fCDM universes. They expand the full-scale factor of the fit to the early universe ΛCDM67 and have the same early scale factor versus time (although one needs to look at the data, not the graph, to see this). However, they exhibit a late universe Hubble parameter that matches ΛCDM73 as long as most of the clusters are developed.

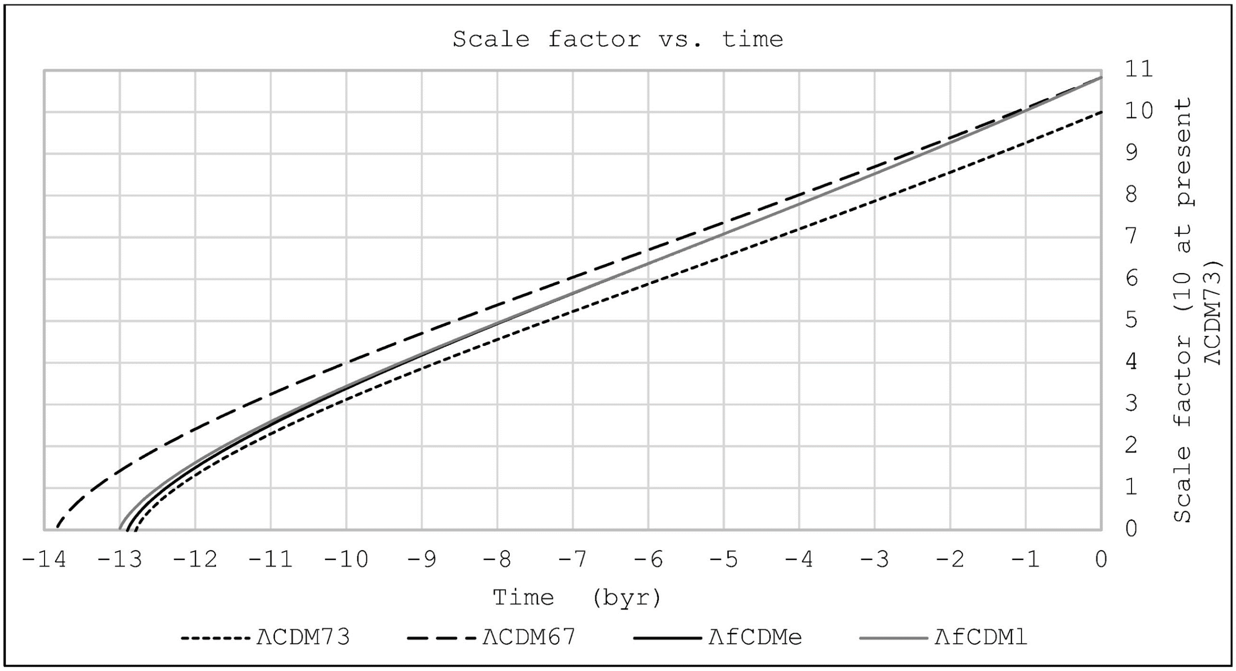

Figure 1. Scale factor vs. time for the various universe models. (Note that the Λ_fCDM models lie virtually on top of each other at this scale, and the gray line hides the black line.)

Figures 2 and 3 respectively plot the Hubble parameter versus time and the percent difference between the parameter for the Λ_fCDM universes and ΛCDM73. Note that in **Fig. 2**, the Λ_fCDM universes lie on top of ΛCDM73, except at the far left. But the differences are well apparent in **Fig. 3**, as clusters disappear back in time. For illustration, the dash-dot curve is for a universe with no clusters. Clearly, the clusters cause the nearly perfect fit. All these graphs are run with the parameters in the prior tables, except the cluster radius was changed to 4.005 Mpc to optimize the fit to H_0=73 km s^{-1} Mpc^{-1} in the late universe. **Figure 4** illustrates the cluster development assumptions for the early and later cluster development cases. Note that until about 4 bya (and before about 10 bya), the two lines are coincident.

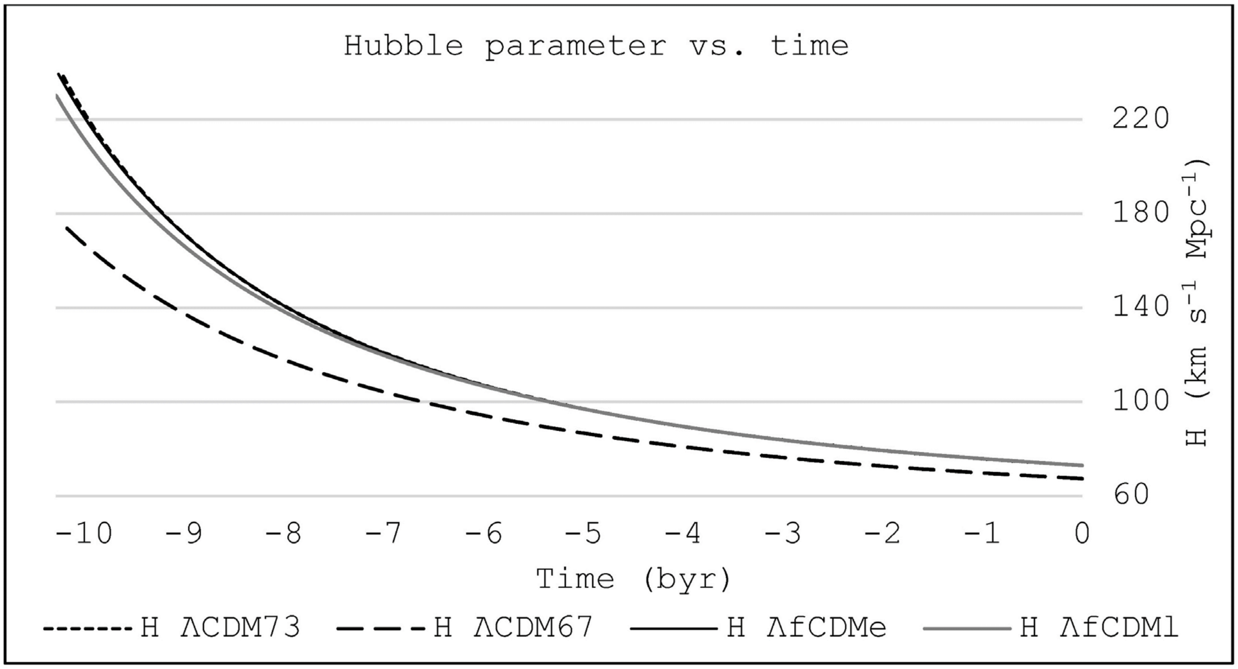

Figure 2. Hubble parameter H vs. time for the various universes.

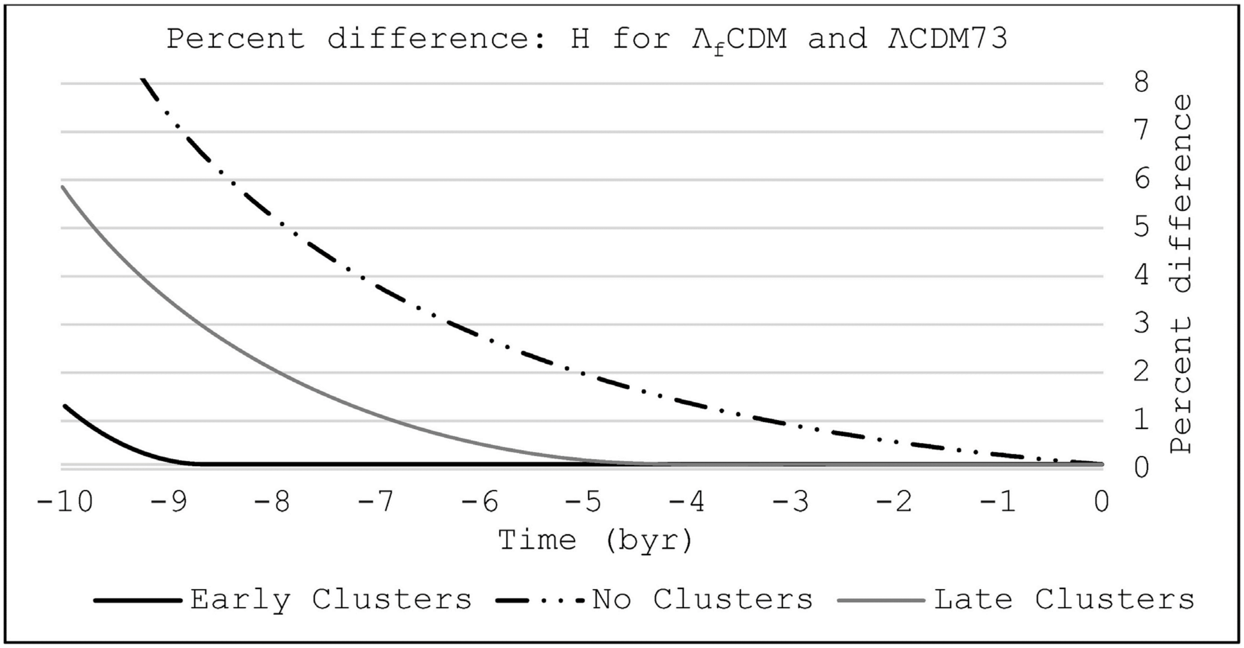

Figure 3. The difference between H for Λ_fCDM and ΛCDM73 universes.

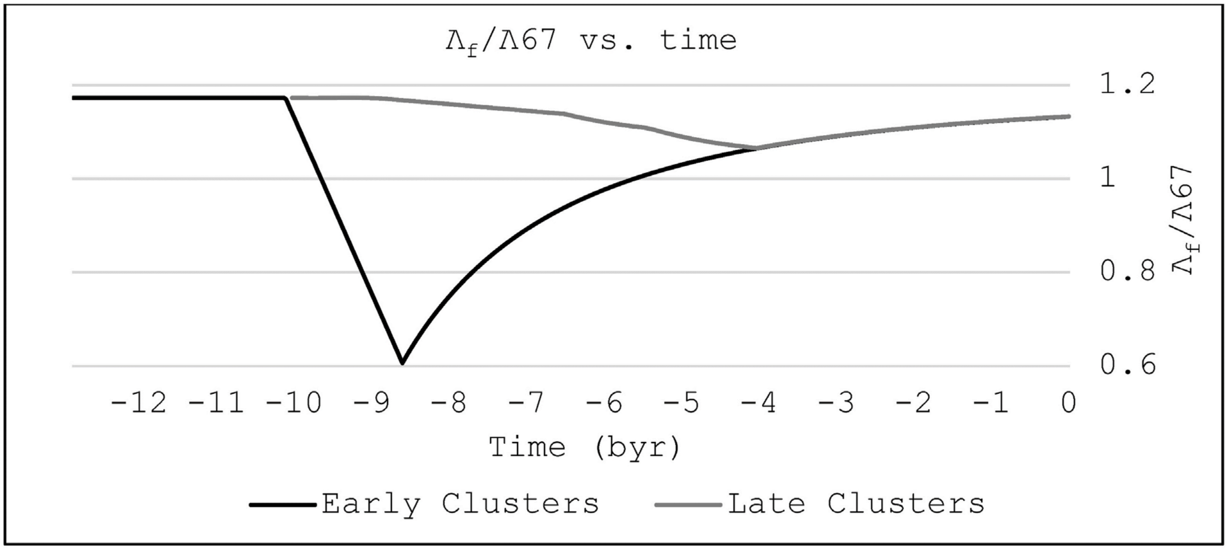

Figure 4. $\Lambda_f/\Lambda 67$ as clusters develop for early and late cases.

Our bookends in **Fig. 4** show that the Hubble parameter for the Λ_fCDM universes matches the ΛCDM73 universe in the last 9 byr for the early cluster development case and the last 5 byr for the late cluster development case. The late cluster development case moves to a lower H universe several billion years back (see **Fig. 2**, where the gray curve is leaning towards a lower H). There is some evidence that the Hubble parameter isn't constant. A survey of distant quasars gravitationally lensed by closer galaxies calculated the Hubble value at six different redshift distances. The uncertainties of these values are fairly large, but the Hubble parameter for closer lensings seems higher than for more distant lensings (Wong et. al. 2020)[58]. This model could fit that data, with some adjustments to the cluster development timing assumptions.

Finally, the model is robust to other assumptions. The total matter in clusters can be decreased or increased with a cube root adjustment to cluster radius to yield identical results. The matter density parameter for ΛCDM73 can be reduced from 0.315 and the cluster radius adjusted to obtain a similar fit. For example, a matter density parameter of 0.308 and a cluster radius of 3.8 Mpc obtain the same fit.

5. Conclusions

Simulations show that a Λ_fCDM model that assumes gravitationally bound space does not contribute to the expansion of the universe can resolve the Hubble tension. In this model Λ is still a constant, but does not affect expansion of space within gravitationally bound structures. Herein, we have matched the new model to the best-fit ΛCDM models for the early universe and late universe observations. The next step is to see whether the new model fits the actual data based on a structure formation timeline that is also consistent with it.

The Λ_fCDM model has a higher Hubble parameter than the ΛCDM67 model, meaning it will lead to a more homogeneous universe locally than the Planck data indicate (without a need to change the matter density parameter). This is moving in the right direction to resolve the S_8 tension (Zohren et al. 2022)[59], which this paper has not addressed directly.

Finally, any universe that attempts to fit a H_0 of 73 km s^{-1} Mpc^{-1}, even for part of its age, will have a shorter age than implied by the Planck data, which fit a H_0 of 67.4 km s^{-1} Mpc^{-1}. Thus, the model herein, although leading to an older age of the universe than a standard ΛCDM73, still has an age of about 13 billion years, with different parameter assumptions (such as a lower matter density parameter of 0.308) making it older by ~100 million years.

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