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On the dynamics of volatile meteorites

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Abstract
Canonical models for bolides in the atmosphere predict that fragile bolides break up at much higher altitudes than those actually observed. Here we investigate the hypothesis that such fragile bolides may survive to low altitudes by a protective outgassing sheath of volatile ices and organics that shields the meteoroid from direct atmospheric heating. Surviving meteorite fragments would be expected to possess higher degrees of porosity than generally acknowledged meteorite classes.

1. Introduction
Observational data of meteoroids show inconsistencies with the models used to predict their behaviour. For millimeter to tens of meter-sized bolides canonical models are unable to account for the survival of very fragile bolides to the lower altitudes as has been observed. The Maribo meteorite that fell in Denmark on January 17, 2009 had an entry velocity of 28.5 $km\ s^{-1}$ and has been linked to the Taurids meteor stream, which itself is thought to be associated with comet Encke (Haack et al., 2011). When recovered, the weak 25g fragment appeared intact but fell apart when touched (Haack et al., 2012). The fragment has now been classified as a CM2 carbonaceous chrondite. This is evidence for the ability of weak and friable material to survive atmospheric entry and fall as recoverable meteorites. A similar example is to be found in the Sri Lankan Polonnaruwa meteorite which has a fluffy porous structure, a very low density and stable oxygen isotopes pointing to their extraterrestrial provenance (Wallis et al, 2013).

Disintegration of meteoroids descending through the atmosphere is usually described by a process of continual ablation where the energy used to heat the bolide is proportional to the cube of its speed (u^3) (Bronshten 1983); or by catastrophic fragmentation when the ram pressure ($\sim u^2$) exceeds the tensile strength of the body (Hills & Goda 1991). Both these models predict that ~ 1m radii, low-density meteoroids must reach a minimum altitude of 80 – 60 km.

Frequently fireballs are observed at altitudes between 90 and 50 km above the Earth; however, other fireballs, such as the Tunguska bolide, appear to survive to much lower altitudes, exploding at ~ 10 km or less (Chyba, Thomas & Zahnle 1993). Here we hypothesise that meteoroids which survive to lower than expected altitudes are composed of volatile ices and organics held within and surrounding a denser core. The gases from outgassing, volatile material form a sheath around the body thus protecting it from direct interaction with the atmosphere as it decelerates.

Studies of cometary meteoroids suggest that rather than being composed of a homogeneous material such as stone or chrondite, they possess both volatile and high density refractory components. Investigation of the tracks in aerogel formed by particles collected from the comet 81P/Wild 2 indicated that the cometary dust consisted of a mixture of cohesive, relatively strong particles as well as particles with a more volatile matrix containing smaller stronger grains (Burchell et al. 2008). Similarly, modelling a Leonid meteoroid, Coulson (2003) predicted ~90% of the initial mass of cometary fragments are a composite of low-density material with the remainder made up of denser carbonaceous material in order to correctly describe its trajectory.

Here we consider a meteoroid consisting of a coherent carbonaceous matrix with pores filled with water ice and volatile organics. In the next section we model the meteoroid in free-space at a solar distance of 1 AU and calculate the rate of sublimation of volatile material prior to collision with the Earth's atmosphere.

2. **A composite bolide in free space**

We assume that the bolide was a typical cometary fragment, composed of volatile ices and organics held within as well as surrounding a denser core of either a stone or chronditic-type material. For simplicity, we suppose that the initial bolide was spherical with a radius $a \sim 1\,m$ with an average density of $0.9\ gcm^{-3}$.

In free-space within the Solar System, such a cometary body is heated by the Sun. At a solar distance R and an angle θ between the Sun and a normal to the surface of the body, the energy balance equation is

$$\frac{F_{\odot} e^{-\tau_T} \left(1 - A(\nu)\right) \cos\theta}{R^2} = \varepsilon\sigma T_B^4 + \frac{Z(\theta) L(T_B)}{N_0} + \kappa \left. \frac{\partial T_B}{\partial r} \right|_{r=a} \quad (1)$$

where $F_{\odot}$ is the energy flux from the Sun, $A(\nu)$ is the effective albedo at a given frequency ν, and τ_T is the total optical depth between the Sun and the body. The energy from the Sun is dissipated through thermal radiation, sublimation of volatile particles from the body and conduction of heat throughout the body – the successive terms on the right-hand side of Equation 1. Here T_B is the equilibrium temperature of the body, $Z(\theta)$ is the sublimation rate of the volatile material with a latent heat of sublimation L (usually a function of temperature), and thermal conductivity κ. N_0, ε and σ are Avogadro's constant, the emissivity and the Stefan-Boltzmann constant respectively.

At a distance of 1 AU, a 1m radius body has a temperature approximately equal the blackbody temperature of ~ 260 – 270 K (Coulson & Wickramasinghe 2003) depending on the effective albedo. Sublimation cooling and the subsequent increase in optical depth from dust production through the release of volatiles lowers typical temperatures of cometary bodies to ~ *200 K* at 1 AU (Keller 1990). At such values of temperatures energy losses from the body occur principally by sublimation (radiation losses are lower by a factor ~ 50).

The saturation pressure of the sublimating grains is given by the Clausius-Clapeyron equation

$$P_{sat} = P_{ref} \exp\left[\frac{H}{R_{gas}}\left(\frac{1}{T_{ref}} - \frac{1}{T}\right)\right] \qquad (2)$$

where R_{gas} is the universal gas constant and H is the enthalpy change of sublimation increased by the enthalpy of vaporisation at temperatures above the melting point of the volatile material (Coulson & Wickramasinghe 2003).

Assuming that the volatile material can be treated as an ideal gas, the number density n of the sublimating gas particles is related to the saturation pressure by

$$P_{sat} \approx n k_B T_B \qquad (3)$$

where k_B is Boltzmann's constant.

In the case of thermodynamic equilibrium, the speed v of the sublimating molecules can be calculated using

$$v = \sqrt{\frac{K_B T_B}{2\pi m_u M}} \qquad (4)$$

where m_u is the atomic mass of the sublimating molecules and M the molecular weight.

From Equations 2 – 4 the rate of sublimation can be found using

$$Z(\theta) = n(\theta)\sqrt{\frac{K_B T_B}{2\pi m_u M}} \qquad (5)$$

For water-ice at a temperature of 200 K, the sublimation rate is $\sim 5 \times 10^{22} m^2 s^{-1}$ and the saturation pressure is ~ 1 torr. For a comet composed of volatile organics rather than ice, both the sublimation rate and the saturation pressure are reduced by a factor of ~ 0.5, if one takes account of the somewhat higher binding energies of the former.

The number density of the sublimating molecules falls off as the inverse square of the distance from the surface of the body. For simplicity we assume that the sublimating molecules form a dense region around the body that is at least one mean free-path in length. The mean free-path of the sublimating gas,

$$\lambda_g = \frac{v}{Z\sigma_T} \tag{6}$$

where $\sigma_T \sim 10^{-19}\,m^2$ is the total scattering cross-section. For water ice at 200 K, $\lambda_g \approx 2\,cm$.

The gases from the sublimating material form a sheath around the body which protects it from direct interaction with the atmosphere as it descends at hypersonic speeds.

3. Modelling the bolide in the Earth's atmosphere

On its fall through the low density atmosphere, the bolide is heated by direct impact from incoming gas molecules from the Earth's atmosphere. These impacting gas molecules deposit energy in the surface as well as sputtering ice molecules (Coulson & Wickramasinghe 2003). If the bolide is travelling through the atmosphere with a speed *u*, the sublimation rate is increased by $\sim 0.5\rho_{atm}u^3L^{-1}$ where ρ_{atm} is the density of the atmosphere (Coulson & Wickramasinghe 2003).

For a body entering the Earth's atmosphere with the minimum initial speed of 12 km s^{-1}, the increased sublimation from collisions with incoming air molecules at an altitude of 10^5 km is $1.5\times 10^{23}\,m^{-2}s^{-1}$, approximately three times greater than the sublimation rate from thermally sublimating grains. For a body entering the atmosphere with the maximum initial speed of 72 km s^{-1} the sublimation rate is increased by two orders of magnitude to $3.1\times 10^{25}\,m^{-2}s^{-1}$.

As the bolide descends, the increasing densities of the atmosphere and the outflowing gas lead to a transition from free molecular flow to hydrodynamic flow. This transition occurs when the total mean free path of atmospheric and sublimated molecules ($\lambda \equiv \lambda_{atm} + \lambda_g$) is less than the bolide radius ($\lambda < a$). In the absence of sublimation, for a bolide with a radius of 1 m, the hydrodynamic region corresponds to an altitude of ~ 80 km, where $\lambda_{atm} \sim 1\,cm$ (Allen 2000). In the case of a sublimating bolide, the "outgas" density increases the altitude at which the transition to hydrodynamic flow occurs. For a water-ice dominated bolide, this occurs at an altitude ~ 100 km.

Within the hydrodynamic flow region, the aerodynamic drag is proportional to u^2 (Coulson 2003). We calculate that the total mass lost through sublimation is $<1\%$ of the original mass of the bolide. Hence the equation of motion for the deceleration of the body can be greatly simplified by assuming that the mass remains essentially constant during deceleration. Solving the equations of motion for a bolide entering the Earth's atmosphere under the influence of atmospheric drag, the velocity profile for the body can be written as a function of its altitude h

$$u(h) = u_0 \exp\left(- \frac{3C_D}{a} \frac{\rho_0}{\rho_m} H e^{-\frac{h}{H}} \right) \qquad (7)$$

where $\rho_0 e^{-\frac{h}{H}}$ is the variation in atmospheric density at a scale-height H (Allen, 2000) and C_D is the atmospheric drag coefficient. We assume here that the value of C_D is unity for consistency with the majority of existing meteoroid entry models; however, studies by Kremeyer et al. (2006) show that an aerosheath around a body travelling at hypersonic speeds significantly reduces the drag coefficient, by up to ~ 90%_compared with a sphere. In the subsonic regime air-layer drag reduction gives ~80% reduction in the coefficient.

In deriving Equation 7, the effect of gravity upon the bolide has been ignored, similar to the approach used by Bronshten (1983) and Ceplecha (1993). This assumption is valid so long as the magnitude of the drag force is greater than the force of gravity. Such a condition is satisfied provided that

$$u(h) > \left(\frac{\rho_m g a}{3\rho_0 C_D e^{-\frac{h}{H}}} \right)^{\frac{1}{2}}$$

For a bolide of radius 1 m and density 0.9 g cm^{-3} at an altitude of 10 km, (i.e after the onset of deceleration), the effect of gravity does not become significant unless the bolide's velocity is less than the minimum in-fall speed, 12 km s^{-1}. Under these conditions the deceleration of the body may be adequately described using Equation 7. In the Appendix, we present an analytical solution for the velocity profile of a meteor when gravity is significant.

Figure 1 shows velocity profiles for bolides of radius 1 m entering the Earth's atmosphere at an angle of $\pi/4$ to the downward vertical and an initial speed of 12 km s^{-1}. The region of maximum deceleration in Figure 1 occurs at an altitude of around 10 km, where mechanical stresses on the body are greatest. If mechanical stresses from deceleration are greater than the macroscopic strength of the body, it will fracture.

After the transition to hydrodynamic flow has occurred, a bow-shock of atmospheric and sublimated gas particles surrounds the forward hemisphere of the bolide at a distance of ~ 0.5 m. There are three distinct regions to consider: (1) a sheath of sublimating particles, (2) a region of shocked atmospheric and sublimated gas particles, and (3) a larger region consisting of unshocked atmospheric gases.

The sheath of sublimating gases behaves like the atmosphere of a comet or non-magnetic planet in the solar wind. The bow shock stands-off ahead of the bolide, diverting the atmosphere around it. The two protective properties compared with a "no sheath" situation are:

- The bow shock is not attached, so fracture due to pressure gradients are much less probable.
- The hot shocked gases make no direct contact and their radiative heating of the bolide is reduced by the optical depth of the sublimated particles.

The saturation pressure of the sublimating molecules is greater than the maximum ram pressure exerted by the bow shock if the bolide temperature exceeds 300 – 400 K. The sheath thickness extends at least one mean free path (~ 10^{-3} m at a temperature 300 K) in front of the body, and from Equation 6 is determined by the speed of flow from the sheath into the tail (~100 m s^{-1}).

Protected from direct impact by incoming air molecules, sublimation is limited by the radiative heating from the shocked gases. The temperature of the shocked gas can be calculated from the pressure and density of the shocked region. Assuming that the atmospheric gases are monatomic, the pressure of the shocked gas is $P_s » \frac{3}{4} r_{atm} u^2$, and the velocity of the shocked gas is $u_s » \frac{1}{4} u$. The maximum temperature of the shocked gas near the stagnation point is of the order

$$\sim \frac{3}{16} \frac{m}{k_B} u^2 \qquad (8)$$

where μ is the mass of the gas molecules.

Using Equations 7 and 8, the temperature of the shocked gas region is around 50,000 K for a 1 m radius bolide entering the Earth's atmosphere with an initial speed of 12 km s^{-1}, and a density of 0.6 g cm^{-3}. The bulk of the kinetic energy from the deceleration of the bolide in the atmosphere is dissipated through the heating of the shocked gas region rather than in heating the bolide. This calculation ignores the effects of ionization of gas molecules which would absorb a fraction of the energy, and so the values of 50,000 K should be considered an upper bound for the temperature of the shocked region. Shocked gas temperatures of $\sim 10^4\,K$ imply that radiant heating through the aerosheath region is the primary means of heat transfer to the body.

4. **Temperature Distribution within the meteoroid**

Radiative heating of the bolide from the shocked gas can be described using a modified from of Equation 1.

$$e^{-\tau_{eff,\gamma}}\sigma T_s^4 = \varepsilon\sigma T_B^4 + \frac{Z\,L\left(T_B\right)}{N_0} + \frac{ak}{3K}\frac{\partial T_B}{\partial t} \tag{9}$$

The last term on the right-hand side of Equation 9, the thermal conduction term, is described by the heat conduction equation, which for spherical geometry takes the form

$$\frac{\partial T}{\partial t} = \frac{1}{r^2}\frac{\partial}{dr}\left(k\left(r\right)\frac{\partial T}{\partial r}\right), \qquad 0 \le r \le a \tag{10}$$

for the internal temperature of a meteoroid of radius a. Where $k\left(r\right) = \sqrt{\frac{K}{C\rho}}$, where K is the thermal conductivity and C the specific heat capacity. If k is assumed to be independent of r, then equation 10 reduces to a linear, parabolic partial differential equation. Solving subject to the boundary conditions

$$\partial_t T\left(a,0\right) = T_0,$$

&

$$\partial_t T\left(a,t\right) = T_S\left(t\right) = \frac{1}{2}\rho_m \Lambda u^3\left(t\right) - \frac{L}{kN_0} n\sqrt{\frac{k_B T}{2\pi m M}}$$

gives

$$T\left(t,r\right) = T_0 + \sum_{n=1}^{\infty}\frac{\left(-1\right)^n}{n}\frac{2a}{\pi r}T_S\left(a,t\right)e^{-\left(\frac{n\pi k}{a}\right)^2 t}\sin\frac{n\pi r}{a} + \int_0^t \sum_{n=1}^{\infty}\left(-1\right)^{n-1}\frac{2n\pi k^2}{ar}T_S\left(t=0\right)e^{-\left(\frac{n\pi k}{a}\right)^2 t'}\sin\frac{n\pi r}{a}T_s\left(t'\right)dt'$$

Hence we can associate a time constant τ with thermal conduction within the meteoroid such that

$$\tau = \left(\frac{a}{k\pi}\right)^2 \tag{11}$$

Inserting suitable values for the density, thermal conductivity and specific heat capacity for a 1 m radius meteoroid composed of ice gives $\tau \approx 5\times 10^4\ s$. Typical meteoroid flight times through the atmosphere ~ 100 s; hence thermal conductivity is not significant in 1 m radius ice meteoroids.

The value of the time constant is not very sensitive to the composition of the meteoroid: $\tau_{graphite} \approx 1.2\times 10^4\ s$ and $\tau_{iron} \approx 4.1\times 10^3\ s$ are still much greater than likely meteoroid flight times for 1 m bolides.

Bodies with radii less than $5\times 10^{-4} m$ are too small to sustain thermal gradients (Coulson & Wickramasinghe 2003). This gives a lower-bound for meteoroid size where thermal conductivity is significant. From equation 11, we note that heat conduction is likely to be important for ice meteoroids with radii $5\times 10^{-4} m < a < 1\times 10^{-2} m$, and for iron and graphite bolides with radii in the range $5\times 10^{-4} m < a < 1\times 10^{-1} m$.

As thermal conductivity is insignificant for 1 m sized meteoroids, radiation emission and ablation are the mechanisms by which heat from the shocked gas is partitioned at the meteoroid surface. While there is sufficient volatile material able to transfer heat in contact with the surface of the bolide, ablation removes the energy from the shocked gas without greatly increasing the temperature of the meteoroid. At temperatures < 1,000 K, sublimation is the dominant mechanism for heat loss. The rise in temperature of the bolide from increased sublimation is discussed in the next section.

5. Radiative Heating of the Meteoroid

Compression of the air molecules forming the bow shock in front of the meteoroid generates temperatures of $\sim 10^4 K$. Heat from the bow shock radiates isotropically, so that a considerable fraction of the thermal energy goes into heating the atmosphere rather than the meteoroid.

For a meteoroid of radius *a*, the region of shocked air is separated by a distance $\approx \frac{3}{2}a$ around the centre of the meteoroid. If the pressure of ablating material forming the aerosheath is greater than the pressure exerted by the shocked gas, the aerosheath separates the bow shock from the surface of the meteoroid. The thickness of the aerosheath is $\sim \lambda$, the mean-free path of the ablating material. From equation 6, $\lambda \sim 1\, mm$ for bolide temperatures *200 – 400 K*, so that the presence of an aerosheath does not significantly extend the stand-off distance of the bow shock. Assuming that the shocked region can be considered as an hemispherical shell of thickness $\frac{1}{2}a$, that emits radiation as a black-body at a constant temperature, for isotropic emission the fraction of radiation emitted into the meteoroid is approximately 4/9.

If there is sufficient volatile material in the bolide, the majority of the energy from radiative heating by the shocked gas goes into sublimating more volatile gases from the bolide. The rise in the temperature of the bolide is strongly dependent on the composition of the molecules of the sublimated material forming the sheath.

Assuming that the bolide is composed purely of water-ice, radiation from the bow-shock will be scattered by the sublimating water molecules within the sheath. For typical bow-shock temperatures $\sim 10{,}000\, K$, the wavelength of the incident UV radiation is $\sim 1\times 10^{-7} m$, much greater than typical molecular radii $\sim 10^{-10} m$. Under these conditions, radiation interacts with the ice molecules through Rayleigh scattering (Wickramasinghe, 1973; van de Hulst 1981). The intensity of the radiation incident on the surface of the bolide is then reduced by a factor of

$$\frac{8\pi N\alpha^2}{\lambda^4 R^2}\left(1+\cos^2\theta\right) \tag{12}$$

where R is the distance between the shocked air and the surface of the bolide and N the number density of the sublimating molecules, approximately equal to the sublimation rate, $Z \sim 1\times10^{23}$ for an ice bolide at 273 K. α is the polarizability of the molecules which can be calculated from the complex refractive index $m(\lambda) = n - ik$ using the equation

$$\alpha = \left(\frac{m^2-1}{m^2+2}\right)a^3 \tag{13}$$

(Wickramasinghe, 1973). Warren and Brandt (2008) have determined the real and imaginary optical constants for ice across the UV and IR wavelengths. Using these values in equation 13, the Rayleigh scattering efficiencies for a pure ice bolide are calculated and shown in Figure 2. The λ^{-4} dependence of the scattering efficiency implies that the effect of scattering is several orders of magnitude greater at the UV wavelengths than the IR. This implies that the sublimating molecules are more efficient at scattering incident UV radiation from the shocked air than IR radiation emitted by the bolide. A resulting inverse greenhouse effect may thus lead to lower than expected bolide temperatures.

Absorption of incident radiation by molecules is proportional to λ^{-1} (van de Hulst 1981), so that scattering is the dominant mechanism by which the intensity of incident radiation is reduced at UV wavelengths.

The energy balance equation for an ice bolide is obtained by modifying Equation 9, so that the incident radiation spectrum is given by the Planck function $B(\lambda, T \sim 10{,}000\ K)$. Integrating (12) over all incident angles θ to obtain the Rayleigh scattering cross-section σ_s gives

$$\int_\lambda \sigma_s(\lambda)\, B(\lambda, T = 10{,}000\ K)\, d\lambda = \varepsilon\sigma T_B^4 + \frac{Z\, L(T_B)}{N_0} \tag{14}$$

and from the results of the previous section the heat conduction term on the right hand side of Equation 9 can be omitted.

In the absence of any sublimating material surrounding the bolide, the incident radiation flux in (14) can be approximated as a blackbody with the temperature equal to that of the bow-shock temperature ($\sim 10{,}000\ K$).

$$\varepsilon\sigma T_S^4 = \varepsilon\sigma T_B^4 + \frac{Z L(T_B)}{N_0} \tag{15}$$

In this case the thermal energy from the bow-shock is balanced by a maximum sublimation rate of $\sim 1\times 10^{27} m^2 s^{-1}$, corresponding to a maximum bolide temperature of ~ 500 K. The energy loss due to radiation from the bolide is significantly less than the sublimation losses and so the $\varepsilon\sigma T_B^4$ term in Equations 14 and 15 can be ignored.

Numerical integration of the incident radiation flux in Equation (14) using the Rayleigh scattering efficiencies calculated in Figure 3 gives a sublimation rate $\sim 1\times 10^{23} m^2 s^{-1}$, corresponding to a maximum bolide temperature of 260 K. Hence a sublimating 1*m* radius pure ice bolide would lose *< 2 %* of its original mass during atmospheric descent.

As the more volatile fractions of the bolide are used up, the bolide temperature rises, the less volatile carbonaceous material also evaporates and the capacity to generate a protective sheath is lost. At this point, the bow shock attaches and the bolide disintegrates explosively.

Sublimation is very effective in cooling the infalling bolide; it also shields the bolide from strong pressure gradients associated with the attached bow (or limb) shock of a sheath-less bolide. Depletion of the surface volatiles reduces the optical depth from the aerosheath, consequently reducing the shielding from radiative heating. Raised surface temperature and attachment of the bow shock to the body may all play a part in deciding the final break up. The bow shock travels through the bolide with a velocity approximately equal to that of the bolide (~ 10 km s^{-1}). The body is compressed by the shockwave and then ruptured by the reflected shock from the rear face of the bolide so that fragmentation occurs within $\sim 10^{-4}\ s$

6. Conclusion

The classical modelling of stony and iron meteorite falls cannot explain the low altitude break-up of a fragile meteorite. We propose an alternative sublimation model, which applies to bolides with substantial fractions of water ice and organics within a low density matrix of porous siliceous material. The presence of a protective layer of sublimating material enables such fragile bodies to withstand high thermal heating rates until either mechanical stress or the loss of volatile material results in catastrophic failure of the body.

If the optical thickness of the ablating material drops, heating of the body may produce gases from vaporisation of volatile components inside the body. Mechanical stress or explosion from ignition of gases may account for the disintegration process in bolides formed of volatile components held within an impermeable porous shell.
Our model assumes that the sublimation pressure increases as the bolide descends through the Earth's atmosphere such that the sublimation pressure ≈ ram pressure and that the mass lost by sublimation << than the original mass of the body. Both these

assumptions are validated by our calculations.

The detailed transition from free molecular flow to hydrodynamic flow is a sensitive function of both the mean free path of the sublimated molecules and the incoming atmospheric gas molecules (Coulson 2006). The presence of an aerosheath is a further factor to be considered in determining the altitude at which meteoroids transition between flow regimes.

We conclude by noting that the model discussed in this paper is applicable to the Maribo meteorite discovered in 2009 (Haack et al, 2011) and more particularly to the case of the Polonnaruwa stones that fell in Sri Lanka in December 2011 (Wallis, et al, 2013). The distribution of stable oxygen isotopes in the latter being non-terrestrial leaves little option but to conclude that these low-density porous fragments define a new type of meteorite resulting from the fragmentation of a predominantly water-ice-organic bolide of the type described in this paper.

Appendix

Consider a spherical, non-ablating meteor of radius a, falling through the atmosphere under the influence of gravity and atmospheric drag. If the speed of the meteor *v* is written as a function of its path length through the atmosphere *x*, the equation of motion of the meteor is

$$v\frac{dv}{dx} = a_2 - a_1 e^{-a_0 x} v^2 \qquad \text{(A1)}$$

with the initial conditions $v = v_0, \quad x = x_0$; i.e. the meteor has an initial speed of $v = v_0$ in free space prior to atmospheric decent.

Here

$$a_0 = \frac{\cos\theta}{H}$$

$$a_1 = 3\frac{C_D}{a}\frac{\rho_0}{\rho_m}$$

$$a_2 = g\cos\theta.$$

θ is the angle the meteor's path makes with the downward vertical and *g* is the acceleration due to gravity.

Writing $f(x) = a_1 e^{-a_0 x}$, the non-linear equation A1 becomes

$$v\frac{dv}{dx} + f(x)v^2 = a_2 \qquad \text{(A2)}$$

which can be solved by means of an integrating factor $\frac{1}{2}e^{2\int f(s)ds}$ to give

$$v(x) = \left(v_0^2 e^{-2\frac{a_1}{a_0}\left(e^{-a_0 x} - e^{-a_0 x_0}\right)} + 2\frac{a_2}{a_0} e^{-2\frac{a_1}{a_0}e^{-a_0 x}} \left[E_i(X) - E_i(X_0)\right] \right)^{\frac{1}{2}} \qquad \text{(A3)}$$

where E_i is the exponential-integral $E_i(x) = -\int_{-x}^{\infty} \frac{e^{-t}}{t} dt$ and X, X_0 are the dimensionless quantities $X = \frac{6}{\cos\theta}\frac{H}{a}\frac{\rho_0}{\rho_m}e^{-\frac{\cos\theta}{H}x}$, $X_0 = \frac{6}{\cos\theta}\frac{H}{a}\frac{\rho_0}{\rho_m}e^{-\frac{\cos\theta}{H}x_0}$

Calculating values for the speed of a 1 m radius Polonnaruwa type bolide with an initial speed of 12 km s^{-1} using Equation A3 is consistent with ignoring the effects of gravity to within 3 decimal places for altitudes above 15 km.

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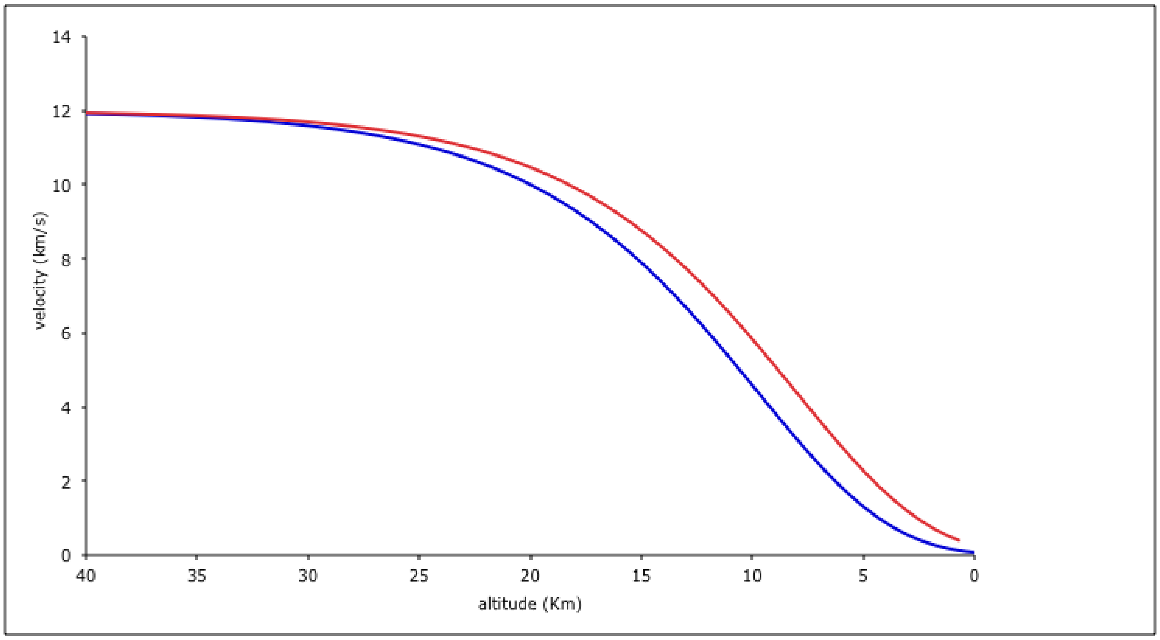

Figure 1: Velocity profiles for 1 m radii bolides entering the Earth's atmosphere at angle of $p/4$ to the downward vertical and an initial speed of *12* km s^{-1}. The blue curve is for a bolide composed of a comet-like ice and organics with a density of 0.6 g cm^{-3}. The red curve is for a higher density bolide of 0.9 g cm^{-3}.

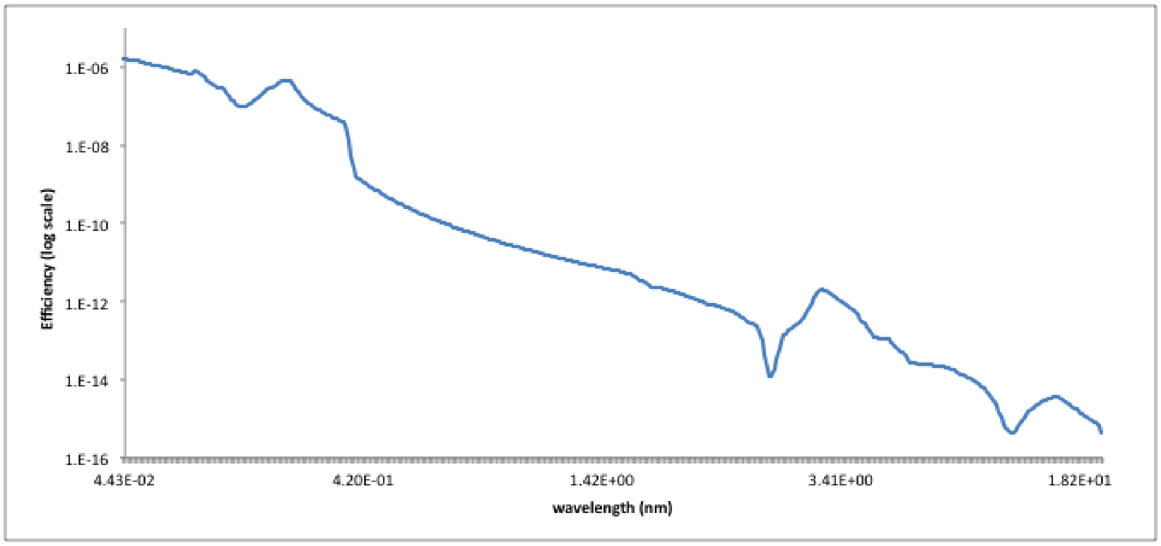

Figure 2: The variation of Rayleigh scattering efficiencies (on a log scale) with wavelength across the UV and IR wavelengths. Calculated from equation 12 for a 1m radius block of sublimating ice, with a sublimated particle number density of $N = 1 \times 10^{23}$, using the optical constants for ice given by Warren and Brandt (2008).