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A novel technique to study the structure of the universe

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ABSTRACT

The universe can be modeled as a gigantic reservoir where only waves that have an integer number of modes are allowed; therefore the spectrum of the cosmic background radiation has missing frequencies. The spacing between frequencies is hopelessly small to be detectable with conventional techniques. However, beats among the frequencies give rise to intensity fluctuations that have much lower frequencies, allowing observations that can measure the spectral structure introduced by the cosmological reservoir. The spectral structure gives information on the large scale structure of the universe. For example, it can give the Hubble constant.

1. INTRODUCTION

Electromagnetic emission (e.g. spontaneous emission) is not a property of an isolated emitter (e.g. an isolated atom) but, rather, of an emitter-vacuum system. The vacuum acts like a vast "reservoir" in which electromagnetic waves are dumped. The reservoir restricts the modes available and therefore the frequencies of allowed electromagnetic waves. These characteristics of the emitter-vacuum system are well-documented and are used in a series of elegant experiments in cavity quantum electrodynamics (Berman 1994).

The universe can be modeled by a gigantic reservoir as shown in figure 5.8 in Peebles (1993). Because only waves that have an integer number of nodes are allowed, the spectrum of a radiating source of electromagnetic radiation has missing frequencies and therefore has a "picket-fence" structure. The spacing between the pickets can be used to determine basic cosmological parameters and the shapes of the pickets give information on large scale structure. This extremely fine structure is totally unobservable with conventional techniques but one can measure extremely fine frequency spacing by observing classical radiation fluctuations. A similar astrophysical application of the technique has been suggested by Borra (1997, 2011, 2014) to measure the time delays between the beams of a gravitational lensed object; because interference between the beams also induces very fine spectral features undetectable by conventional spectroscopy.

2. RADIATION FLUCTUATIONS AND MISSING FREQUENCIES

Let us consider a wave packet propagating in a closed universe. Open universes will be discussed later. Let us assume a uniform distribution of matter so that the geometry is smooth. As depicted in figure 5.8 in Peebles (1993), the boundary conditions are such that only modes that have an integer number of oscillations around the "circle" of radius a(t) are allowed. Therefore, given a(t) the scale factor of the universe, the allowed modes are only those that satisfy the relation

$$n\lambda = ka(t), \tag{1}$$

where n is an integer number, λ is the wavelength of the mode and k a geometry-dependent factor not significantly larger than 1. Equation 1 predicts that the energy distribution of any radiating electromagnetic source in free space is made of discrete frequencies. This predicts a spectrum made of monochromatic spikes. Measurements of the separation between the spikes allow us thus, in principle, to measure a(t) and therefore solve the cosmological problem. Unfortunately, the separation between the spikes is so small that it is hopelessly beyond the reach of standard spectroscopic techniques. However, as we shall see, measuring classical radiation fluctuations allows detection of such spacing.

Let us consider a wave packet of amplitude, as a function of time, V(t) having the frequency spectrum $G(\omega)$ given by the Fourier transform of V(t). The radiation signal is thus made by the interference of a continuous distribution of monochromatic modes. The discontinuous nature of the frequency spectrum can be mathematically represented by the shah function III(x) (Bracewell 1986) which is made by an infinite number of Dirac δ functions spaced by unit intervals. In practice, however, we should not expect the sampling function to be the shah function which predicts a spectrum made of purely monochromatic waves. For example, a real universe has a clumpy matter distribution and observations sample electromagnetic beams of finite extent containing radial rays which travel distinct geodesics having different lengths distorted by warped space-time. This can be modeled with a picket-fence function made of the convolution of the shah function with a continuous function S(x)

$$p(\omega a(t)/c) = \mathrm{III} * S \quad , \qquad (2)$$

where the symbol * signifies the convolution of the functions III and S. For example, S could be a Gaussian function.

The frequency spectrum $H(\omega)$ is now given by

$$H(\omega) = p(\omega a(t)/c)\, G(\omega). \qquad (3)$$

We thus see that there is a periodic function p(ωa(t)/c) that allows to distinguish between the spectrum G(ω) made of a smooth distribution of frequencies and a H(ω) spectrum which is not smooth.

Although the spacing between the spikes of the function is hopelessly out of reach of conventional spectroscopic techniques, an experimental solution is suggested by the ingenious experiment devised by Alford & Gold (1958) to measure the speed of light, which leads to a similar difficulty. For brevity, I shall not discuss in details the Alford & Gold experiment. It suffices to say that the experiment measures I(t), the current from a square law detector (a photomultiplier in their setup), determining τ, a time interval that plays the role of the D/c term in Equation 3, from the spectrum I(ω) of I(t) which is modulated by the function cos(ωτ/2). We can then use the current I(t) to get the spectral minima of its fluctuations that are present at frequencies that much lower than those of H(ω).

Givens (1961) analyzed the Alford & Gold experiment as a wave propagation effect and our analysis closely follows his. What allows easy comparison to Givens (1961) is the similarity of our Eq. 3 to his Eq. 4 which gives the frequency spectrum

$$H(\omega) = 2\cos(\omega\tau/2)G(\omega) \quad . \tag{4}$$

The periodicity of the cosine term in Eq. 4 is analogous to the periodicity given by the shah function in Eq. 2.

The function p(ωa(t)/c) is periodic and can be expanded in Fourier series of frequency c/ a(t) . Because there are no negative frequencies, we are free to choose the form of p(ωc/ a(t)) for ω<0 and can therefore assume that p(ωc/ a(t)) is an even function which has therefore real coefficients and can be represented, with τ=2 π a(t)/c, by the cosine Fourier series

$$p(\omega\tau) = \sum_{n=0}^{\infty} a_n \cos(n\omega\tau). \qquad (5)$$

Following Givens (1961) we write that the current measured by a square law detector is given by

$$I(t) = \int\int \left[f(\omega) f^*(\phi) G(\omega) G^*(\phi) \sum_{n=0}^{\infty} a_n \cos(n\omega\tau) \sum_{n=0}^{\infty} a_n \cos(n\phi\tau) \right] e^{i(\omega-\phi)t} d\phi d\omega. \qquad (6)$$

The product of the two sums in the Fourier series gives a sum of terms containing a_0^2 , "pure" terms like $a_0 a_n$ cos(nωτ), and $a_0 a_m$ cos(mΦτ) as well as mixed terms like $a_m a_n$ cos(nωτ)cos(mΦτ). Now, let us make the substitution ω = Φ + ω'. Equation 6 is then made of the sum of an infinite number of integrals containing a_0^2, $a_0 a_m \cos(m\Phi\tau)$, $a_0 a_n \cos(n(\Phi\tau + \omega'\tau))$ and $a_m a_n \cos(n(\Phi\tau + \omega'\tau))\cos(m\Phi\tau)$ terms. Decomposing the product $a_m a_n \cos(n(\Phi\tau + \omega'\tau))\cos(m\Phi\tau)$ as a sum of cosines and following a reasoning similar to the one in Givens (1961), we obtain that all the terms in the sum of integrals that are functions of $m\Phi\tau$ are approximately zero, since $f(\Phi)$ and $G(\Phi)$ are both slowly varying functions of Φ, compared to the rapidly oscillating $\cos(m\Phi\tau)$ or $\cos((m\pm n)\Phi\tau + n\omega'\tau)$ terms. There is only left the term containing a_0^2 and terms containing $1/2a_n^2 \cos(n\omega'\tau)$. Defining the periodic function

$$P(\omega'\tau) = a_0^2 + \sum_{n=1}^{\infty} 1/2a_n^2 \cos(n\omega'\tau), \tag{7}$$

where the coefficients a_n are the same as in Equation 5, we have that

$$I(t) = \int \left[P(\omega'\tau) \int f(\phi + \omega') f^*(\phi) G(\phi + \omega') G^*(\phi) d\phi \right] e^{i\omega' t} d\omega' . \tag{8}$$

The current I(t) is related to its frequency spectrum I(ω) by

$$I(t) = 1/\sqrt{2\pi}\int I(\omega')e^{i\omega' t}d\omega' . \qquad (9)$$

Comparing Equation 8 to Equation 9, we see that

$$I(\omega') = \sqrt{2\pi}\big(P(\omega'\tau)\big)\int f(\phi+\omega')f^{*}(\phi)G(\phi+\omega')G^{*}(\phi)d\phi . \qquad (10)$$

The integral gives the autocorrelation function of the product f(ω')G(ω'). We see that the frequency spectrum is modulated by the periodic function P(ω'τ) whose Fourier expansion gives coefficients simply related to those of p(ωτ).

Astronomical instruments use a bandpass that is small compared to its central frequency so that we can, for discussion purposes, use

$$f(\omega')G(\omega') = g\exp(-(\omega'-\omega_0)^2/\sigma^2) . \qquad (11)$$

This allows evaluating the integral in Equation 10 with

$$I(\omega') = KP(\omega'\tau)\exp(-1/2\omega'^2/\sigma^2), \qquad (12)$$

where all the constants are contained in K using the fact that the error function, obtained from the integral in equation 10 with equation 11, tends to ± 1 for ω' that tends to ± ∞. Equation 12 shows that all information about the central frequency ω_0 has been lost, that I(ω') is not zero at arbitrarily low frequencies and is periodic with periods = n τ . It illustrates the important effect which shows that a spectral modulation happens at frequencies that are considerably outside of the spectral bandpass of the spectral frequency measured. The frequency spacing thus becomes measurable.

One can intuitively understand Equation 12 by seeing that the signal fluctuations are caused by wave beats among all the frequencies in the beam, like wave beats modulate the carrier frequency in a radio detector. An unmodulated source differs from a modulated source because its frequency spectrum H(ω) is modulated by the picket fence function (e.g. some frequencies are missing); consequently the beats of a spectrally modulated and an unmodulated source will have different power spectra. Because beats generates lower frequencies given by the difference among the beating frequencies, closer beating frequencies give lower will frequencies of the fluctuations. This therefore allows generating very high resolution spectroscopy (Mandel 1962a). Consequently, the extremely fine spectral features predicted by Equation 3 can be detected as current fluctuations that have much lower frequencies.

3. DISCUSSION AND CONCLUSION

A determination of p(ωa(t)/c) gives information on the large scale structure of the universe. The scale parameter a(t) can be obtained from the separation Δλ between two peaks; therefore giving us all the information needed to derive the principal cosmological parameters. The function S in Equation 2 gives information on the structure of the universe along a line of sight and its determination along different lines of sight gives information on the isotropy of the universe. To some reader it may not be obvious what is the usefulness of the scale parameter a(t) that can be obtained from the separation Δλ between two peaks; Section 5 in Peebles (1993) shows that all cosmological parameters can be obtained from a(t). For example Equation 5.4 in Peebles (1983) gives the Hubble parameter H as time derivative of a(t) (da(t)/dt) divided by a(t).

$$H = a(t)/(da(t)/dt) \qquad (13)$$

If, on the one hand, determining the rate of separation of the peaks of p(ωa(t)/c) gives information on a(t), on the other, the rate of separation must be small enough that it is detectable within the time resolution of the instrumentation. We obtain a criterion of detectability by imposing that the change in the position of the peak of a given order n, δλ, caused by the expansion of the universe, in the time of observation Δt, be smaller than the separation between two peaks Δλ. Assuming uniform expansion one obtains the criterion

$$\delta\lambda/\Delta\lambda = H_0\, a(t)\Delta t/\lambda << 1 \,. \tag{14}$$

Because a(t) is of order c/H_0, the criterion will become

$$\delta\lambda/\Delta\lambda \approx \Delta t/T << 1, \tag{15}$$

where T is the period of variation of the fluctuation. Equation 12 shows that the power spectrum of the fluctuations has power up to a period $T = a(t)/c$, of the order of the age of the universe. There is therefore power for periods considerably greater than one second. Typical square law detectors have resolution times significantly lower than one second, ensuring that the criterion can be met.

The fundamental assumption of this work is that an electromagnetic wave packet can be modeled by the interference of infinitely long monochromatic waves. One must question whether this is simply a mathematical construct or whether it truly reflects reality. The experimental evidence supports the reality of the mathematical model. First, as pointed out by Givens (1961), the Alford & Gold experiment requires interference over the distance of the experiment (about 60 meters), although the coherence length of the white light used was far less than 60-m, which is only possible if there are monochromatic waves significantly longer than the coherence length. Second, in a series of experiments, Delisle and coworkers (e.g. Cielo, Brochu, & Delisle 1975) demonstrated spectral modulation with a Michelson interferometer and incoherent white light over distances as large as 300-m. Given the short coherence time of the white light, interference could only take place in that experiment by considering the constructive or destructive interference of the monochromatic modes composing the wave packet. Third, in a more telling and remarkable experiment, Chin et al. (1992) demonstrated spectral modulation of femtosecond laser pulses in a Michelson interferometer where the path difference was greater than the spatial extent of the pulses: There is interference between pulses that do not spatially overlap. Chin et al. (1992) conclude that the experiment shows that the Fourier components of the short pulse do exist as infinite electromagnetic waves.

We shall now discus the effect of noise. The noise to consider (kind and magnitude) depends on the instruments and techniques that are used, which depend on the region of the electromagnetic spectrum used. Its discussion is therefore extremely complex and beyond the scope of this paper. In the ideal case, the instrumental or background noise can be made negligible but photon shot noise cannot be eliminated and gives a limit that can be estimated. Borra (1997) used Mandel's (1962b) discussion of the effect of shot noise on the Alford & Gold experiment to estimate the fluxes at which the shot noise contribution becomes comparable to the radiation fluctuations in astronomical sources. Assuming that the observations are done at the frequency ν = 1 GHz and α=0.5 Borra (1997) finds that shot noise becomes noticeable for sources below 10 milliJansky observed with a 100-m diameter radio-telescope. One can obtain better results at lower frequencies since the effect of shot noise decreases ~ ν for a given flux. Several publication (Hanbury Brown, Jennison, & Das Gupta 1952; Jennison, & Das Gupta 1956) that discuss observations of angular sizes and structure of radio sources with intensity interferometry show that wave-interaction effects are detectable in astronomical radio sources. In this respect, it must be noted that wave interaction observations require telescopes that have far lower surface qualities than those used for conventional observations (Hanbury Brown, 1968). This signifies that one could use large inexpensive low-surface-quality telescopes.

Discussion of experimental setups is beyond the scope of this work, but we can use it to obtain some experimental guidelines. Shot noise considerations clearly favor observations at long wavelength. The observational setup could involve observing a bright astronomical radio source. However, considering that the theory assumes radiation from an isolated emitter, one should observe the transparent lobes of extended radio sources, rather than their opaque compact cores.

The discussion assumes a closed universe. For an open universe one could, for computational purposes, use periodic boundary conditions and repeat the same treatment as for a closed universe. However, it is not clear to this writer whether the spacing then becomes infinitely close rendering it impossible to distinguish the spectrum from a continuous one. If it is the case, the success or failure to detect the spacing would allow distinguishing between an open or a closed universe.

The theory used in this paper and discussed in section 2 models the universe as a gigantic reservoir where only waves that have an integer number of modes are allowed: One may of course wonder how realistic this model is. The use of this model is validated in section 5 of the book by Peebles (1993). The discussion in Peebles (1993) is very long and will take considerable time to read it and understand it. However, Figure 5.8 in Peebles (1993) gives a simple, and rapid to understand, illustration of the validity of this model. The main message is that observations of radiation fluctuations in radiating sources in free space contain information on the large scale structure of the universe. All the cosmological parameters (e.g. the Hubble constant H0) can be obtained from the function a(t) discussed in section 2.

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