Bianchi type I dust filled universe with variable bulk viscosity and vacuum energy density in C-field cosmology

by

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Abstract

A cosmological model which admits dust filled universe and a negative energy massless scalar field as a source in the presence of bulk viscosity and vacuum energy density (Λ) in Bianchi Type I space-time is investigated. It has been shown that creation field (C) increases with time which matches with the result as obtained by Hoyle and Narlikar (1964). The model has no real singularity initially and particle horizon exists in the model. The model represents constant expansion but accelerating universe. The spatial volume increases exponentially representing inflationary scenario. The value of State Finder parameters agrees with ΛCDM model. It has been shown that to what extent this model with time dependent vacuum energy density and variable bulk viscosity in creation filed cosmology matches with the results of Hoyle and Narlikar (1964) of creation field cosmology.

Key Words: Bianchi Type I, Dust, Variable bulk viscosity, Vacuum energy density, Creation field.

Introduction

Bianchi type I cosmological models in the Einstein system of field equation create more interest in the study because these models provide us a systematic way to obtain cosmological models more general than FRW (Friedmann-Robertson-Walker) models. Also as pointed out by Patridge and Wilkinson (1967), FRW models are unstable near the singularity. Therefore, spatially homogeneous and anisotropic Bianchi type I space-time is undertaken to study the universe in its early stages of evolution. The existence of anisotropic universe that approaches to isotropic one, has been pointed out by Land and Magueijo (2005).

The introduction of viscosity in the cosmic fluid content has been very useful in explaining many physical aspects of dynamics of homogeneous cosmological models. The dissipative mechanism not only modify the nature of singularity but also successfully explains the large entropy per baryon in the present universe. The remarkable degree of the isotropy of the cosmic microwave background radiation (CMBR) reveals the significance of dissipative effects in cosmology. Heller and Klimek (1975) have investigated viscous fluid cosmological model without initial singularity in which it has been shown that bulk viscosity removes the initial singularity effectively. Chimento et al. (1997) studied the cosmological solutions with non-linear viscosity. Gron (1990) reviewed the research on viscous cosmological models and also studied the cosmological models in Bianchi Type I space-time with shear, bulk and non-linear viscosity. Zimdahl (1996) investigated that sufficiently large viscous pressure leads to inflationary like solution in flat FRW model. The effect of viscosity on cosmological models has also been investigated by many authors viz. Bali (1984), Sahni and Starobinsky (2000), Saha (2005), Bali et al. (2012), Brevik and Gron (2013) in different contexts.

In the early universe, the non-trivial role of vacuum generates a cosmological constant (Λ) term in Einstein field equation which leads to inflationary scenario as pointed out by Abers and Lee (1973) in which it has been pointed out that during an early exponential phase, the vacuum energy is treated as large cosmological constant as was expected by Glashow-Salam-Weinberg model (Langacker (1981)) and Grand unified field theory (Sakharov (1968)). Therefore, the present day observations of smallness of cosmological constant ($\simeq 10^{-122}$) support to assume that cosmological constant is time dependent. Gibbons and Hawking (1977) conjectured that any cosmology with positive cosmological constant (Λ) would asymptotically approach to a de-Sitter space-time. Therefore, the cosmological models linking the cosmological term leaving the form of Einstein's field equations unchanged and preserving the conservation of energy momentum tensor of matter content, have been investigated. A number of cosmological models in which cosmological constant (Λ) decays with time have been investigated by many authors viz. Abdel Rahman (1990), Beesham (1994), Berman (1990), Bali and Singh (2008), Singh et al. (2012). In the late eighties, Astronomical observations revealed that the predictions of FRW models do not always meet our requirements as was believed earlier (Smoot et al. (1992)). Also gravitational collapse of a massive bodies is an unavoidable consequence of General Relativity (Penrose (1965), Hawking (1965)). So alternative theories of gravity were proposed from time to time. The most well known theory was steady state theory of Bondi and Gold (1948). In this approach, the universe does not have any singular beginning nor an end on the cosmic time scale. To maintain constancy of matter density, they considered a very slow but continuous creation of matter in contrast to explosive creation of standard model. But it suffered a serious disqualification by not giving any physical justification for continuous creation of matter and principle of conservation of energy was sacrificed in this formalism. To oversome this difficulty, Hoyle and Narlikar (1964) adopted a field theoretic approach introducing a massless and chargeless scalar field in the Einstein-Hilbert action to account for the matter creation. In C-field theory, there is no big-bang type singularity as in Steady State Theory of Bondi and Gold (1948). If a model successfully explains the creation of positive energy without violating the conservation of energy then it is necessary to have some degree of freedom which acts as a negative energy mode. Thus a negative energy field provides a natural way for creation of matter. Narlikar and Padmanabhan (1985) have investigated the solution of modified Einstein field equations in presence of C-field. They have shown that cosmological models based on this solution is free from singularity, particle horizon and also provides a natural explanation to the flatness problem. Bali (2012) investigated avoidance of singularity in the exterior of spherically symmetric metric in C-field cosmology. Recently Bali and Saraf (2015) investigated C-field cosmological model for barotropic fluid distribution with variable bulk viscosity and vacuum energy density (Λ) in FRW model.

Regarding avoidance of singularity, the alternative ideas about black hole were also evolving. Observations of the inner quasar structure at scales of several R-G became technically feasible for quasars, using techniques of gravitational microlensing and reverberation. Thus studying the UV- optical luminous inner structure immedialy produced evidence of magnetic propeller and favoured the MECO (Magnetic Eternally Collapsing Object) family of solutions that followed the basic Mitra (2000,2002) ECO solutions of the Einstein-Maxwell field equations for an object with finite mass and an intrinsic dipole magnetic field (Schild, Leiter & Robertson 2006,2008; Lovegrove, Schild & Letelier 2011). Similar models had been proposed by Robertson and Leiter (2002,2003,2004) for BH binary star candidates. This new direction of black hole theory is based upon a deeper consideration of Quantum Electrodynamics and its role in producing a pressure at the event horizon that halts the collapse to singularity and produces instead a surface of very high redshift.

The Metric and Field Equations

We consider Bianchi Type I metric in the form

$$ds^2 = dt^2 - A^2 dx^2 - B^2 dy^2 - C^2 dz^2 \qquad \ldots(1)$$

where A, B, C are metric potentials and are functions of t-alone.

Einstein modified field equations by introduction of C-field with bulk viscosity are given by

$$R_i^j - \frac{1}{2} R g_i^j = -\left[\underset{(m)}{T}{}_i^j + \underset{(c)}{T}{}_i^j\right] - \Lambda g_i^j \qquad \ldots(2)$$

(in the geometrized unit 8πG=1, c = 1)

where

$$\underset{(m)}{T}{}_i^j = (\rho + p) v_i v^j - p\, g_i^j - \xi\theta(v_i v^j - g_i^j) \qquad \ldots(3)$$

where ξ the coefficient of bulk viscosity given by Landau and Lifshitz (1963) and

$$\underset{(c)}{T}{}_i^j = -f\left(C_i C^j - \frac{1}{2} g_i^j C^\alpha C_\alpha\right) \qquad \ldots(4)$$

given by Hoyle and Narlikar (1964) with $f > 0$, a coupling constant between matter and creation field and $C_i = \frac{dC}{dx^i}$. We assume the coordinates to be comoving so that

$$v^1 = 0 = v^2 = v^3, v^4 = 1, v_4 = 1$$

We interpret ρ and p as the energy density and isotropic pressure. It is, however, possible to generalize the form of T_i^j to include contributions from bulk viscosity, without violating the symmetries in Bianchi Type I space-time.

The non-vanishing components of $\underset{(m)}{T}{}_i^j$ are given by

$$\underset{(m)}{T}{}_1^1 = -p + \xi\theta = \underset{(m)}{T}{}_2^2 = \underset{(m)}{T}{}_3^3 \qquad \ldots(5)$$

$$\underset{(m)}{T}{}_4^4 = \rho \qquad \ldots(6)$$

The non-vanishing components of $\underset{(c)}{T}{}_i^j$ are given by

$$\underset{(c)}{T}{}_1^1 = \frac{1}{2} f \dot{C}^2 = \underset{(c)}{T}{}_2^2 = \underset{(c)}{T}{}_3^3 \qquad \ldots(7)$$

$$\underset{(c)}{T}{}_4^4 = -\frac{1}{2} f \dot{C}^2 \qquad \ldots(8)$$

The modified Einstein field equations (2) for the metric (1) lead to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = \left(-p + \xi\theta + \frac{1}{2} f\dot{C}^2\right) + \Lambda(t) \qquad \ldots(9)$$

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4C_4}{AC}=\left(-p+\xi\theta+\frac{1}{2}f\dot{C}^2\right)+\Lambda(t) \qquad \ldots(10)$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}=\left(-p+\xi\theta+\frac{1}{2}f\dot{C}^2\right)+\Lambda(t) \qquad \ldots(11)$$

$$\frac{A_4B_4}{AB}+\frac{B_4C_4}{BC}+\frac{A_4C_4}{AC}=\left(\rho-\frac{1}{2}f\dot{C}^2\right)+\Lambda(t) \qquad \ldots(12)$$

Solution of Field Equations

To get the deterministic and inflationary scenario, we assume that

$$\xi\theta = \text{constant} = k \qquad \ldots(13)$$

as considered by Zimdahl (1996) and the scale factor R for the metric (1) is given by

$$R^3 = ABC = e^{3H_0t} \qquad \ldots(14)$$

where H_0 is Hubble constant and A,B,C are metric potentials. Equations (9) and (10) lead to

$$\left(\frac{A_{44}}{A}-\frac{B_{44}}{B}\right)+\frac{C_4}{C}\left(\frac{A_4}{A}-\frac{B_4}{B}\right)=0 \qquad \ldots(15)$$

Thus, we have

$$\left(\frac{A_4}{A}-\frac{B_4}{B}\right)_4+\left(\frac{A_4}{A}-\frac{B_4}{B}\right)\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)=0 \qquad \ldots(16)$$

which on integration leads to

$$\left(\frac{A_4}{A}-\frac{B_4}{B}\right)=\frac{\ell}{ABC}=\ell e^{-3H_0t} \qquad \ldots(17)$$

where ℓ is constant of integration and using (14). Similarly equations (10) and (11) lead to

$$\left(\frac{A_4}{A} - \frac{C_4}{C}\right) = m\, e^{-3H_0 t} \quad \ldots(18)$$

where m is constant of integration and using (14) also.

Now from equation (14), we have

$$\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} = 3H_0 \quad \ldots(19)$$

Equations (17), (18) & (19) lead to

$$\frac{A_4}{A} = \left(\frac{\ell + m}{3}\right) e^{-3H_0 t} + H_0 \quad \ldots(20)$$

Similarly equations (17), (18) and (20) lead to

$$\frac{B_4}{B} = \left(\frac{m - 2\ell}{3}\right) e^{-3H_0 t} + H_0 \quad \ldots(21)$$

and

$$\frac{C_4}{C} = \left(\frac{\ell - 2m}{3}\right) e^{-3H_0 t} + H_0 \quad \ldots(22)$$

Thus, we have

$$A = e^{H_0 t} \exp\left[-\left(\frac{\ell + m}{9H_0}\right) e^{-3H_0 t}\right] \quad \ldots(23)$$

$$B = e^{H_0 t} \exp\left[-\left(\frac{m - 2\ell}{9H_0}\right) e^{-3H_0} t\right] \quad \ldots(24)$$

$$C = e^{H_0 t} \exp\left[-\left(\frac{\ell + 2m}{9H_0}\right) e^{-3H_0 t}\right] \quad \ldots(25)$$

Therefore the metric (1) leads to

$$ds^2 = dt^2 - e^{2H_0 t} \exp\left[2\left\{-\left(\frac{\ell + m}{9H_0}\right)e^{-3H_0 t}\right\}\right]dx^2$$

$$+ e^{2H_0 t} \exp\left[2\left\{-\frac{(m - 2\ell}{9H_0} e^{-3H_0} t\right\}\right]dy^2$$

$$+ e^{2H_0 t} \exp\left[2\left\{-\frac{(\ell - 2m)}{9H_0}\right\}e^{-3H_0 t}\right]dz^2 \qquad \ldots(26)$$

Conservation Equation

The conservation equation

$$\left(\underset{(m}{T}{}_i^j + \underset{(c)}{T}{}_i^j + \Lambda g_i^j\right)_{;j} = 0 \qquad \ldots(27)$$

leads to

$$(\dot{\rho} - f\,\dot{C}\ddot{C}) + (\rho - \zeta\theta - f\dot{C}^2)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) + \dot{\Lambda} = 0 \qquad \ldots(28)$$

where Λ is vacuum energy density. The conservation equation (28) shows that a decaying vacuum energy term Λ transfers energy continuously to the material component. The effective time dependent cosmological term is regarded as second fluid component with energy density

$$\rho_{vac} = \Lambda(t) \qquad \ldots(29)$$

where we have assumed p = 0 following Hoyle and Narlikar (1964). We also assume that vacuum energy density Λ is given by

$$\Lambda = \frac{b}{R^6} \qquad \ldots(30)$$

where the scale factor R is given by equation (14).

Now equation (12) to determine matter density ρ leads to

$$\rho=\frac{A_4B_4}{AB}+\frac{B_4C_4}{BC}+\frac{A_4C_4}{AC}+\frac{1}{2}f-\Lambda \qquad \ldots(31)$$

where $\dot{C}=1$ is assumed as considered by Hoyle and Narlikar (1964). Equation (31) leads to

$$\rho=3H_0^2+\frac{1}{2}f-4b\,e^{-6H_0t} \qquad \ldots(32)$$

where

$$\ell^2+m^2-\ell m=9b \qquad \ldots(33)$$

and

$$\Lambda=\frac{b}{R^6}=b\,e^{-6H_0t} \qquad \ldots(34)$$

Thus, we have

$$\dot{\rho}=24\,H_0 b\,e^{-6H_0t} \qquad \ldots(35)$$

To Determine Creation Field C

Conservation equation (28) leads to

$$(\dot{\rho}-f\,\dot{C}\,\ddot{C})+(\rho-k-f\dot{C}^2)(3H_0)-6bH_0e^{-6H_0t} \qquad \ldots(36)$$

where

$$ABC=e^{3H_0t} \qquad \ldots(37)$$

$$=\Lambda=b\,e^{-6H_0t} \qquad \ldots(38)$$

Equation (36) leads to

$$\frac{d}{dt}(\dot{C}^2)+6H_0\dot{C}^2=\frac{2\dot{\rho}}{f}+\frac{6H_0}{f}(\rho-k)-\frac{12bH_0e^{-6H_0t}}{f} \qquad \ldots(39)$$

which leads to

$$\frac{d}{dt}(\dot{C}^2) + 6H_0\dot{C}^2 = \frac{48H_0 b e^{-6H_0 t}}{f} + \frac{6H_0}{f}\left(3H_0 - 4b\, e^{-6H_0 t} + \frac{1}{2}f\right)$$

$$-\frac{12bH_0 e^{-6H_0 t}}{f} \qquad \ldots(40)$$

Thus, we have

$$\frac{d}{dt}(\dot{C}^2) + 6H_0\dot{C}^2 = \frac{12bH_0}{f}e^{-6H_0 t} + \frac{3H_0}{f}(6H_0 + f) \qquad \ldots(41)$$

which leads to

$$\dot{C}^2 = \frac{6H_0 + f}{2f} \qquad \ldots(42)$$

where $\ell^2 + m^2 - \ell m = 0$ i.e. $b = 0$.

Equation (42) leads to

$$C \,\alpha\, t \qquad \ldots(43)$$

where

$$\frac{6H_0 + f}{2f} = 1 \qquad \ldots(44)$$

Thus creation field (C) increases with time which matches with the result as obtained by Hoyle and Narlikar (1964).

Particle Horizon

$$\int_{-\infty}^{\infty} \frac{dt}{R^3} \qquad \ldots(45)$$

$$= \int_0^{\infty} e^{-3H_0 t}\, dt$$

$$= \frac{1}{3H_0} = \text{finite} \qquad \ldots(46)$$

Thus Particle Horizon exists i.e. the observers are in Communicable region.

State Finder Parameters {γ,s}

The state finder parameters effectively differentiate between forms of dark energy and provide simple diagnosis whether a particular model fits into the basic observational data. Following Sahni et al. (2003), the state finder diagnostic pair {γ,s} is given by

$$\gamma = 1 + \frac{3\dot{H}}{H^2} + \frac{\ddot{H}}{H^3} = 1 \quad \text{as } H = \frac{\dot{R}}{R} = H_0 \,(\text{constant}) \qquad \ldots(47)$$

$$s = \frac{\gamma - 1}{3\left(q - \frac{1}{2}\right)} = 0 \qquad \ldots(48)$$

which agrees with ΛCDM model.

Physical and Geometrical Aspects

The homogeneous matter density (ρ), the scale factor (R), the expansion (θ), the coefficient of bulk viscosity (ξ), the vacuum energy density for the model (26), the deceleration parameter (q) are given by

$$\rho = 3H_0^2 - 4b\, e^{-6H_0 t} + \frac{1}{2} f$$

$$R = e^{H_0 t}$$

$$\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} = 3H_0$$

$$\xi = \frac{k}{3H_0}$$

$$\Lambda = b\, e^{-6H_0 t}$$

$$q = -\frac{\ddot{R}/R}{\dot{R}^2/R^2} = -1$$

The matter density (ρ) > 0 leads to

$$6H_0^2 + f > 8b\, e^{-6H_0 t}$$

For large values of t, ρ > 0 leads to

$$6H_0^2 + f > 0$$

For large values of t, the matter density is constant in presence of C-field. Thus in Creation field cosmology, there is neither beginning nor end of universe.

Conclusion

The Creation field increases with time which matches with the result as obtained by Hoyle and Narlikar (1964). The vacuum energy density (Λ) decreases with time and it tends to zero for large values of t. The scale factor increases exponentially representing inflationary scenario. There is no real singularity initially in the model (26). The deceleration parameters (q) < 0 which shows that the model represents accelerating universe. Also since the deceleration parameter q = −1 which shows that the model leads to de-Sitter universe. In the model, particle horizon exists i.e. the observations are in communicable region. The model represents constant expansion but the universe is accelerating. The coefficient of bulk viscosity is inversely proportion to expansion.

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